INTERFERENCE TEST ANALYSIS METHOD IN THE PRESENCE OF A LINEAR BOUNDARY

Ryuichi ITOI and Michihiro FUKUDA

Geothermal Research Center, Kyushu University, Kasuga, JAPAN 816

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ABSTRACT

A method has been developed for analyzing interference test data in the presence of a hydrologic boundary. Kalman filtering algorithm has been applied as a parameter estimating method to the line source solution that expresses pressure response caused by fluid production and/or injection at active wells. This solution is modified to express the effects of the presence of the boundary on pressure response. The analysis method comprises two steps. First, an infinite reservoir model will be used for analyses, and performances of the estimated values of parameters, transmissivity and storativity, with time would provide information on the presence of the boundary and its type:an impermeable boundary or a constant pressure boundary. On the basis of this result, an appropriate model will be assigned to reanalyze the data. Synthetic data was analyzed to validate the use of the method, and the estimated values of the parameters are to be satisfactory.

1. INTRODUCTION

Interference tests are widely conducted at many geothermal fields in order to estimate reservoir parameters such as transmissivity and storativity by analyzing the test data(Leaver, *et al.,* 1988: Gotoh, 1990). The tests also give information on the presence of hydrologic boundaries in the field. The hydrologic boundaries are roughly classified into two types: one is impermeable boundary and the other is constant pressure boundary. The impermeable boundary acts as a no-flow boundary and it would limit the aerial extension of the reservoir. The constant pressure boundary may play a recharge fault. The presence of these hydrologic boundaries would give vital influences on reservoir performance upon exploitation of the field. Therefore, the location of these boundaries should be detected during exploration stage through production or operation stage. This can be realized by interpreting geological data in combination with well testing data, which would lead to more reliable results. These boundaries are commonly modeled as vertical, fully penetrating linear boundaries. The effects of the presence of the boundary on pressure response in reservoirs can be expressed by locating an image well symmetrically opposite side of the reservoir with respect to the linear boundary(Earlougher, 1977).

Graphical methods have been reported to locate the position of a boundary(Sageev, *et al.* ,1985:Duong, 1990), but these methods only estimate the distance between a well and a boundary while the strike of the boundary has been given as initial conditions for the analysis. In order to uniquely determine the position of the boundary, three and more observation wells for pressure measurement are required (McEdwards and Benson, 1981:Sageev, *et al.* , 1985). A computer assisted method is a powerful tool for analyzing interference test data with multiple observation wells as well as under actual conditions such as variable flow rate history at active wells. McEdwards and Benson(1981) developed a computer code to locate a boundary as well as to estimate transmissivity and storativity by applying a non-linear least square regressions method. In this program, the location of a boundary has been expressed by using two parameters:azimuthal angle to the vertical line to the boundary and the distance to the boundary from a given origin. Itoi *et al.*(1990, 1992) developed methods to analyze interference test data using the line source solution to which Kalman filtering has been applied. Parameter estimating method using Kalman filtering can be defined as an on-line analysis method as it provides the best estimates of unknown parameters when the new measurement data become available at each time step(for example, Brown, 1983).Thus, this method has an advantage over a conventional least square regressions method in such a manner that a forced match to unrealistic values for paramters can be avoided when a wrong model is assigned for analysis and/or data involves serious measurement error. In this paper, we have developed a method to analyze interference test data for estimating reservoir parameters, transmissivity and storativity, and the location and the strike of the hydrologic boundary simultaneously. Synthetic data has been analyzed to verify the use of **this** method.

2. PRESSURE RESPONSE AT OBSERVATION WELLS

Pressure response at an observation well as a result of production and/or injection of fluid at an active well can be expressed by the line source solution on the basis of isotropic, homogeneous, and infinite porous type of reservoir. The effects of the presence of a single linear boundary on the pressure response can be explained by locating an image well symmetrically opposite side across the boundary. Thus, the reservoir can be handled to be of infinite type with a pair of active and image wells. Here, we handle the case of multiple observation wells and a single active well. The pressure response at the n-th observation well under a constant flow rate at an active well in the presence of the linear boundary can be expressed

$$\Delta p_{n,k} = -\frac{M}{4\pi T}\left\{\mathrm{Ei}\,(-u_{n1,k}) \pm \mathrm{Ei}\,(-u_{I1,k})\right\} \quad (1)$$

where $\Delta p_{n,k}$ is the pressure change at time t_k ($=p_o - p_{n,k}$, Pa), p_o is the initial pressure(Pa), $p_{n,k}$ is the measured pressure at the n-th observation well(Pa), M is the flow rate(m^3/s), T is the transmissivity($=kh/\mu$, $m^3/Pa \cdot s$), k is the permeability(m^2), h is the thickness of the reservoir(m), μ is the viscosity of fluid(Pa•s), Ei($-u$) is the exponential function of u. The positive sign in the bracket is for the impermeable boundary and the negative one the constant pressure boundary. Variables $u_{n1,k}$ and $u_{I1,k}$ are defined as:

$$u_{n1,k} = \frac{Sr_n^2}{4Tt_k} \quad , \quad u_{I1,k} = \frac{Sr_{nI}^2}{4Tt_k}$$

where S is the storativity($=\phi ch$, m/Pa), ϕ is the porosity(-), c is the compressibility(1/Pa). r_n is the distance between the n-th observation well and the active well, r_{nI} is that between the n-th observation well and the image well(m), t_k is the time when the pressure values are measured at the observation wells. Subscript k denotes time step.

When the fluid production is stopped at time t_s, the subsequent pressure responses at the observation wells are:

$$\Delta p_{n,k} = -\frac{M}{4\pi T}\left[\left\{\mathrm{Ei}\,(-u_{n1,k}) \pm \mathrm{Ei}\,(-u_{I1,k})\right\} - \left\{\mathrm{Ei}\,(-u_{n2,k}) \pm \mathrm{Ei}\,(-u_{I2,k})\right\}\right]$$

where $u_{n2,k}$ and $u_{I2,k}$ are given as

$$u_{n2,k} = \frac{Sr_n^2}{4T(t_k - t_s)} \quad , \quad u_{I2,k} = \frac{Sr_{nI}^2}{4T(t_k - t_s)}$$

The location of the linear boundary can be expressed by the angle, α, and the perpendicular distance, d, from the origin of the Cartesian coordinate used to define the location of wells to the boundary. The angle is measured clockwise from the y axis as shown in Fig.1 The distances between the observation and the active wells are expressed using coordinate on the second Cartesian coordinate whose y axis is set to be parallel to the linear boundary. Hence, the distance between the n-th observation well and the image well is:

$$r_{nI} = \sqrt{(x_n' - x_I')^2 + (y_n' - y_I')^2} \quad (4)$$

where x_n', y_n', x_I' and y_I' are expressed in the matrix form:

$$\begin{bmatrix} x_n' \\ y_n' \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} x_n \\ y_n \end{bmatrix}$$

$$\begin{bmatrix} x_I' \\ y_I' \end{bmatrix} = \begin{bmatrix} 2d - x_A' \\ y_A' \end{bmatrix}$$

where x,y is the coordinates of wells, subscripts A and I represent the active well and the image well, respectively, and superscript ' indicates the second Cartesian coordinate. Description of locating the boundary in detail is found in the paper by McEdwards and Benson(1981).

Therefore, Eqs.(1) and (3) represent the relation between the measured pressures at observation wells and the reservoir parameters, T and S, and the location of the linear boundary implicitly expressed by two parameters: α and d. These equations correspond to the observation equation to introduce Kalman filtering for parameter estimating.

3. PARAMETER ESTIMATING METHOD USING KALMAN FILTERING

We assume that a reservoir system can be described by a set of equations:the state equation and the observation equation. The state equation expresses transient behavior of reservoir parameters to be identified in the form of discrete-time. During the interference tests, the reservoir parameters can be assumed to be time invariant, then the state equation can be expressed as follows:

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{w}_k \quad (5)$$

where $\mathbf{x}_k$ is the state vector at time t_k.

The measurement equation is linearly correlated with the state vector and expressed as

$$\mathbf{y}_k = H_k \mathbf{x}_k + \mathbf{v}_k \quad (6)$$

where $\mathbf{y}_k$ is the measurement vector at time t_k, H_k is the observation matrix , and $\mathbf{v}_k$ and $\mathbf{w}_k$ are the measurement and the state noise vectors assumed to be a white with known covariance V_k and W_k.

On the basis of the system expressed by Eqs.(5) and (6), the algorithm of Kalman filtering provides the best estimates of state vector at time t_k when a new measurement becomes available. By denoting the best estimate at t_k as $\hat{\mathbf{x}}_{k/k}$ where the "hat" indicates estimate, this will be given by updating the previous estimate, $\hat{\mathbf{x}}_{k/k-1}$, as follows:

$$\hat{\mathbf{x}}_{k/k} = \hat{\mathbf{x}}_{k/k-1} + K_k(\mathbf{y}_k - H_k \hat{\mathbf{x}}_{k/k-1}) \quad (7)$$

where K_k is the Kalman gain matrix and is given as

$$K_k = P_{k/k-1} H_k^{\mathrm{T}} \left[H_k P_{k/k-1} H_k^{\mathrm{T}} + V_k \right]^{-1} \quad (8)$$

Superscripts -1 and T denote inversion and transposition, respectively. P is the error covariance matrix associated with the present estimates and expressed as

$$P_{k/k} = P_{k/k-1} - K_k H_k P_{k/k-1} \quad (9)$$

For subsequent parameter estimation at the next time step, the state vector is predicted as

$$\hat{\mathbf{x}}_{k+1/k} = \hat{\mathbf{x}}_{k/k} \quad (10)$$

and the error covariance matrix is given as

$$P_{k+1/k} = P_{k/k} + W_k \quad (11)$$

Kalman filtering can be applied to the linear system: the measurement values are linearly correlated with the parameters. However, the observation equations, Eqs.(1) and (3), are apparently nonlinear with respect to the parameters to be identified. Thus, the measurement equation should be linealized as follows. Further formulation is developed on the basis of Eq.(1) for the simplicity. Thus, Eq.(1) can be written in a vector form as

$$\mathbf{y}_k = h(\mathbf{x}_k) + \mathbf{v}_k \quad (12)$$

$$\mathbf{y}_k = \left[\Delta p_{1,k}, \Delta p_{2,k}, \ , \Delta p_{n,k}, \ , \Delta p_{N,k} \right]^{\mathrm{T}}$$

$$\mathbf{x}_k = \left[T, S, a, d \right]^{\mathrm{T}}$$

where subscript N indicates the total number of observation well. $h(\mathbf{x}_k)$ is a nonlinear function of $\mathbf{x}$. This function can be approximated with Tavlor's series expansion about the prior estimate, $\hat{\mathbf{x}}_{k/k-1}$, and is retainedonly the first-&der term. The result is:

$$Y_k = H_k \mathbf{x}_k + \mathbf{v}_k \quad (13)$$

$$Y_k = \mathbf{y}_k - h\left(\hat{\mathbf{x}}_{k/k-1}\right) + H_k \hat{\mathbf{x}}_{k/k-1} \quad (14)$$

$$H_k \left| \begin{matrix} \frac{\partial h_1}{\partial T} & \frac{\partial h_1}{\partial S} & \frac{\partial h_1}{\partial \alpha} & \frac{\partial h_1}{\partial d} \\ \frac{\partial h_2}{\partial T} & \frac{\partial h_2}{\partial S} & \frac{\partial h_2}{\partial \alpha} & \frac{\partial h_2}{\partial d} \\ \frac{\partial h_3}{\partial T} & \frac{\partial h_3}{\partial S} & \frac{\partial h_3}{\partial \alpha} & \frac{\partial h_3}{\partial d} \end{matrix} \right|_{:/k-1} \quad (15)$$

for N=3

The measurement vector, Y_k, comprises three terms including $\mathbf{y}_k$ whose components are equal to $\Delta p_{n,k}$ $(n=1,2,\cdot,\cdot N)$. Other two terms in Eq.(14) are known at time t_k since they are evaluated with predicts of $\mathbf{x}$ at previous time, t_{k-1}. Observation matrix expressed by Eq.(15) should be evaluated at each time step. Therefore, Kalman filtering can be formed using the linealized equation, Eq.(13), instead of Eq.(12) as an observation equation. This technique is called as an extended Kalman filtering (Brown, 1983).

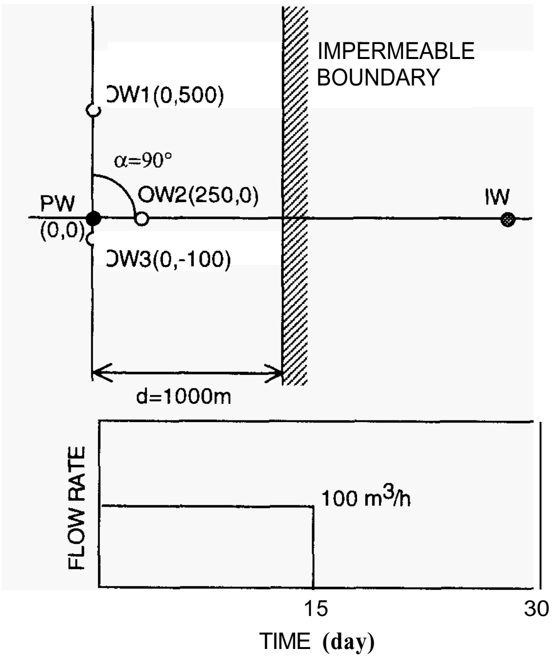

Fig.1 Location of wells and an impermeable boundary together with flow rate history at production well(PW) to generate synthetic data. Three observation wells(OW1,OW2,OW3) are indicated by open circles, and an image well by dotted circle.

4. EXAMPLE OF ANALYZING SYNTHETIC DATA

The synthetic data of interference test was generated for a reservoir of porous type with an arrangement of wells and an impermeable boundary as shown in Fig.1. Coordinate values of wells are indicated in bracket (unit:m) right after the well name. Pressure responses at three observation wells(OW1, OW2, and OW3) upon producing fluid at the production well(PW) under constant flow rate, 100 m^3/h, are calculated using Eq.(1). The fluid production has been continued for fifteen days, then it is stopped completely, but the pressures at three observation wells are further calculated for subsequent 15 days using Eq.(3); 30 days in total with an interval of 4 hours. The values of reservoir parameters are given as T=1x10^{-7} m^3/Pa•s, S=1x10^{-7} m/Pa, α=90 degree, and d=1000 m to compute the synthetic data.

Analysis procedure with the method above consists of two steps. First, data are analyzed by assigning an infinite reservoir model in order to detect the presence of boundaries. If pressure responses at observation wells are affected by the presence of a boundary, estimated values for parameters(T and S) will be time variant since the method yields the best estimates to a given model at each moment of pressure measurements. These performances of estimated values allow us to judge a type of the boundary. Then we can select an appropriate reservoir model for reanalyzing the data. We have, therefore, first analyzed the synthetic data using an infinite reservoir model by giving the initial guesses for reservoir parameters as T=1.0x10^{-8} m^3/Pa•s and S=1.0x10^{-8} m/Pa. During the course of the analysis. the initial values of diagonal and nondiagonal components of the matrix P are given to be 1.0x10^{-9} and 0, respectively. The matrix elements of covariance of noise vector, W_k, are set to be time invariant as 2.5x10^{-17} and 0 for diagonal and nondiagonal parts, respectively. Those of V_k are also given to be time invariant as 100 and 0 for diagonal and nondiagonal parts, respectively.

Fig.2 presents estimated values of transmissivity(T) and storativity(S) with time. Both estimates quickly reach the correct values(T=1x10^{-7} m^3/Pa•s and S=1x10^{-7} m/Pa) as fast as the sixth time step. The estimated values of T show being stable up until the time when the fluid production was stopped on the 15th day of fluid production, then they start decieasing with time. On the other hand, estimated values of S start decreasing soon after it reached to the correct value, then it increases with time after the fluid production stopped. These performances in the estimates violate the assumptions of heing time invariant, suggesting a choice of a wrong model.

Fig.3 shows the comparison hetween the synthetic pressure drawdown, Δp_k, and the predicted one, $\Delta \hat{p}_{k/k-1}$, at the observation well OW3 on the semi-log paper. The predicted pressure drawdown at time t_k is calculated using the predicted estimate, $\hat{x}_{k/k-1}$. Discrepancies between the two kinds of pressure drawdown are large during very early period, but they drastically decrease within a short period corresponding to quick approaches of T and S to the correct values in Fig.2. Then, they show good matches in the subsequent periods. This implies that the infinite reservoir model explains well the synthetic data with estimated values of T and S at each time step. The performances of cstimated values, however, violate the assumptions that T and S are time invariant As Sageev *et al.*(1985) pointed out, pressure drawdomn affected by the presence of an impermeable boundary branch off ahnve that for an infinite reservoir model. and discrepancies in the pressure drawdown between two kinds of models will expand with time. An increase in pressure drawdown with time for the impermeable boundary model can be explained by estimating smaller values of S at each time with the infinite reservoir model as shown in Fig.2 while estimated values of T remain constant. Therefore, from this performance of estimated S, we can judge that the pressure responses at observation wells are affected by the presence of the impermeable boundary.

In a simiiar way, we can detect the presence of a constant pressure boundary from analysis using the infinite reservoir model. When this kind of the boundary presents, pressure drawdown at observation wells will approach to be almost constant after a certain elapsed time(Sageev, *et al.*, 1985). Then, the pressure drawdown for the constant pressure boundary model during the subsequent period will be smaller than that for the infinite reservoir model. We may figure out that estimated values of S derived from the method above by use of the infinite reservoir model will depart off to larger values of estimates during this perind. Then, it should be noted, for example, tha: a reservoir with a small value of storativity would receive significant effects of the presence of boundary at very early stage of the test. Thus the differences in pressure response between the models will become large even in early stage of the test. tinder this kind of conditions, parameter estimating using the infinite reservoir model may fail due to a diversion in the estimated values of S .

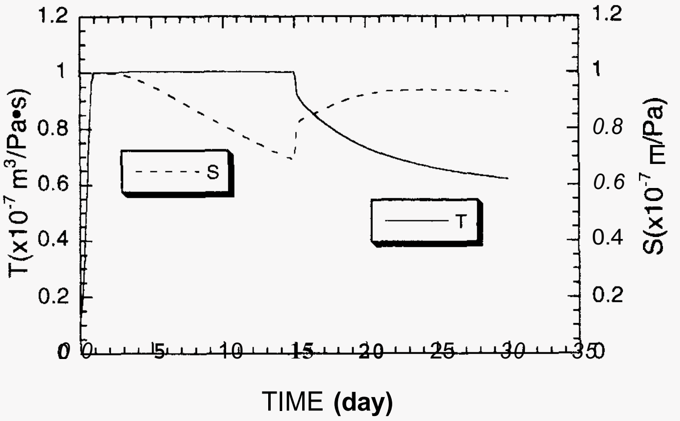

Fig.2 Performances of estimated transmissivity (T) and storativity(S) with time when synthetic data in the presence of an impermeable boundary was analyzed using an infinite reservoir model.

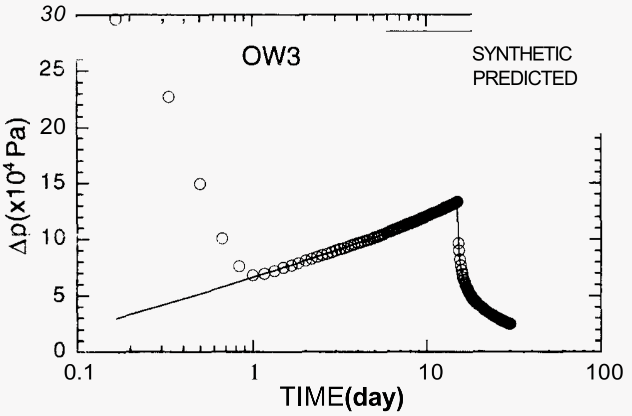

Fig.3 Comparison between synthetic pressure drawdown at OW3 with predicted ones, $\Delta \hat{p}_{k/k-1}$.

From these results, we may well choose a model in the presence of a impermeable boundary, and we reanalyze the data by giving initial guesses for four parameters;T, S, α and d. Initial values for the parameters are given as T=1.002x10^{-7} m^3/Pa•s, S=6.862x10^{-8} m/Pa, α=60 degree(1.05 rad), and d=600 m. The initial values of T and S are those of estimated at the time when the fluid production was stopped in Fig.2. Diagonal part of the matrix P associated with T, S, α and d are given as the squares of the 10% of the initial values such that 2.5x10^{-15}, 1.2x10^{-15}, 0.27 and 9x104, respectively, and nondiagonal parts are given to be 0. The matrix elements of covariance of noise vector, W_k, associated with T, S, α and d are set to be time invariant as 2.5x10^{-19} , 1.18x10^{-17}, 2.74x10^{-3}, and 9x102. respectively, and 0 for nondiagonal parts. Those of V_k are also given to be time invariant as 100 and 0 for diagonal and nondiagonal parts, respectively. Figs.4, 5, 6 show the estimated values of T, S, α and d along with open circles indicating the correct values. Rapid changes occur in the estimated values for three kinds of parameters(S, α, and d) in early period, then they gradually approach to the correct values with time, in other words with an increase in the number of pressure data. Changes in estimated values are small until the date when the fluid production stopped. During the latter half of the pressure measurement period, corresponding to pressure recovery period, all parameters show about the same values as the correct ones. This suggests that pressure measurement during pressure recovery period provide important information for estimating parameters with this method. The estimated values when the last pressure values obtained at the observation wells are T=9.995x10^{-8} m^3/Pa•s,S=9.985x10^{-8} m/Pa,α=90.01 degree, d=1000.2 m, with errors less than 0.2%.

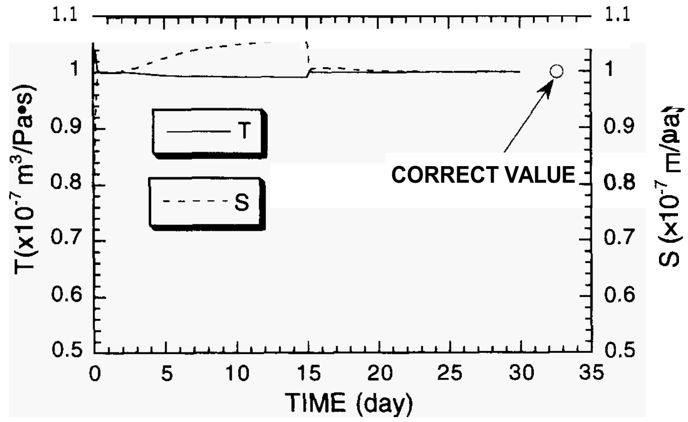

Fig.4 Performances of estimated transmissivity(T) and storativity(S) during analysis using a reservoir model in the presence of an impermeable boundary.

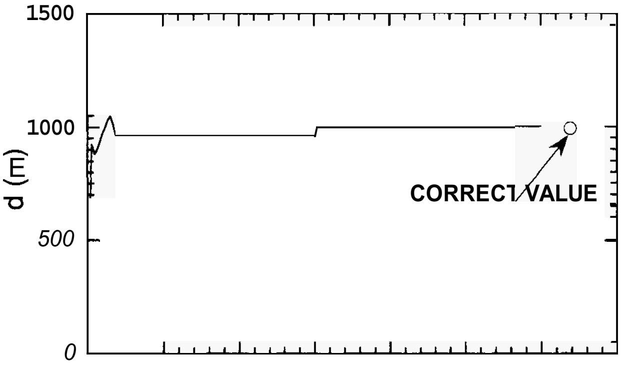

Fig.6 Performance of estimated distance(d) during analysis using a reservoir model in the presence of an impermeable boundary.

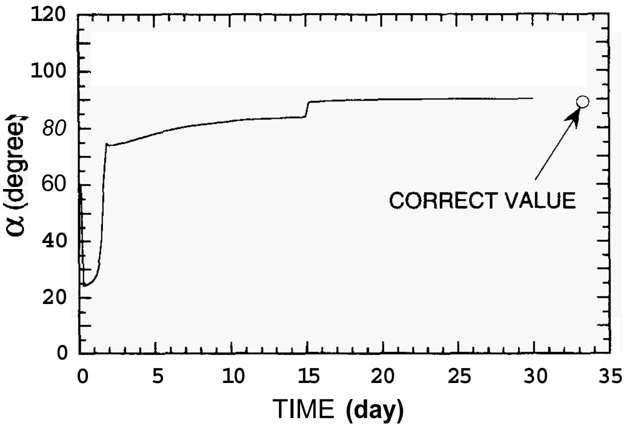

Fig.5 Performance of estimated angle(α) during analysis using a reservoir model in the presence of an impermeable boundary.

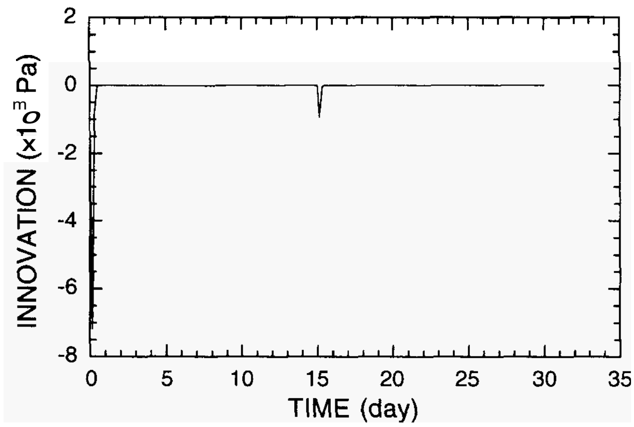

Fig.7 Change in innovation with time.

Fig.7 shows the change in innovation, defined by the difference between the synthetic pressure drawdown and the predicted one at time t_k for the well OW3. Absolute values of innovation quickly approach to almost zero and remain as such throughout the observation period except a short time when the fluid production stopped. This implies that the estimated values of parameters at each moment can successfully predict the synthetic pressure values for the next time step as well as they almost satisfy the assumptions of being time invariant as shown in Figs. **4, 5,** and ***6.*** Further studies, however, are necessary to investigate the sensitivity of choice of initial guesses on the parameter estimating performances.

5. CONCLUSIONS

1) **A** method to analyze pressure interference test data in the presence of a hydrologic boundary was presented. It uses extended Kalman filtering algorithm for parameter estimating that is applied to the line source solution modified to express the effects of the boundary.

2) In order to identify an appropriate reservoir model to be assigned for analysis, it is recommended to analyze the data first with the solution for an infinite reservoir model. As reservoir parameters to be identified are assumed to be time invariant, performances of the estimated values of parameters(transmissivity and storativity)imply the presence of boundaries and also provide information on type of the boundary. Particularly, performance of estimated storativity seems to be very sensitive to the selection of reservoir models.

3)Synthetic data in the presence of an impermeable boundary was analyzed using the method for four parameters(transmissivity, storativity, azimuthal angle, and distance to the boundary) simultaneously. Very good estimates can be attained during early period and these estimates approach to the correct values with time, in other words, as the number of the observed pressure values increases.

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