Temperature and Response Time of Magma Hydrothermal Systems, with an Example from Kilauea, Hawaii

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The time required for cooling of a rapidly emplaced magma body is inversely related to the vigor of hydrothermal circulation, as quantified by the Rayleigh number. Yet, many, if not most, silicic volcanic centers persist much longer than the thermal pulse created by invasion and cooling any single magma body. Accordingly, it seems appropriate to consider the thermal characteristics of a system continuously resupplied with magmatic heat. In this scheme, temperature evolves toward steady-state values that balance the rate of heat input and removal, the former by continuous intrusion of magma and the latter by conduction and ground-water convection. The time required to reach steady-state, which we refer to as the response time, is inversely proportional to the square-root of a "heat-flux" Rayleigh number, which differs from the Rayleigh number used to examine cooling of a rapidly emplaced magma body. Steady-state temperatures themselves exhibits the same proportionality.

We obtain simple expressions among averages of temperature, heat-input rate, hydrothermal circulation, regional ground-water flow, and physical properties of reservoir and aquifer rocks in and around a magma-driven hydrothermal-convection system. We use this framework to evaluate the geothermal field of the lower east rift zone of Kilauea Volcano, Hawaii. The analysis suggests that heat supplied by intermittent intrusion of magma is probably inadequate to maintain the hydrothermal reservoir at observed drill-hole temperatures, which exceed −300°C. More likely, heat delivered from depths greater than −4 km maintains the reservoir at those temperatures. This deeper hot-rock-and-magma system is, in turn, maintained by magma continuously intruded at a rate that is small in comparison to the long-term eruption rate of Kilauea; limits on intrusion rate imposed by the modest present-day estimates of extension across the lower east rift zone probably allow for even greater rates of magma accumulation. Low ground-surface heat flux is explained by the vigorous ground-water flow across the top of the reservoir.

1 Introduction

Temperatures and reservoir-response behavior of hydrothermal aquifers depend upon a variety of factors, including physical reservoir properties, regional geologic, tectonic, and hydrologic settings, and, most importantly, rates of heat input from magmatism. Yet, these factors are typically poorly quantified due to lack of data. Even for those shallow regions of hydrothermal reservoirs well explored by drilling, bulk permeability remains uncertain; properties of the deeper and distal regions of geothermal systems are even more poorly known.

We characterize the natural-state of a reservoir by relating average temperatures, heat fluxes and "thermal response times" through spatial averaging of transport properties across geologically and hydrologically determined length scales. In so doing, a geothermal system is divided into regions of heat input and loss and into discharging and recharging aquifers, each able to convect heat by regional horizontal flow and by thermally buoyant flow. Fluid mass, momentum, and heat are conserved among these regions with magmatic heat supply and with rates of accumulation and loss within the aquifers. The method employs "lumped-parameter" or "macroscopic-balance" approximations and leads to relatively simple expressions for average temperatures in a system that conducts heat in all three directions and convects heat upward by buoyant hydrothermal circulation and horizontally in one direction by a regional ground-water flow.

2 Overview

Cooling histories of intrusions were first analyzed by the theory of heat conduction (summarized by Jaeger, 1964, 1968). Results were used to estimate cooling times and sizes of thermal aureoles and to interpret igneous textures and facies of contact metamorphism. Where heat transfer is dominated by conduction, time is typically nondimensionalized as $\tau = \kappa t/L^2$, where t is time, κ is thermal diffusivity, and L is a length, usually a half-thickness or radius. An empiricism (Delaney, 1987), consistent with results derived below, is that cooling effects of wall rocks are felt at the intrusion center when $\tau \sim 0.1$, as much heat has been lost to country rocks as is retained by the intrusion when $\tau \sim 1$, and cooling is largely finished when $\tau \sim 10$. Diffusivity does not vary by more than a factor of two among common rock types, and so one needs only know L to estimate the duration of cooling. Because dimensionless time τ is sensitive to L^{-2}, minor uncertainties in the characteristic length introduce substantial errors in estimates of the duration of cooling. Also, existing mathematical solutions are valid only if the time required for magma emplacement is small in comparison to any subsequent time of interest.

More recent studies have focused upon hydrothermal cooling (Cathles, 1977; Norton and Knight, 1977; Parmentier and Spooner, 1978) and the accompanying solute transfer (Norton and Taylor, 1979; Cathles, 1981; Parmentier, 1981) driven by buoyancy of hot meteoric water. These studies were prompted by the growing appreciation of the role of hot-spring systems in ore deposition (*e.g.,* White, 1955, 1974) and the necessity for meteoric water to account for hydrogen- and oxygen-isotope characteristics of some igneous rocks (*e.g.,* Taylor and Forester, 1979). In contrast to conduction, where heat diffuses in all directions from the intrusion, hydrothermal circulation carries heat upward. The buoyant plume is replenished by a deeper, lateral flow from the surrounding aquifer. This cool inward flow of water maintains substantially lower temperatures near the sides of the intrusion than would be calculated from heat-conduction theory, just as buoyant upflow supports relatively high temperatures above the intrusion.

The thermal structure of hydrothermal systems is particularly sensitive to the permeability structure of the intrusive and country rocks. A single parameter that incorporates permeability and related properties is the Rayleigh number *Ra,* which is defined differently by different workers and for different system configurations but always measures the importance of cooling by buoyant convection relative to that by conduction. Estimates of Ra for modern and fossil systems are rare (although see Norton and Taylor, 1979).

Many geothermal systems are situated in hilly or mountainous terrain. Horizontal gradients in hydraulic head, therefore, can force convective removal of heat (Forster and Smith, 1989). The parameter that measures the relative importance of this type of convection relative to heat conduction is the Peclet number *Pe.* Although it is likely that ground-water flow across hydrothermal reservoirs is important in some instances, estimates of *Pe* are rare.

Petrologic studies suggest that many of the igneous systems that feed volcanic fields are characterized by addition over time of multiple batches of magma, as well as multiple episodes of removal by eruption. Moreover, many volcanic fields remain active for periods that far exceed the cooling time of a single magma intrusion. Yet the thermal behavior of such systems are often modeled under the assumption of rapid intrusion followed by magmatic quiescence (*e.g.,* Smith and Shaw, 1978). We suggest that many systems, therefore, are better characterized by a continuous resupply of magmatic heat. In such a scheme, the magma-hydrothermal system develops a steady-state temperature that reflects an interplay among its size, rate of magmatic heat input, and hydrology. If hydrothermal circulation is a dominant heat-transfer mechanism, the steady-state temperature and the time required to achieve it depend upon a "heat-flux" Rayleigh number that differs from the one employed when treating cooling of a single suddenly emplaced intrusion. We derive both types of Rayleigh numbers below.

In summary, magma-hydrothermal systems are generally comprised of numerous intrusions, irregular in shape, and not conforming to the idealized geometries of dikes, sills, stocks, and so forth. Magmatism often involves a long history of impulsive and varying rates of magma resupply. Heat in such systems can be carried with ground water flowing in response both to regional hydrologic conditions and to buoyant circulation induced by elevated the temperatures. These factors are commonly poorly constrained by data. We describe a method to develop simple thermal models prior to acquisition of data that would justify a sophisticated numerical analysis. The central simplification is that the shape of the magma/hydrothermal system is given by its height, breadth, and width; the distribution of igneous and hydrothermal components within the system need not be specified.

3 Heat Balance

We idealize a magma-hydrothermal system that cools by conduction and convection and maintains itself by replenishment of heat at a rate Q. Heat accumulates with time t, conducts in the X, Y, and Z directions, and convects with Darcian velocities u in the horizontal X direction and v in the vertical Z direction:

$$(\rho C)_b \frac{\partial \Theta}{\partial t} = k_b \nabla^2 \Theta - (\rho C)_f \left[u\frac{\partial \Theta}{\partial X} + v\frac{\partial \Theta}{\partial Z} \right] + Q.$$

Here, Θ is temperature, $(\rho C)_f$ and $(\rho C)_b$ are volumetric heat capacities of the pore water and bulk rock-and-water composite, respectively, and k_b is bulk thermal conductivity. Each term has units of energy rate per unit volume.

Considerable simplification results if spatial derivatives are replaced by differences. That is, $\partial/\partial X \simeq -L_x^{-1}$ and $\partial/\partial Z \simeq -L_z^{-1}$ and $\nabla^2 \simeq -\left(L_x^{-2}+L_y^{-2}+L_z^{-2}\right)$, where L_x and L_y are the half-thickness and half-width of the system and L_z is its height. After some rearrangement and using the thermal diffusivity $\kappa = k_b/(\rho C)_b$, we have:

$$\frac{d\Theta}{dt} = -\kappa \frac{\Theta}{L^2} - \frac{(\rho C)_f}{(\rho C)_b}\left[u\frac{\Theta}{L_x} + v\frac{\Theta}{L_z} \right] + \frac{Q_{total}}{(\rho C)_b V_{total}}$$

where

$$L = \left(L_x^{-2}+L_y^{-2}+L_z^{-2}\right)^{-1/2}$$

and where Q_{total} and V_{total} are the total heat supply rate and volume of the system, respectively. Temperature Θ is now an average value; for those comprised of multiple intrusions, Θ is temperature averaged across the intrusive and host rocks. Parameter L is an effective thermal diffusion distance that takes into account conduction in each of the three directions. The form of L confirms that the smallest dimension contributes most to the rate of conductive cooling; a narrow intrusion cools predominantly from its sides, for instance, and an equant intrusion cools in all directions.

In the absence of ground-water flow, $u = v = 0$, the energy equation has two important solutions. For consistency with the remainder of this paper, we identify the temperature of the intruded region as Θ_1 and that of the far field, which remains constant, as Θ_0. For the instance where the intrusion of magma is sudden and there is no subsequent heat addition ($Q = 0$), the solution is:

$$\Theta_1 - \Theta_0 = \hat{\Theta}\, e^{-\kappa t/L^2}, \qquad \text{with} \qquad \hat{\Theta} = \Theta_i - \Theta_0,$$

a simple expression for cooling as a function of the size of the system and the initial, imposed temperature rise 6. Temperature decays exponentially from its initial value Θ_i to its final value Θ_0.

A more useful solution, we feel, describes thermal evolution after magma begins to feed a developing system. For the case where the rate of addition of magmatic heat is constant, the solution is

$$\Theta_1 - \Theta_0 = \hat{\Theta}_q \left(1 - e^{-\kappa t/L^2}\right), \qquad \text{with} \qquad \hat{\Theta}_q = \frac{Q_{total} L^2}{V_{total} k_b} - \Theta_0.$$

In this instance, the temperature rises exponentially from an initial value of Θ_0 by an amount $\hat{\Theta}_q$. The parameter $\hat{\Theta}_q$ is the steady-state temperature rise of a conduction-dominated system receiving heat Q_{total} within the volume $V_{total} \simeq 2L_x 2L_y L_z$.

The first solution above, for a rapidly emplaced igneous system that cools quiescently, allows one to estimate the duration of what has been called the "thermal perturbation." The second solution, for a continuously active intrusion, allows one to estimate both the temperature and the time required to attain that temperature. We now treat instances where heat transfer is affected by flow of ground water, as well as conduction.

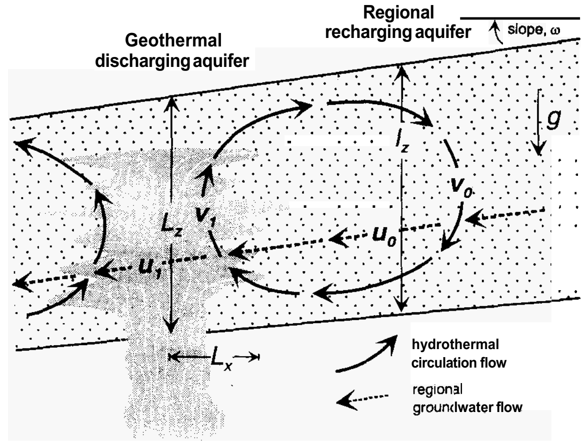

Figure 1: Definition sketch of geometric reservoir parameters and idealized flow paths for a geothermal system situated in a sloping ground-water aquifer. Total flow path would be the sum of those arising from the regional ground-water flow and the buoyant hydrothermal circulation.

4 Hydrologic Balance

Heated water imposes a buoyant force in relation to cooler water nearby. The proportionality between water density and temperature is the volumetric thermal expansion coefficient a. Treating the coefficient as a constant and referencing variable densities and temperatures of the hot water, ρ_1 and Θ_1, to constant ambient conditions, ρ_0 and Θ_0, respectively, we have

$$\rho_1 = \rho_0 \left[1 - \alpha(\Theta_1 - \Theta_0)\right].$$

If the temperature difference is $\sim$100°C, then $\alpha(\Theta_1 - \Theta_0) \sim 10^{-1}$ and the vertical pressure gradient is -1 MPa/km; for steam or supercritical-water, $\alpha(\Theta_1 - \Theta_0) \sim 10^0$ and the pressure gradient can, in principle, approach -10 MPa/km.

For simplicity, we treat an aquifer that can have properties different from those in the surroundings. We examine a configuration (Figure 1) where hydraulic head is generated by a uniform slope ω so that regional ground-water flow sweeps across the geothermal reservoir. Many geothermal fields are in low, structural grabens or within high volcanic constructs, and one can also develop equations that have regional ground-water flows directed into or away from these features, respectively.

There are two components of a hydrothermal circulation cell, a downward flow of cold water in the far-field, or regional, aquifer and an upward flow of hot water in the near-field geothermal aquifer. Concomitantly, there can be a horizontal regional flow of water across the geothermal system. The motion equations for these flows have the form

$$u = -\frac{K}{\mu}\frac{\partial P}{\partial X} \simeq -\frac{K}{P} pg \sin w,$$
$$v = -\frac{K}{\mu}\left(\frac{\partial P}{\partial Z} - \rho g\right) \simeq -\frac{K}{\mu}\left(\frac{\Delta P}{H} - \rho g\right)$$

Darcian velocities are sensitive to pressure gradients $\partial P/\partial X$ and $\partial P/\partial Z$, permeabilities K, and viscosities μ. The recharging, or regional, flows have velocities u_0 and v_0; the corresponding temperature Θ_0 and water density ρ_0 vary only slightly and are treated as constants. The hot, discharging, or geothermal, flows have velocities u_1 and v_1, average temperature Θ_1, and density ρ_1. The gravitational body force $\rho_0 g$ acting on the cool water of the recharging aquifer exceeds that of the discharging geothermal aquifer $\rho_1 g$ by $\alpha(\Theta_1 - \Theta_0)$. Pressure gradients are approximated by their finite differences across vertical length scales, the aquifer depths $H_0 \simeq l_z$, and $H_1 \simeq L_z$, and by the aquifer slope ω.

Pressures must be balanced and mass fluxes conserved between the recharging and discharging flows. Although a variety of situations can be envisioned, they all generate similar results. Here, we assume that the mass flux of water flowing across the discharging, geothermal aquifer is the same as that flow across the recharging aquifer, that the mass flux ascending buoyantly through the geothermal reservoir is the same as that descending in the recharging aquifer, and that the pressure differences and depths of both aquifers are the same:

$$A_0^u \rho_0 u_0 = A_1^u \rho_1 u_1,$$
$$A_0^v \rho_0 v_0 = A_1^v \rho_1 v_1,$$
$$\Delta P_0 / l_z = \Delta P_l / L_z.$$

The parameters A_0^u, A_1^u, A_0^v, and A_1^v are areas through which the water leaves the recharging aquifer and gains access to the geothermal aquifer. These parameters are important, for instance, when the recharge is from or to a fault zone or a graben.

After combining the above equations and neglecting small terms, the velocities of the hot discharging water can be written

$$u_1 = -\frac{K_1}{p_0}\frac{2\rho_0 g \sin\omega}{\mu_1/\mu_0 + K_0 A_0^u / K_1 A_1^u},$$
$$v_1 = -\frac{K_1}{p_0}\frac{\rho_0 g \alpha(\Theta_1 - \Theta_0)}{\mu_1/\mu_0 + K_0 A_0^v / K_1 A_1^v}.$$

Velocities are sensitive to properties of both the regional and geothermal aquifers. For instance, if the geothermal-aquifer permeability is much less than that of the regional **aquifer,** $K_0/K_1 \gg 1$, then there is a relatively abundant supply of recharging water, and the discharging flow of the geothermal aquifer limits the efficiency of convective cooling. Similarly, the viscosity of cool recharging water is generally greater than that of the hot discharging water and therefore limits convective cooling.

Although, in general, the hydraulics of a geothermal reservoir are strongly affected by its surroundings, it is useful to examine the case where the properties of the regional recharging aquifer and geothermal discharging aquifers are the same. Assuming also that $K_0 = K_1$, $\mu_1 = \mu_0$, $A_0^u = A_1^u$ and $A_0^v = A_1^v$, we have

$$u_1 = -\frac{K_1}{\mu_0}\rho_0 g \sin\omega,$$
$$v_1 = -\frac{K_1}{\mu_0}\rho_0 g\, \alpha(\Theta_1 - \Theta_0)/2,$$

which differs from the result obtained by Cathles (1981) by a factor of 2. Cathles considered the viscous resistance of the hot discharging flow but neglected that of the recharging flow. For simplicity in the following sections we shall use these simplified equations.

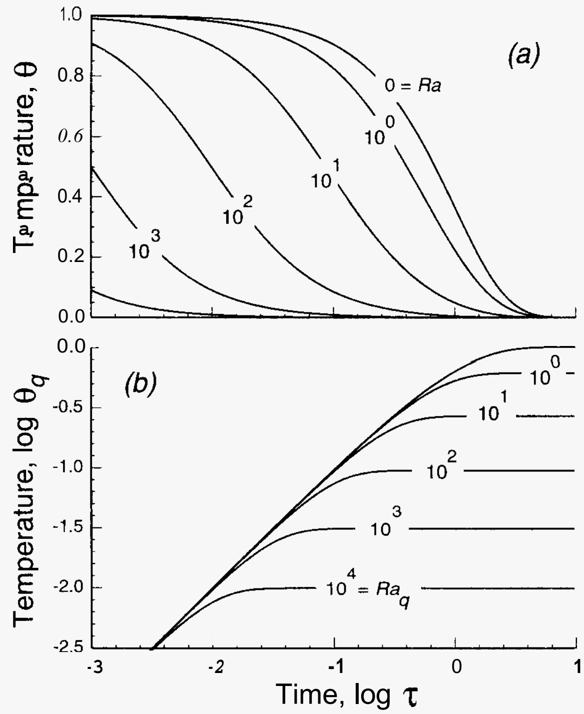

Figure **2:** Temperature as a function of time for various Rayleigh numbers and $Pe = 0$. ***a.*** Cooling associated with sudden emplacement of a intrusion. *b.* Heating associated with constant long-term magma-heat supply.

5 Rapid Intrusion

We now focus on the problem of intrusions cooling in the presence of lateral ground-water flow driven by a regional slope of the aquifer and of vertical flow driven by hydrothermal buoyancy. We define a nondimensional temperature difference and a nondimensional time:

$$\theta = \frac{(\Theta_1 - \Theta_0)}{\hat{\Theta}} \qquad \text{and} \qquad \tau = \frac{\kappa t}{L^2}.$$

The vigor of the horizontal and vertical convective heat flows, relative to total heat conduction, are defined by

$$Pe = \frac{(\rho C)_f}{k_b}\frac{K_1}{\mu_0}\frac{L^2}{L_x}\rho_0 g \sin\omega,$$
$$Ra = \frac{(\rho C)_f}{k_b}\frac{K_1}{\mu_0}\frac{L^2}{L_z}\rho_0 g \alpha\hat{\Theta}/2,$$

respectively. The ratio of horizontal to vertical heat **flows,** is

$$\frac{Pe}{Ra} - 2\frac{L_z}{L_x}\frac{\sin w}{\alpha\Theta}$$

Generally, the slope of a ground-water system is modest, no more than several degrees, so that it would often be the case that $\sin w < \alpha\hat{\Theta}$. On the other hand, the vertical extent of a magma-hydrothermal system within some shallow portion of the upper crust may be greater than the horizontal extent, $L_z > L_x$. One can, therefore, not assume that the regional flow of ground water sweeps away less heat than is carried by buoyant circulation of hydrothermal waters.

Inserting the equations above for 6, τ, *Ra,* and *Pe* , as well as the equations for u_1 and v_1 derived in the previous section into the energy equation, and setting $Q_{total} = 0$ we obtain the nondimensional energy equation

$$\frac{d\theta}{d\tau} = -\theta - Pe\,\theta - Ra\,\theta^2.$$

The rate of cooling by conduction is proportional to temperature 6; cooling rate is similarly proportional to the vigor of regional flow of ground-water flow as quantified by the Peclet

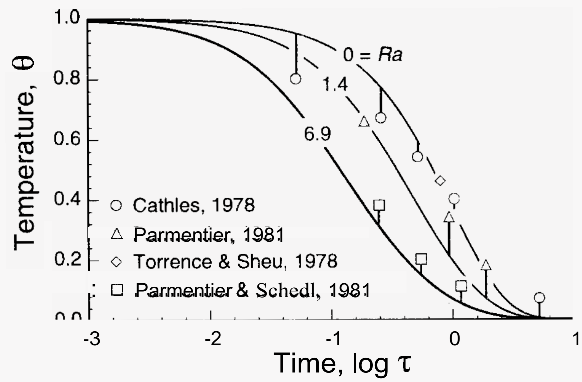

Figure 3: Temperature as a function of time for Rayleigh numbers corresponding to parameters used in numerical solutions for pluton cooling.

number. In contrast, the rate of cooling by buoyant hydrothermal circulation is proportional to θ^2. For the initial condition $\theta(\tau = 0) = 1$, the solution for instantaneous intrusion is

$$\theta(\tau) = \frac{(1+Pe)\,e^{-(1+Pe)\tau}}{(1+Pe) + Ra\left[1 - e^{-(1+Pe)\tau}\right]}$$

We show temperature as a function of time for various *Ra* and $Pe = 0$ in Figure *2a*. If buoyant hydrothermal circulation is the predominate cooling process, then $\theta(\tau) \simeq (1 + Ra\tau)^{-1}$.

6 Numerical Results

Out method of calculating average temperature as a function of time yields results comparable to those obtained from two-dimensional numerical models (Figure *3)* published by Cathles (1978), Torrence and Sheu (1978), Parmentier (1981), and Parmentier and Schedl (1981). We determined average temperatures from their published figures by spatial averaging of isotherms; values of *Ra* were calculated from their model parameters, as were the values of nondimensional time corresponding to stage of cooling portrayed in their figures. Ambient host-rock temperature Θ_0 was taken as the value at the tnidheight of the intrusion and water and thermal properties estimated for that temperature and hydrostatic pressure. Cathles (1978; circles, Figure **3)** treated the conductive cooling of a 2.25-km-high and 1.5-km-wide intrusion, showing isotherms at 1, 5, 10, 20, and 100 ka. Temperatures calculated from expressions above agree to within 15% of the overall temperature difference at each of these times. Torrence and Sheu (1978) treated the cooling of a square, impermeable intrusion surrounded by permeable rocks; they provide one illustration of the isotherms, and the temperature calculated for it (diamond, Figure 3) agrees to within 1%. Parmentier (1981) treated a convectively cooling intrusion with the same thermal and hydraulic properties as the host rocks. Temperatures estimated from expressions above agree to within 12% for the illustrations (triangles, Figure 3) they published. Parmentier and Schedl (1981) treat a similar geometry as Parmentier (1981), and the agreement is better than 7% for each of their three illustrations. Taken together, it appears that our estimates tend to be relatively high during the early stages of cooling and relatively low thereafter.

Another important finding emerges from the comparison with previously published numerical results. Values of the Rayleigh numbers *Ra* are <7 for all of the numerical models. That is, none of the numerical results were obtained for situations where hydrothermal cooling vastly predominated over conductive cooling, even though the purpose of the models was to illustrate the importance of the former relative to the latter. Because one's choice of length scale used to define a Rayleigh number is somewhat arbitrary, the resulting interpretation varies from one study to the next. Results published by Parmentier (1981), for instance, employ a Rayleigh number, as defined by him, of 50; using the definition above, we obtain $Ra = 1.4$ for the same model parameters (triangles, Figure **3).** Moreover, many workers, especially those employing numerical methods of solution, neither define nor estimate any Rayleigh number. It is important, we feel, to estimate the importance of heat transfer by buoyant hydrothermal circulation relative to heat conduction. Our definition of *Ra,* and, for that matter, *Pe,* are superior to others because they incorporate the appropriate convective length scales and the total heat transfer by conduction.

7 Steady Intrusion

As explained earlier, it is reasonable to interpret many geothermal systems as being maintained by steady, or, at least, intermittent resupply of magmatic heat. For such systems, we define a nondimensional temperature difference and a "heat-flux" Rayleigh number,

$$\theta_q = \frac{\Theta_1 - \Theta_0}{\hat{\Theta}_q}, \qquad \text{with} \qquad Ra_q = \frac{(\rho C)_f}{k_b}\frac{K_1}{\mu_0}\frac{L^2}{L_z}\rho_0 g\alpha\hat{\Theta}_q/2.$$

Temperature rise θ_q and Rayleigh number Ra_q depend upon Θ_q, the steady-state conduction temperature for the system when subject to heat addition at rate Q_{total}.

Inserting the equations above for θ_q, *i-*, *Ra,,* *Pe,* u_1, and v_1 into the energy equation, we obtain the nondimensional energy equation

$$\frac{d\theta_q}{d\tau} = -\theta_q - Pe\,\theta_q - Ra_q\,\theta_q^{\,2} + 1.$$

The solution for $\theta_q(\tau = 0) = 0$ is

$$\theta_q(\tau) = \frac{2\left(1 - e^{-G\tau}\right)}{[(1+Pe)+G] \quad [(1+Pe)-G]\,e^{-G\tau}},$$

where

$$G = \sqrt{(1+Pe)^2 + 4Ra_q}.$$

We show temperature rise as a function of time for various *Ra,* and $Pe = 0$ in Figure 2*b*, where it is apparent that the temperature rises during a transient and then reaches the steady value.

$$\theta_q(\tau \to \infty) = \frac{\sqrt{(1+Pe)^2 + 4Ra_q} - (1+Pe)}{2Ra_q}$$

If heat transfer is dominated by buoyant hydrothermal circulation, then

$$\theta_q(\tau) = \frac{\tanh\sqrt{Ra_q}\,\tau}{\sqrt{Ra_q}} \qquad \text{and} \qquad \theta_q(\tau \to \infty) = \frac{1}{\sqrt{Ra_q}}$$

As can be verified in Figure 2*b*, both the steady state temperature and the time required to achieve it, which we refer to as the thermal response time, are sensitive to $Ra_q^{-1/2}$ This response time differs considerably from the cooling time of a suddenly emplaced intrusion (compare Figures *3a,* b). If heat transfer is dominated by horizontal ground-water flow, then

$$\theta_q(\tau) = \frac{1 - e^{-Pe\tau}}{Pe} \qquad \text{and} \qquad \theta_q(\tau \to \infty) = \frac{1}{Pe},$$

and the response time and steady-state temperature are sensitive to Pe^{-1}.

8 East Rift Zone, Kilauea Volcano

The lower east rift zone of Kilauea Volcano, Hawaii, (Figure 4) has been the site of repeated fissure eruptions fed by dikes that traverse the depths of interest for geothermal exploration. These dikes, however, are unlikely to be the sole or even dominant source of heat for the high-temperature (—300°C) hydrothermal system known to exist there at depths of less than a few kilometers (Kauahikaua, 1993; Moore and Kauahikaua, 1993). Although there is evidence for small local magma reservoirs along the rift zone, there are no estimates of their size, depth, and rate of replenishment. Nevertheless, data from geothermal wells along the lower east rift zone indicate bottom-hole heat flows in the range of 370–820 mW/m^2 (Kauahikaua, 1993). Conductive heat flow at the surface is essentially zero, there is little fumarolic activity, and there are no thermal springs along

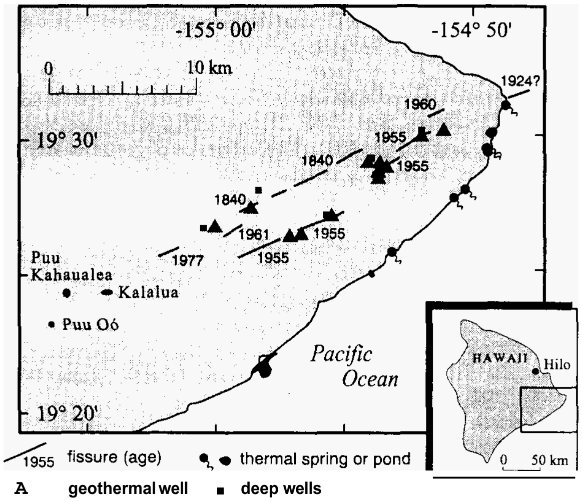

Figure 4: Lower east rift zone of Kilauea Volcano, Hawaii, showing locations of eruptive vents of the the 19th and 20th centuries, thermal springs or ponds, and geothermal drill holes.

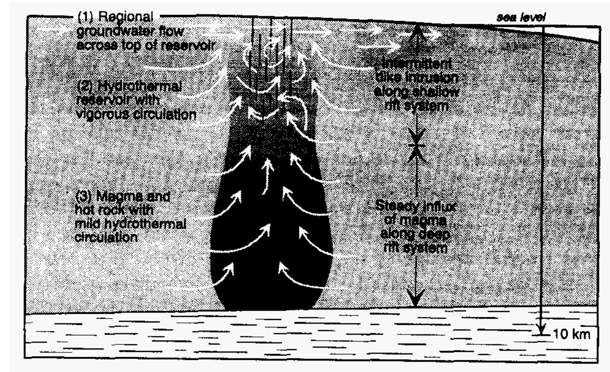

Figure 5: Schematic section across the lower east rift zone with ground-water flow paths due to weak hydrothermal circulation at depth, strong circulation in the overlying hydrothermal reservoir, and vigorous near-surface flow of ground water across the top of the reservoir.

the rift zone. Rather, heat rising from the hydrothermal reservoir apparently is swept laterally by vigorous ground-water flow to produce a few warm-water discharges along the nearby seacoast.

The lower east rift zone was the site of eruptions in 1840, 1955, 1960, and 1961. Abundant earthquakes and formation of a graben and related fissures along the axis of the rift zone suggest a shallow intrusion of magma in 1924 (Swanson *et al.*, 1976) and, perhaps, a small submarine eruption offshore. Eruptions in 1790 and 1750 are the only activity documented for the 18th century (Moore, 1992). Overall, Moore (1992) estimated an average interval of 14 years between eruptions during the past 1500 years. The magma supply rate to Kilauea Volcano is thought to be 100×10^{-3} km^3/yr (Swanson, 1972; Dzurisin *et al.*, 1984), approximately the effusion rate for long-lived eruptions. Moore (1992) estimated that —5 km^3 of lavas accumulated during the past 1500 years and notes that many flows reached the seacoast and delivered an undetermined volume of lava to the submarine slope of Kilauea. Nonetheless, a minimum average rate of lava accumulation along the lower east rift zone is 3×10^{-3} km^3/yr. Virtually all eruptions on Kilauea are fed by dikes that traverse the uppermost 2–4 km of the edifice. Although many rift-zone dikes propagate down rift from magma reservoirs near the summit, the 1955 and 1960 eruptions were fed by dikes that probably originated from magma bodies beneath them within the lower east rift zone (Delaney *et al.*, 1990).

We divide a vertical section of the lower east rift zone into three regions (Figure 5). From the base of the volcano at —9 km upward to —4 km, we postulate a low-permeability hot-rock-and-magma environment. Weak hydrothermal circulation and conduction carry heat upward to the overlying hydrothermal reservoir that is presently being explored and exploited for purposes of electric-power generation. The top of that reservoir is characterized by very strong lateral ground-water flow across and along the rift zone. This water flows seaward from regions of high rainfall further upslope. Although the permeability structure of Kilauea is poorly determined (Ingebritsen and Scholl, 1993), very high permeabilities of surface lavas do not persist at depth. Most drill holes along the lower east rift zone, for example, encounter high-temperature fluid and low-permeability rocks owing to hydrothermal alteration and sealing of fractures.

We first inquire into the addition of heat to the reservoir from a hot-rock-and-magma system that we suppose to exist at depths greater than —4 km. What is the rate of heat replenishment that can maintain it? We choose a 4-km width of the heated region and a vertical extent of 5 km, which, together with the shallow hydrothermal reservoir spans the entire volcanic edifice. At these depths, we assume that the horizontal component of ground-water flow is small, so $Pe = 0$. The overall conductive geothermal gradient for Hawaii is ~40°C/km, and so we estimate regional background temperatures of ~160–360°C at depths of 4–9 km. In order to supply sufficient heat to maintain the overlying hydrothermal reservoir, we assume that the rift-zone hot-rock-and-magma system at 4–9-km depths has temperatures 500°C higher. Similarly, we let the permeability of this system be 10^{-17} m^2, which is probably a high estimate, at least in the absence of tectonic effects that might tend to create permeability. Using the result derived above for $Pe = 0$, we obtain a Rayleigh number $Ra_q = 1 \times 10^{-1}$ and a heat-supply rate $Q_{total} = 6$ kW/m along the rift zone. For magma initially invaded at 1200°C and releasing all of its heat of crystallization, the equivalent intrusion rate is 9 m^3/yr/m along the rift zone, or 2×10^{-3} km^3/yr along the 25 km of lower east rift zone, 2% of the estimated magma-supply rate for Kilauea (Swanson, 1972; Dzurisin *et al.*, 1984). This volume of intrusion implies rift-zone extension of less than 2 cm/yr at 4–9-km depths. Temperatures of —650–850°C within this magma-and-hot-rock system exceed those able to support differential stresses that can lead to nucleation of earthquakes. Indeed, earthquakes of $M<2$ at 4–9-km hypocentral depths are rare along the east rift zone of Milauea; in contrast, they are common at those depths south of Kilauea's rift system and at shallower depths within the rift system.

If a —650–850°C hot-rock-and-magma body persists at depths greater than —4 km, what temperatures might exist in the overlying hydrothermal reservoir? This component of the system resides below sea level, and so we set $Pe = 0$. We seek conditions allowing a temperature of 175°C above ambient, where ambient temperatures range from 15–175°C. Using a permeability of 1×10^{-15} m^2 (Ingebritsen and Scholl, 1993) and a rift-zone width of 2 km, we obtain $Ra_q \simeq 2$. These conditions are accounted for by heat carried upward from the deeper system.

We add to this estimate the heat that would result from intrusion of 10-km long dikes that average 0.5 m in thickness (Walker, 1987) every 15 years. If this heat were added steadily and homogeneously, then the average rate of heat addition would be 7 kW/m of rift zone, about equal to that supplied from the deeper system and thus doubling the estimated temperature rise. This approximation is probably much too large because dike injections are highly localized both spatially and temporally. If the thermal anomaly due to a dike injection is only, say, 10 m wide, then intense hydrothermal circulation adjacent to it would cool it to only a few percent of its initial temperature during the 15-year average recurrence interval. Although the heat added to the hydrothermal reservoir by dike intrusion is probably removed quickly, fractures that accompany dike intrusion may comprise an important drilling target. Taken together, then, heating from a deep reservoir continuously supplied with magma and from shallow dikes invaded intermittently along the rift zone, can account €or the temperatures observed in the drill holes with the permeability estimates obtained by Ingebritsen and Scholl (1993).

Finally, the shallowest rocks of the system are extremely permeable; values of $K_1 = 10^{-10}$ m', or even higher, reflect the porous nature of young, shallow lavas. Using the relation for $Ra_q = 0$ and $Pe >> 1, \theta_q(\tau \rightarrow \infty) = Pe^{-1}$, the heat flux of 6 kW/m of rift zone can be removed at temperature increases of less than 1°C by a ground-water table sloping only 0.5°(Sorey and Colvard, 1994). It is this strong flow of water recharged by heavy rainfall upslope of the rift zone and discharging nearby at sea level that accounts for the absence of significant surface heat flow in the rift zone itself.

9 Conclusions

We developed a method to relate the most important thermal and hydrologic factors acting upon a magmatic hydrothermal system in a fashion allowing estimation of temperature, hydrologic and thermal response time, and the rate of heat replenishment that maintains the system. The method is appropriate for poorly explored systems, where the value of numerical models is uncertain owing to the lack of data to constrain the shape, magmatic history, physical properties, and regional hydrologic conditions. We chose a simple system configuration (Figure 1) to illustrate the method and to obtain the most basic results.

For dominantly conductive cooling of a rapidly emplaced intrusion, an accurate generalization is that as much heat is lost to the surroundings as has been retained by the intrusion when $\tau \sim 1$ and cooling is essentially complete when $\tau \sim 10$. Our results also indicate that, if cooling is dominated by hydrothermal circulation, as has been lost to the surroundings as is retained by the intrusion when $\tau \sim 1/Ra$ and cooling is essentially complete when $\tau \sim 10/Ra$. Similarly, if cooling is dominated by the regional flow of ground water, as much heat has been lost to the surroundings as is retained by the intrusion when $\tau \sim 1/Pe$ and cooling is essentially complete when $\tau \sim 10/Pe$.

More importantly, many hydrothermal systems are maintained by a supply of magmatic heat that is prcbably more nearly temporally continuous than discrete. Where this is the case, hydrothermal reservoir temperature Θ is critically related to the rate of heat addition Q_{total} and average permeability K, such that $\Theta \propto \sqrt{Q/K}$. That temperatures of interest for electrical generation span a range of only a couple hundred degrees places severe constraints on the combinations of reservoir properties to support such a system. Paradoxically, increased permeability, which is attractive for maximizing the rate of production, causes decreased temperatures. We conclude, therefore, that most commercially attractive hydrothermal reservoirs are anomalies maintained by much larger, and, generally, poorly explored, hot-rock-and-magma systems.

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10 References

Cathles, L.M., (1977). An analysis of the cooling of intrusives by ground-water convection which includes boiling. *Econ. Geol.,* Vol. 72, pp. 804–826.

Cathles, L.M., (1981). Fluid flow and genesis of hydrothermal ore deposits. In: *Econ. Geol. 75th Anniv. Vol.,* B. J. Skinner Jr. (Ed.). Economic Geology Publishing Co., New Haven, pp. 424–457.

Delaney, P. T., (1987). Heat transfer during emplacement and cooling of mafic dykes. In: *Mafic Dyke Swarms,* H.C. Halls and W.F. Fahrig (Eds.). *Geol. Assoc. Canada Sp. Paper 34,* pp. 31–46.

Delaney, P.T., R.S. Fiske, A. Miklius, A.T. Okamura, MX. Sako (1990). Deep magma body beneath the summit and rift zones of Kilauea volcano, Hawaii. *Science,* Vol. 247, pp. 1311–1316.

Delaney, P.T., A. Miklius, T. Arnadottir, A.T. Okamura, M.K. Sako (1993). Motion of Kilauea volcano during sustained eruption from the Puu OOand Kupaianaha vents, 1983–1991. *J. Geophys. Res.,* Vol. 98, pp. 17,801–17,820.

Dzurisin, D., R.Y. Koyanagi, T.E. English (1984). Magma supply and storage at Kilauea volcano, Hawaii, 1956–1983. *J. Volc. Geotherm. Res.,* Vol. 21, pp. 177–206.

Forster, C., L. Smith (1989). The influence of groundwater flow on thermal regimes in mountainous terrain: a model study. *J. Geophys. Res.,* Vol. 94, pp. 9439–9451.

Ingebritsen, S.E., M.A. Scholl (1993). The hydrogeology of Kilauea volcano, *Geothermics,* Vol. 22, pp. 255–270.

Jaeger, J.C., (1964). Thermal effects of intrusions. *Rev. Geophys.,* Vol. 2, pp. 443–466.

Jaeger, J.C., (1968). Cooling and solidification of igneous rocks. In: *Basalts: Poldervaardt Treatise on Rocks of Basaltic Composition,* H.H. Hess, A. Poldervaardt (Eds.). John Wiley, New York, pp. 504–536.

Kauahikaua, J.P., (1993). Geophysical characteristics of the hydrothermal systems of Kilauea Volcano, Hawaii. *Geothermics,* Vol. 22, pp. 271–299.

Moore, R.B., (1992). Volcanic geology and eruption frequency, lower east rift zone of Kilauea volcano, Hawaii. *Bull. Volc.,* Vol. 54, pp. 937–981.

Moore, R.B., J.P. Kauahikaua (1993). The hydrothermal-convection systems of Kilauea: an historical perspective. *Geothermics,* Vol. 22, pp. 233–241.

Norton, D., J. Knight (1977). Transport phenomena in hydrothermal systems: cooling plutons. *Am. J. Sci.,* Vol. 277, pp. 937–981.

Norton, D., H.P. Taylor (1979). Quantitative simulation of the hydrothermal systems of crystallizing magmas on the basis of transport theory and oxygen isotope data: an analysis of the Skaergaard intrusion. *J. Petr.,* Vol. 20, pp. 421–486.

Parmentier, E.M., (1981). Numerical experiments on ^{18}O depletion in igneous intrusions cooling by groundwater convection. *J. Geophys. Res.,* Vol. 86, pp. 7131–7144.

Parmentier, E.M., A. Schedl (1981). Thermal aureoles of igneous intrusions: some possible indications of hydrothermal convective cooling. *J. Geol.,* Vol. 89, pp. 1–22.

Parmentier, E.M., E.T.C. Spooner (1978). A theoretical study of hydrothermal convection and the origin of the ophiolitic sulphide ore deposits of Cyprus. *Earth Planet. Sci. Lett.,* Vol. 40, pp. 33–44.

Smith, R.L., H.R. Shaw (1979). Igneous-related geothermal systems. In: *Assessment* of *Geothermal Resources of the United States—-1978,* L.J.P. Muffler (Ed.). U. *S. Geol. Sum. Circ. 790,* pp. 12–17.

Sorey, M.L., E.M. Colvard (1994). Potential effects of the Hawaii geothermal project on ground-water resources on the island of Hawaii. *U.S. Geol. Sum. Water Res. Inv. Rept. 94-4028,* 35 p.

Swanson, D.A., (1972). Magma supply rate of Kilauea volcano, 1952–1971. *Science,* Vol. 175, pp. 169–170.

Swanson, D.A., W.A. Duffield, R.S. Fiske (1976). Displacement of the south flank of Kilauea volcano: the result of forceful intrusion of magma into the rift zones, *U.S. Geol. Sum. Prof. Paper 963.*

Taylor, H.P., (1974). The application of oxygen and hydrogen isotope studies to problems of hydrothermal alteration and ore deposition. *Econ. Geol.,* Vol. 69, pp. 843–883.

Taylor, H.P., R.W. Forester (1979). An oxygen and hydrogen study of the Skaergaard intrusion and its country rocks: a description of a 55-m.y. old fossil hydrothermal system. *J. Petr.,* Vol. 20, pp. 355–419.

Torrence, K.E., J.P. Sheu (1978). Heat transfer from plutons undergoing hydrothermal cooling and thermal cracking. *Numerical Heat Transfer,* Vol. 1, pp. 147–161.

Walker, G.P.L., (1987). The dike complex of Koolau volcano, Oahu: internal structure of a Hawaiian rift zone. U.S. *Geol. Sum. Prof. Paper 1350,* pp. 961–993.

White, D.E., (1955). Thermal springs and epithermal ore deposits. *Econ. Geol. 50th Anniv. Vol.,* Economic Geology Publishing Co., New Haven, pp. 99–154.

White, D.E., (1974). Diverse origins of hydrothermal ore fluids. *Econ. Geol.,* Vol. 69, pp. 954–973.