

Analytical and Numerical Modelling of a Coaxial Borehole Heat Exchanger to Extract Geothermal Energy

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ABSTRACT

Unused wells in petroleum fields are readily available as a potential source of geothermal energy. To produce geothermal heat in a viable manner, borehole heat exchangers may be used in some of the available deep wells (> 500 m). Therefore, we aim to develop a rigorous modelling technique to evaluate such systems in deep oil and gas wells.

We examine a mathematical model based on unsteady-state heat conduction for the coupled borehole-reservoir system. Using the analytical technique, our paper presents cases evaluated using the Method of Characteristics. We briefly discuss the challenges encountered while using the above method for a non-linear hyperbolic system of partial differential equations in a practical scenario. Finally, we adopt a numerical technique based on the Finite Difference Scheme to analyse transient heat transfer in a coupled wellbore-reservoir system. The paper also discusses a hypothetical case that uses borehole heat exchangers to supply geothermal heat in a greenhouse.

We conclude that the numerical technique based on the finite difference scheme is appropriate to study deep borehole heat exchanger systems. The hypothetical case study of a greenhouse demonstrates the feasibility of using such systems to supply geothermal heat. Future work should perform a detailed economic study and develop a more rigorous analytical method to optimise the design of multi-borehole systems.

1. INTRODUCTION

1.1 Direct Heat Use

Geothermal energy offers a potential solution to substitute for fossil fuels owing to its abundance and renewable nature. Apart from electricity generation, direct use of heat is a popular application of geothermal energy (Burnell et al., 2021; Climo et al., 2020; Climo, Blair, et al., 2021; Climo, Milicich, et al., 2021). Global thermal energy use has increased significantly over the last seven years (1,020,887 TJ/annum (72.3% increase) (Lund & Toth, 2021)). New Zealand is witnessing a growth in the use of shallow geothermal resources with the help of Geothermal (ground source) Heat Pumps (GSHPs) (Seward & Carey, 2021). GSHPs utilising aquifer water energy in New Zealand are mostly concentrated in the south and constitute about 0.5 PJ/annum (<1.5% of the total) geothermal energy (Climo, Milicich, et al., 2021).

In most places, adequate water energy sources are rarely available; the closed-loop GSHPs coupled with Borehole Heat Exchangers (BHEs) are a proven way to produce geothermal heat in such cases (Li & Lai, 2015; Spitler &

Bernier, 2019). Such GSHPs constitute 58.8% of global geothermal energy use (Lund & Toth, 2021). Moreover, GSHPs have the lowest CO₂ emissions and the lowest overall environmental impacts, according to the US Environment Protection Agency (L'Ecuyer et al., 1993; Li & Lai, 2015). GSHPs may contribute to New Zealand's move towards a low-carbon economy in the growing commercial process-heat sector (Climo, Milicich, et al., 2021).

BHEs in shallow geothermal systems (usually between 50 m to 200 m) have been extensively modelled in the past (Cui et al., 2018; Li & Lai, 2015; Spitler & Bernier, 2019). In contrast, modelling BHEs for deep geothermal resources (with greater energy potential, usually greater than 1000 m depth) is an ongoing topic of research (Pan et al., 2019). Wells available in mature oil and gas fields worldwide offer one such potential location for the application of BHE technology to extract energy (Duggal et al., 2022). In New Zealand, the Taranaki region has some fields with attractive geothermal gradient (Duggal, Burnell, et al., 2021). Moreover, electricity generation may not be economically viable in such fields (Duggal, Rayudu, et al., 2021b, 2021a). Therefore, we perform deep borehole heat exchanger modelling in this study to analyse likely energy output from mature wells.

1.2 Borehole Heat Exchangers

A schematic for a coaxial borehole heat exchanger is shown in Figure 1. The thermal exchange in a BHE takes place with the help of a flowing fluid.

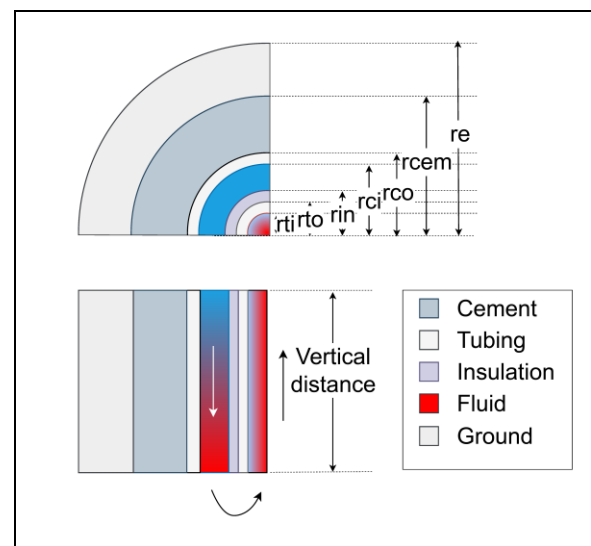


Figure 1: Schematic representation (plan and elevation) of a coaxial Borehole Heat Exchanger.

During heat extraction, cold fluid is sent to exchange heat with the ground down either the inner or outer annulus space. The fluid becomes hot as it reaches the bottom of the well.

This hot fluid then flows up the other annulus to the surface for direct use. For simplicity, we refer to the annulus spaces as pipes. Either the inner or the outer pipes can be used to produce the fluid. In this work, we choose the inner pipe as a producer of the fluid while the outer pipe is an injector in our case.

A mathematical model consisting of a set of Partial Differential Equations (PDEs) is usually developed to fully capture the heat transfer phenomenon occurring in an operational BHE system. In terms of modelling, the constant wellbore wall temperature is a common assumption with shallow BHEs (Li & Lai, 2015), whereas, the wall temperature varies with depth for deep BHEs (Claesson & Javed, 2019). Numerical methods are used to solve the system of PDEs due to the range of time and length scales involved in the study of a BHE system (Li & Lai, 2015; Spitler & Bernier, 2019). Commercial or custom software models implement these numerical methods to support the design of a heat extraction system. This software methodology, however, is computationally demanding (~several days depending upon the application), and hence, is inconvenient for engineers and designers. Alternate solutions are preferred to analyse the energy, optimum control, and operation of BHEs (Cui et al., 2018; Li & Lai, 2015; Pan et al., 2019; Spitler & Bernier, 2019).

Analytical techniques are popular in analysing deep BHE systems (Li & Lai, 2015). Techniques such as Laplace transform (Beier, 2014; Beier et al., 2014), Convolution theorem, Green's function, and Duhamel's theorem (Li & Lai, 2015) have mostly been applied. However, most of the above techniques aim to solve the system of PDEs by making assumptions such as simplifying the model using a steady flux-state for the BHE operation (Li & Lai, 2015). The underlying heat transfer mechanism still invites further investigation. Relaxing some of these assumptions is a continued work in the literature (Li & Lai, 2015; Spitler & Bernier, 2019). The present work tries to look for an alternate method of solving the heat transfer in a wellbore, which may simplify the calculations and analysis.

1.3 Method of Characteristics

The Method of Characteristics (MoC) is one such tool that is used to solve the system of PDEs by converting it into Ordinary Differential Equations (ODEs) and offering an easy-to-manipulate mathematical model (Cheng & Wang, 2007). Moreover, the MoC is a common tool to solve several problems in porous media, and supersonic flow (Cheng & Wang, 2007; Stopa & Wojnarowski, 2006). Stopa and Wojnarowski (2006) applied the MoC to determine the cold water front velocity during reinjection in a geothermal reservoir. A similar extensive study was conducted in the case of CO₂ injection in an EGS reservoir (Laforce et al., 2015).

In this work, we attempt to relax the common assumption of a steady flux-state and analyse an unsteady-state BHE system using the MoC. Our initial work concludes that MoC is unable to appropriately handle the coupled thermal interactions among the different wellbore components in a BHE system (Duggal et al., in review). The issues were mainly: a) the occurrence of the solution discontinuities due to the opposite nature of the fluid flow inside the pipes, and b) the prediction of multiple temperature values at a single point along the wellbore depth in a bounded domain. Here we study theoretical cases and address the known issues.

1.4 Objectives

In this paper, we extend our previous work of analysing a BHE system using the MoC. Our main objective is to address the issues faced in our initial study. We address the main issue by presenting limiting cases in the BHE operation. A potential method to prevent the occurrence of solution discontinuities is discussed briefly. Finally, we capture the coupled thermal interactions using the MoC by presenting a theoretical scenario of practical importance.

The next section, Section 2, briefly describes the methodology used in this paper. The results are discussed in Section 3 of the document. Conclusion and Future work are presented in the subsequent section, Section 4, at the end of this document.

2. METHODOLOGY

Method of Characteristics, an analytical technique, is first used to solve the mathematical model of a borehole heat exchanger. A numerical technique (Finite Difference Method (FDM)) is then applied to analyse the BHE system. We use a single benchmark analytical solution available in the literature to validate our solution using both techniques (Tang et al., 2019). We used wellbore parameters given in Tang et al., (2019) in our present work too.

To analyse a BHE, it is a common practice to divide the whole system into two regions (R. Hasan & Kabir, 2018; Tang et al., 2019). The first region comprises the wellbore having the injection pipe and the production pipe. The other comprises the infinite ground as a heat sink or a heat source. In the next sections, we consider the wellbore section only.

2.1 Analytical Technique: Method of Characteristics

We know from our previous work (Duggal et al., in review) that MoC is unable to appropriately handle coupled dynamics of a wellbore heat transfer. We adopt certain assumptions to avoid challenges faced while applying MoC (Zhu et al., 2004) that are explained with each case.

2.1.1 Fluid flowing without the influence of the surroundings

Our first case assumes the flux from either the production pipe or reservoir to not interfere with the fluid flowing down the injection pipe, see Figure 2. We further assume a perfectly insulated production pipe. Thus, we produce the same bottommost temperature at the surface, and therefore, discuss only the injection pipe next.

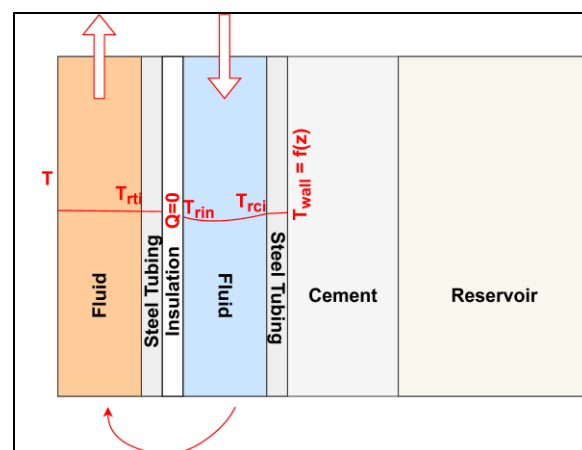


Figure 2: Schematic of heat transfer in a wellbore with a perfectly insulated production pipe.

Applying the energy balance principle to a wellbore results in a system of partial differential equations describing the BHE as (A. R. Hasan & Kabir, 2012):

$$\frac{\partial T_i}{\partial t} + v_{fi} \frac{\partial T_i}{\partial z} = Q(T_i, t, z) \quad (1)$$

In the above equation, T_i is the temperature of the fluid, t , and z are temporal and spatial variables, v_i is the fluid velocity (0.2 m/s), Q is the function describing the thermal interactions, and i , is the subscript for the injection pipe, and fi , the subscript is for the fluid in the injection pipe. All units are in SI unless otherwise stated. The boundary and initial conditions, where g_G is the geothermal gradient, are given as:

$$T_i(z, t) = \begin{cases} T_0 = T_{inj}, & z = 0, t = 0 \\ T_s + g_G z, & z = z, t = 0 \end{cases} \quad (2)$$

MoC based on the above-mentioned assumption of $Q=0$ gives the solution as:

$$T_i(z, t) = T_s + g_G (z - v_{fi} t) \quad (3)$$

where $v_{fi} t < z < L + v_{fi} t$

The equation becomes quasi-linear in case of fluid properties varying with the temperature, or in case of the variable heat capacity (Stopa & Wojnarowski, 2006). Solution of such equations may include violations of the material balance resulting in solution discontinuities, known as shocks (Zhu et al., 2004). We can follow a standard procedure to address the material balance violation (Stopa & Wojnarowski, 2006; Zhu et al., 2004). Therefore, we consider the non-homogeneous hyperbolic advection partial differential equation in our next case.

2.1.2 External interactions influencing the fluid flow

In this case, we assume heat exchange from both sides of the injection pipe, i.e., production pipe and reservoir, see Figure 3. The model also includes conduction heat transfer from the rest of the components (insulation, tubing, cement, etc.) of a BHE system.

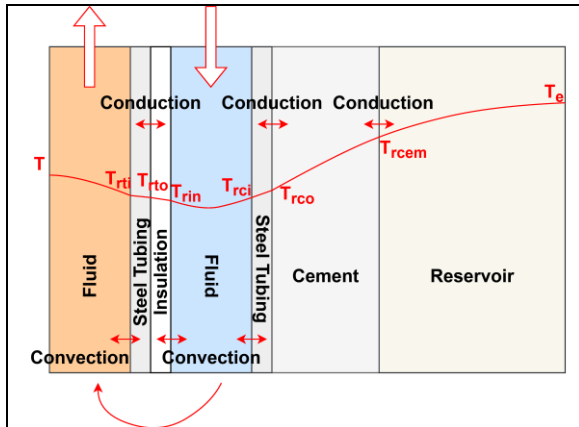


Figure 3: Schematic of heat transfer in a BHE system.

The PDE describing the energy balance in such a case can be written in a compact form as (Abdollahi & Dubljevic, 2013; Tang et al., 2019):

$$\frac{\partial T_i}{\partial t} + F(T, t, z) \frac{\partial T_i}{\partial z} = Q(T_i, t, z) \quad (4)$$

The non-linear nature of the hyperbolic PDE may cause shock (solution discontinuity) while MoC violates the energy conservation criteria (Stopa & Wojnarowski, 2006). We initially proposed a solution to consider the same fluid velocity in both the pipes (Duggal et al., under review). Although the approach avoids solution discontinuities, the BHE configuration restricts to limited radii for one of the pipes. In this work, we test another potential solution where we first decouple both the pipes at the bottom of the wellbore and consider each pipe separately. We only consider one of the pipes to apply to test the potential solution. We identify the development of a more rigorous analytical method considering both pipes and reservoir in a coupled continua as future work.

The above approach requires a weak solution of the hyperbolic PDEs for the non-homogeneous part to avoid multiple solutions at a single point (Chevarunotai et al., 2015; Panini et al., 2019; Ramazanov et al., 2010). Authors have suggested different procedures in the literature to select the auxiliary function (LaForce, Mijić, et al., 2014; Stopa & Wojnarowski, 2006; Zhu et al., 2004). We obtain the weak solution by applying regression to one of the models of practical importance available in the literature (Tang et al., 2019).

The known function, $F(T, t, z)$, in terms of the time (t) and the spatial (z) variables, simplifies the original PDE to a constant velocity hyperbolic advection equation. The characteristic equation now obtained produces a single curve for each pair of z and t values, and does not violate the energy conservation criteria (LaForce, Ennis-King, et al., 2014; Stopa & Wojnarowski, 2006). The general form for the characteristic function is:

$$\frac{dz}{dt} = F(T, t, z) \quad (5)$$

The following boundary and the initial conditions are assumed.

$$T_i(z, t) = \begin{cases} T_0 = T_{inj}, & z = 0, t = 0 \\ T_s + g_G z, & z = z, t = 0 \end{cases} \quad (6)$$

Once the auxiliary function is known the general solution can be written as:

$$T_i(z, t) = f\left(z - \frac{dz}{dt} t\right) \quad (7)$$

The right-hand side of the above equation means that the initial solution which is a function of z , t , and T is advected by the velocity dz/dt in the t - z phase plane.

2.2 Numerical Technique: Finite Difference Method

Numerical techniques can handle PDEs of any nature accurately and reliably. Moreover, numerical approaches can address some of the inherent limitations in analytical techniques such as non-homogeneity, anisotropy, etc. Thus, we use the numerical Finite Difference Method (FDM) to discretise and evaluate a BHE system.

The discretised grid blocks are distributed logarithmically in the radial direction, with equally sized grid blocks in the vertical direction for the reservoir and wellbore. We choose a fully implicit method to discretise the temporal variable. The

implicit approach is more suitable because it is fast, unconditionally stable, and relatively less sensitive to the time step size.

The numerical approach is also validated with the benchmark model selected in this paper. Detailed discussion of the numerical scheme is out-of-scope for our current paper and the readers are referred to Tang et al., (2019) for further information. We adopt a numerical technique only to address the challenges faced while applying MoC in case of a coupled wellbore-reservoir system under unsteady-state heat conduction scenario.

3. RESULTS & DISCUSSION

The section discusses results based on the application of MoC to the cases presented above. We observe that both PDE formulations: quasi-linear and non-linear, require a suitable weak solution to achieve practically meaningful results. We also note a strong dependence of MoC on the initial function in case of homogeneous formulation of PDEs. In contrast, numerical technique can handle coupled and transient wellbore system reliably. Our study also discusses a hypothetical greenhouse case study at the end. See Appendix for modelling parameters.

3.1 Analytical Technique: Method of Characteristics

3.1.1 Fluid flowing without time-varying external influence

Figure 4 shows the temperature profile along the wellbore depth. This case ignores external interactions changing with time and assumes a perfectly insulated production pipe.

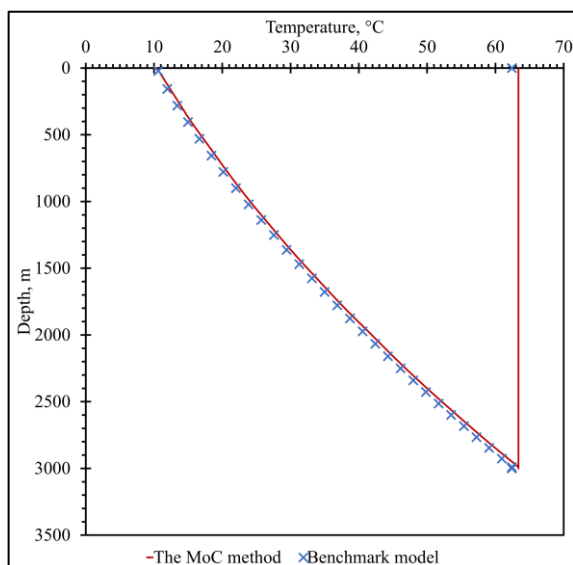


Figure 4: Temperature profile in the injection and the production pipe using the Method of Characteristics (MoC) and the Benchmark model.

We match our MoC results using Eq. (3) with a benchmark model in the literature (Tang et al., 2019). The characteristic equation provides solution at the boundaries. Two main observations are: a) MoC relies strongly on the initial conditions, and b) a numerical scheme should be used to study coupled heat transfer in a borehole system.

3.1.2 The influence of interactions

The hyperbolic formulation of PDEs is non-linear during the influence of external interactions on the fluid temperature inside the injection pipe, see Eq. (4). Figure 5 shows the

variation in fluid temperature with wellbore depth and 3 years operational time of the BHE, using Eq. (4) and Eq. (7). It can be observed that after three years the temperature at the bottom reduces by 18% to that of the initial period of BHE operation due to significant cooling of the adjacent reservoir.

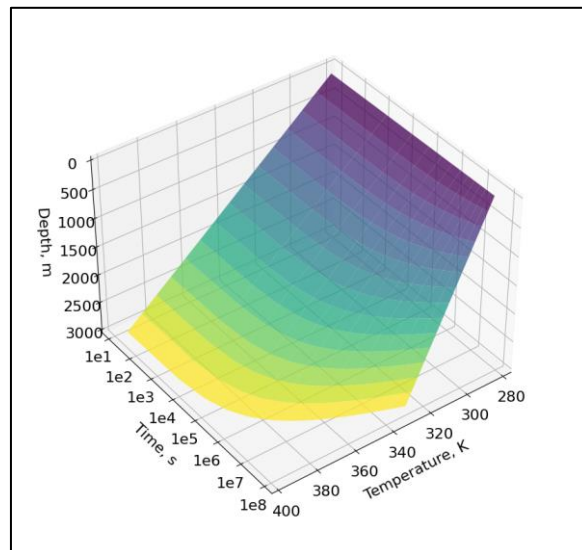


Figure 5: Injection pipe temperature distribution with thermal interactions from either side of the pipe using the MoC.

Figure 6 is another representation of the temperature distribution with the BHE operation time for two different locations, i.e., 2,000 m and 3,000 m.

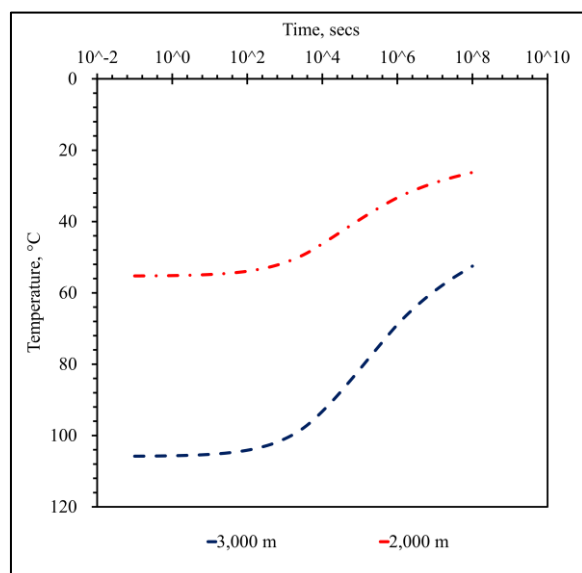


Figure 6: Change in temperature with time at different wellbore locations under the influence of external interactions.

It can be observed that the overall temperature at each point along the wellbore depth reduces significantly at the end of the BHE operation because of the heat extraction from the nearby reservoir region. Note that the above solution is valid only when the initial function is known, and $F(T, t, z)$ is a monotonically decreasing function. Different wellbore configuration and parameters require different auxiliary function, for example, the case of higher or variable flow rate as reported in the literature (Al Saeedi et al., 2018). In such

cases, the additional computation involved with this procedure is probably not worth the marginal gain in evaluation time. Therefore, we explore the numerical modelling to analyse borehole heat exchanger system.

3.2 Numerical Finite Difference Method

We first validate results of our numerical code with the benchmark model (Tang et al., 2019). Figure 7 presents our validation exercise; numerical results comply well with the benchmark model results. Therefore, the numerical model is suitable for the following analysis in our paper.

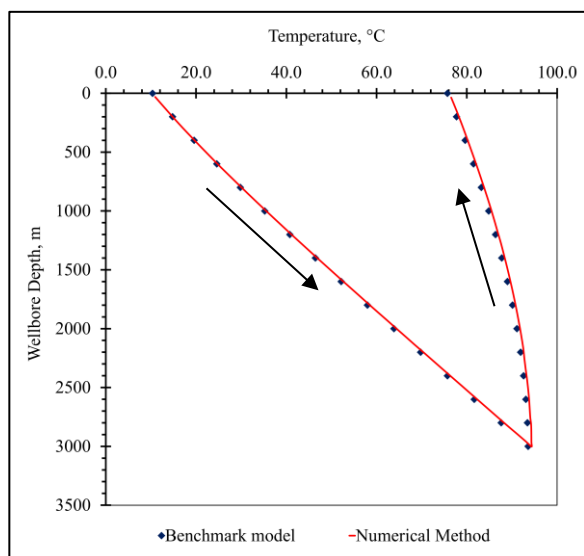


Figure 7: Validation of FDM numerical method against Benchmark model.

Figure 8 presents the temperature distribution in the reservoir section when different mass flow rates are used to extract heat from the adjacent reservoir. We perform a sensitivity study to analyse heat transfer while increasing the fluid mass flow rate and extending operation time of BHE system.

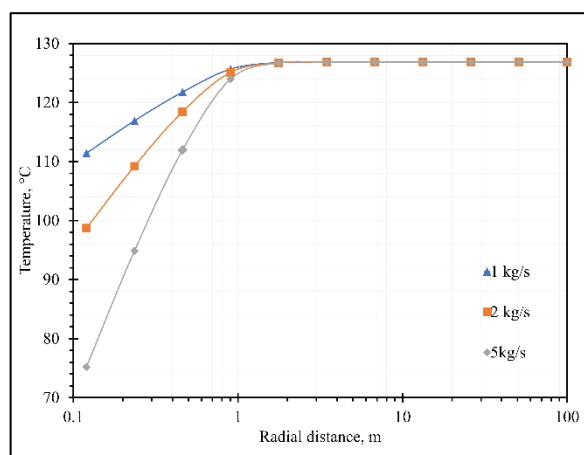


Figure 8: Temperature distribution in the reservoir section obtained using the FDM method after 1 day.

The figure shows that cooling has been observed within 1 m radius of the wellbore after 1 day. We have found all systems to reach quasi-steady state.

We then consider the case of injecting 5 kg/s, and plot instances of the temperature distribution in reservoir after 1

day, 10 days and 100 days of operation, see Figure 9. Two key observations are the cooled region around the wellbore expands from 3 m to around 13 m, and rate of heat extraction reduces as the time of operation increases. The later phenomenon occurs due to a reduced temperature difference between wellbore fluid and the outside reservoir. We conclude that it is necessary to use numerical modelling as compared to the analytical approach when accuracy is a prime requirement. Our findings agree with literature, for example, Giordano et al., (2019) also report 5% to 10% uncertainties in certain cases using analytical heat loss models, so, the numerical models offer a clear advantage (Giordano et al., 2019).

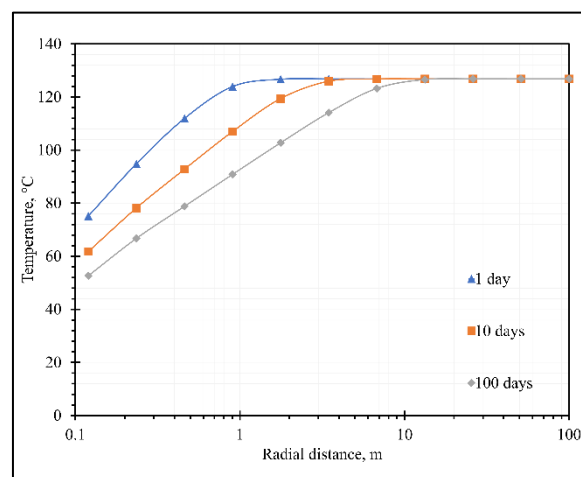


Figure 9: Reservoir temperature at the bottom after multiple days of operation.

4. HYPOTHETICAL GREENHOUSE CASE APPLICATION

We adopted the numerical FDM approach to study the hypothetical case of heating a greenhouse, see the Appendix for details. We follow the methodology suggested in the literature (Thain et al., 2006). Our study assumes the greenhouse is in the New Plymouth area. We consider parameters for our greenhouse including area, size, volume, materials, etc. similar to the Rotorua reference case study available in the literature (Zuquim & Zarrouk, 2021). Note that except the depth of well (4.300 m – similar to New Plymouth-2 well), the wellbore parameters are consistent with Table 1 in Appendix A-1.

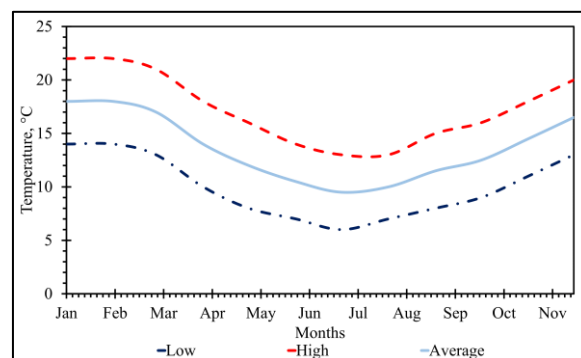


Figure 10: Outside air temperature around the New Plymouth region.

We consider the same area and volume of the greenhouse complex, i.e., 3,468.7 m² and 10,463.7 m³, respectively. Figure 10 shows the annual temperature distribution around

the New Plymouth region. We use this information to calculate the heat loss from the greenhouse to the outside air. It also establishes the required thermal energy to efficiently run the greenhouse complex.

The Rotorua case study uses secondary heating inside the complex because an open system supplies geothermal heat. In contrast, only a primary heating layout is sufficient when using BHE to produce geothermal heat. This is because BHE systems are highly optimisable in terms of operations. Figure 11 presents the required annual thermal load for the greenhouse operation and the available thermal energy after optimised operation of the BHE system. We conclude that for a greenhouse with typical dimensions of 25 m × 6.5 m × 3.8 m, a fluid flowing at less than 2.5 kg/s is sufficient to supply the required thermal load for a crop with growing temperature of 24 °C. Mass flow rate must be varied to keep the heat supply constant. A detailed economic and operational optimisation study may be carried out in the future.

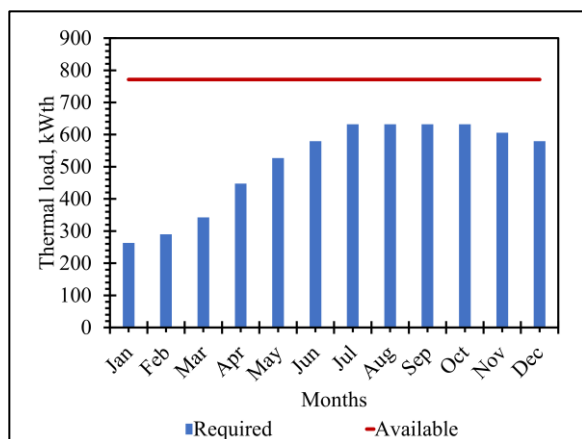


Figure 11: Required vs available thermal load for the greenhouse complex.

5. CONCLUSION

Borehole Heat Exchangers offer a low-carbon and low operational cost option to extract heat for direct uses. We apply the Method of Characteristics to evaluate heat exchanger systems in deep petroleum wells. Our analysis concludes MoC can only be used to evaluate such systems for a limited number of cases. In this paper, we presented two theoretical cases with assumed simplifications, i.e., homogeneous, and non-homogeneous formulations of the mathematical model. Since the non-homogeneous formulation is closer to the practical application, it would be concluded to be less favourable to decouple the system components and perform an MoC analysis. The challenges could be addressed in the future with a more rigorous analytical model considering multi-borehole installations. Overall, we conclude:

1. Non-linear hyperbolic PDEs for a practical case of the BHE operation are difficult to solve using MoC. A few simple cases are easily handled using this analytical method.
2. The numerical finite difference approach addresses the transient short-term response of a BHE system reliably.
3. It is technically feasible to supply sufficient geothermal heat produced directly from a borehole heat exchanger system to real world applications.

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APPENDIX

A-1

The wellbore parameters are presented in the table below.

Table 1: Input parameters for wellbore heat exchanger analysis.

Parameters	Value	Units
Length of wellbore	3000	m
Injection temperature	10.35	°C
Reservoir extent	100	m
Reservoir temperature at the bottom	126.85	°C
Thermal conductivity of rock	1.8	W/mK
Thermal diffusivity of rock	1e-6	m ² /s
Geothermal gradient	0.035	°C/m
Inner tubing radius	0.03	m
Inner tubing thickness	0.005	m
Insulation thickness radius	0.04	m
Annulus inside radius	0.065	m
Annulus tubing thickness	0.005	m
Cement outside diameter	0.12	m
Thermal conductivity of steel	57	W/mK
Thermal conductivity of cement	2.3	W/mK
Thermal conductivity of insulation	0.023	W/mK

A-2

We select greenhouse case parameters similar to the ones considered in Rotorua greenhouse case study (Zuquim & Zarrouk, 2021).

$$T_{diff} = T_{grow} - T_{min} \quad A-1$$

We obtain the total required thermal load for the greenhouse complex as:

$$Q_{total\ thermal\ load} = Q_{loss,trans} - Q_{loss,venti} \quad A-2$$

where,

$Q_{total,thermal,load}$ = Total required thermal load, kW_{th}

$Q_{loss,trans}$ = Transmission heat losses, kW_{th}

$Q_{loss,venti}$ = Ventilation heat losses, kW_{th}

$$Q_{loss,trans} = \frac{A \times T_{diff} \times Q_{mat}}{3600} \quad A-3$$

And

$$Q_{loss,venti} = \frac{V \times T_{diff} \times U_{air}}{3600} \quad A-4$$

Where A is Area exposed to outside air, m², and V is internal greenhouse complex volume, m³. U_{air} is the overall heat transfer coefficient, W/m²K, and Q_{mat} is heat loss due to material for given conditions, W/m²K.

We assume only primary heating loop because BHE systems are customisable according to the application. Therefore, the required mass flow rate becomes

$$M_{geo} = \frac{Q_{total\ thermal\ load}}{h_{out} - h_{in}} \quad A-5$$

where M_{geo} is the required mass flow rate, kg/s, h_{out} is enthalpy of the fluid at the outlet, kJ/kg, and h_{in} is the enthalpy of the fluid at the inlet, kJ/kg.