INVESTIGATION OF PARAMETER UNCERTAINTY FOR AN IDEALIZED GEOTHERMAL MODEL USING LINEAR ANALYSIS

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Keywords: *Sensitivity analysis, parameter estimation, inverse modelling, uncertainty analysis, reservoir modelling, PEST, linear uncertainty analysis.*

ABSTRACT

The parameters of a geothermal model are inherently uncertain. Model parameters are usually estimated by calibrating the model against observations. However, the information contained in the data is typically insufficient for determining the parameters uniquely. Therefore, parameter uncertainty still remains after calibrating a model. The present study, considers linear analysis as a way of quantifying the uncertainty of common geothermal parameters. A synthetic 2D slice model was constructed as test a case for this study. We also considered observation types that are typically used during calibration.

Linear analysis can be used to estimate the posterior covariance matrix that contains the uncertainty information for the model parameters in a linear model. The main disadvantage of linear analysis is the fact that geothermal models are highly nonlinear. This was considered somewhat indirectly by looking at the variation in the model outputs with varying the parameters. The slopes of the relevant plots give the elements of the Jacobian matrix. This has enabled us determine the extent at which the model nonlinearity compromises the results obtained from linear analysis.

We used a quantity called "relative parameter uncertainty reduction" and the correlation coefficient between pairs of parameters as a measure of how well the parameters were constrained by the calibration dataset. Both of these are easily obtained from the posterior covariance matrix. The results obtained from this study suggest that there were parameters which were highly correlated with other parameters despite the simple rock-type distribution used in the model. By mapping out the values of the relative parameter uncertainty reduction, we were also able to infer areas of the model where the observations constrain the permeability and porosity values.

1. INTRODUCTION

Parameter estimation is an integral part of the standard work flow of geothermal model development. There is usually no standard method to measure the parameters directly hence modellers resort to model calibration to infer the parameters indirectly from observations. In the past, the process of geothermal model calibration has been generally treated as deterministic. However, geothermal model parameters are typically non-unique. Therefore, there is growing interest in using uncertainty quantification methods to improve the characterization of geothermal models.

Parameter uncertainty arises because the measured data used to calibrate the model are typically incomplete or are uninformative of one or more parameters in the model. For example, well test data from a well can provide information about the physical properties of the rocks in its immediate surrounding but is unlikely to be useful when estimating rock properties in areas far from the well. O'Sullivan (2001) also noted that natural state temperatures and production pressures can constrain permeability parameters but cannot constrain porosity parameters. Additionally, even when data are available, they are unavoidably subject to measurement noise thus providing some wiggle room for variability in the parameter value. It is possible to improve the measurement practice and obtain more types of data to be able to obtain a more unique estimate of the parameters however the cost and effort involved in obtaining them can make it impractical or economically unfeasible.

There were several well-researched methods developed for quantifying model uncertainty (see Floris *et al*., 2001, Oliver and Chen, 2011). Popular methods for quantifying uncertainty of numerical model parameters and predictions include rejection sampling, randomized maximum likelihood and Markov chain Monte Carlo. These methods give a collection of feasible parameter values (which describe the posterior parameter probability distribution) instead of a single parameter estimate (as would be obtained using the classical model calibration process). The distribution of the generated values for the parameters and/or predictions describes the uncertainty associated with them. The major issue of these methods is that they require running the model several times to properly sample the posterior parameter distribution and obtain accurate statistics. When model runs are long, the large amount of computational work required can limit the practicality of such methods.

Linear analysis provides an alternative method for estimating parameter and prediction uncertainties of a numerical model when the relationship between the model parameters and output is linear or near linear. Linear analysis is a derivative based uncertainty quantification technique that requires a minimal number of forward model runs. The derivative/sensitivity information is commonly obtained using a finite difference method which requires (at the minimum) as many runs as the number of parameters. The computational requirements can be reduced by using adjoint or direct sensitivity methods described by Bjarkason, *et al.* (2016a, 2016b) thereby making linear analysis more attractive.

The work presented in this paper investigates parameter uncertainty in an idealized geothermal model. The objective of this study is to obtain an insight into how well the permeability, porosity and upflow parameters are constrained when they are estimated using data typically used during calibration (i.e. natural state temperatures, production pressures and enthalpy trends). To achieve this, we applied linear uncertainty analysis on a synthetic 2D slice geothermal model. We are particularly interested in how much the parameter uncertainty decreases from the prior (or pre-calibration) estimate when subjected to calibration constraint. Additionally, we are also interested in determining if correlation exists (either inherently or as a result of the limited amount of measured data) between the parameters being estimated.

2. LINEAR UNCERTAINTY ANALYSIS

Press, *et al.* (1996), Finsterle (1999), Tarantola (2005), Oliver, *et al.* (2008) and Doherty (2015) provide detailed discussions on the theory of linear analysis for uncertainty quantification. The method has been used in different applications to assess the predictive uncertainty of numerical models. For example, Dausman et al. (2010), Tiedeman *et al.* (2004) and Finsterle (2015) demonstrated the applicability of linear analysis for evaluating the worth of data and how such data worth information can be used as basis for optimizing data acquisition. Lepine *et al.* (1998) and McVay *et al.* (2005) used linear analysis to estimate the 90% confidence interval of the prediction made by their respective models of oil reservoirs. Omagbon, *et al.* (2015) used the estimated posterior parameter distribution obtained through linear analysis to generate several samples of near calibrated models of a synthetic geothermal system to estimate the potential variability of the model prediction under different production scenarios.

The work presented here focuses on quantifying the identifiability of parameters in a model. A more popular way of achieving this is through direct sensitivity analysis. For example, the work presented by Omagbon and O'Sullivan (2011) and Moon *et al.* (2014) used sensitivity coefficients to determine which parameters are constrainable by the available measured data and used them as basis for selecting parameters that are to be adjusted in the calibration stage. Finsterle (2015), however, pointed out that using sensitivity analysis to quantify parameter identifiability may provide misleading results because they do not account for parameter correlations and redundancies in the observation data.

Doherty and Hunt (2009) used linear analysis and described parameter identifiability in terms of uncertainty reduction from the pre-calibration estimate to the post-calibration value. They suggested the quantities which they refer to as "parameter identifiability" and "relative parameter uncertainty reduction" as two statistics that quantify the capability of a model calibration to constrain the model parameters in place of the sensitivity coefficient. In addition, they demonstrated the use of such quantities in a groundwater flow simulation with 18 parameters. Using those quantities, they were able to identify five parameters which were highly constrained by the calibration dataset. A similar assessment was also carried out by Knowling and Werner (2016) to determine the identifiability of the recharge and hydraulic parameters of a groundwater model.

Linear analysis uses the linear error propagation equation to estimate parameter uncertainty. A linear model given by:

$$\vec{\boldsymbol{h}} = A\vec{\boldsymbol{k}} \tag{1}$$

has an uncertainty described by the covariance matrix:

$$C_h = AC_kA^T \tag{2}$$

In (1), $\vec{\boldsymbol{h}}$ and $\vec{\boldsymbol{k}}$ are normally distributed random vectors while A is the linear transformation matrix. C_h and C_k in equation (2) are the covariance matrices of vectors $\vec{\boldsymbol{h}}$ and $\vec{\boldsymbol{k}}$, respectively. A numerical model may be calibrated using derivative based minimization algorithm with objective function given by:

$$\theta = \left(\vec{\boldsymbol{y}}_{obs} - \vec{\boldsymbol{y}}_{guess}\right)^T C_{y_{obs}}^{-1}\left(\vec{\boldsymbol{y}}_{obs} - \vec{\boldsymbol{y}}_{guess}\right) + \left(\vec{\boldsymbol{x}}_{prior} - \vec{\boldsymbol{x}}_{guess}\right)^T C_{x_{prior}}^{-1}\left(\vec{\boldsymbol{x}}_{prior} - \vec{\boldsymbol{x}}_{guess}\right) \tag{3}$$

where θ is the objective function evaluated at parameters $\vec{\boldsymbol{x}}_{guess}$, $\vec{\boldsymbol{y}}_{obs}$is the measured data, $\vec{\boldsymbol{y}}_{guess}$ is the simulated data corresponding to $\vec{\boldsymbol{y}}_{obs}$, $\vec{\boldsymbol{x}}_{prior}$ is the preferred parameter value (or statistical mean of the prior parameter distribution), $C_{y_{obs}}$ is the covariance matrix of the measurement noise and $C_{x_{prior}}$ is the prior parameter covariance matrix. For this case, the value of the parameter $\vec{\boldsymbol{x}}$ that minimizes (3) is given by (see Tarantola, 2005):

$$\vec{\boldsymbol{x}} - \vec{\boldsymbol{x}}_{guess} = \left(J^T C_{y_{obs}}^{-1} J + C_{x_{prior}}^{-1}\right)^{-1} J^T C_{y_{obs}}^{-1}\left(\vec{\boldsymbol{y}}_{obs} - \vec{\boldsymbol{y}}_{guess}\right) \tag{4}$$

In (4), J is the Jacobian or sensitivity matrix evaluated at $\vec{\boldsymbol{x}} = \vec{\boldsymbol{x}}_{guess}$. This is of the same form as equation (1) where $\vec{\boldsymbol{h}} = \vec{\boldsymbol{x}} - \vec{\boldsymbol{x}}_{guess}$, $A = \left(J^T C_{y_{obs}}^{-1} J + C_{x_{prior}}^{-1}\right)^{-1} J^T C_{y_{obs}}^{-1}$ and $\vec{\boldsymbol{k}} = \vec{\boldsymbol{y}}_{obs} - \vec{\boldsymbol{y}}_{guess}$. The post calibration parameter covariance matrix is therefore obtained from equation (2) which simplifies to:

$$C_x = \left(J^T C_{y_{obs}}^{-1} J + C_{x_{prior}}^{-1}\right)^{-1} \tag{5}$$

The matrix C_x contains the standard deviations of the parameters (the square root of the diagonal elements) and the correlation information (from the normalized off-diagonal elements) between pairing parameters.

We used the "relative uncertainty variance reduction" as suggested by Doherty and Hunt (2009) to quantify the ability (or otherwise) of the calibration process to constrain the values of the adjustable parameters. This quantity is calculated using the equation:

$$f_i = \frac{\left[C_{x_{prior}}\right]_{i,i} - \left[C_x\right]_{i,i}}{\left[C_{x_{prior}}\right]_{i,i}} \tag{6}$$

f_i takes values between 0.0 and 1.0, with a value of 0.0 indicating a complete absence of any information in the calibration dataset for that parameter. In contrast, a value approaching 1.0 indicates that the information content of the calibration dataset is such as to almost eliminate the post-calibration uncertainty of the pertinent parameter.

3. WORKING MODEL

In the present work, we used a simple idealized model of a geothermal system based on the model used by Bjarkason, *et al.* (2016a,b). The model uses a 2-dimensional vertical slice grid system with a dimension of 2.0 km (x-direction) x 20m (y-direction) x 1.6 km (z-direction). The grid has a total of 8000 blocks with 20m x 20m x 20m dimension. The boundary conditions include the following:

1. constant input (0.1 kg/s) of hot (1500 kJ/kg) fluid in the upflow location

2. constant heat flux (80mW/m^2) on the blocks at the bottom-most layer
3. constant temperature (15 ^{o}C) and pressure (1 bar) condition at the top-most layer
4. closed side boundaries

There were 5 wells defined in the present model (see Figure 1); three of which were production wells (wells PRO1, PRO2 and PRO3) and the rest were injection wells (INJ1 and INJ2). Synthetic data was generated to serve as constraint for the calibration. This includes natural state downhole temperature for all the wells and transient pressure and enthalpy for the production wells. For the production history simulation, the model was set to run for 10 years. During this period, the production wells PRO1, PRO2 and PRO3 are producing at constant rates of 0.5 kg/s, 0.5 kg/s and 0.4 kg/s, respectively. The two injection wells were each accepting 0.56 kg/s of water flow at 40 ^{o}C. The model used the six rock-types described in Table 1 to represent the permeability and porosity distribution in the true field. The distribution of these rock-types is shown in Figure 1.

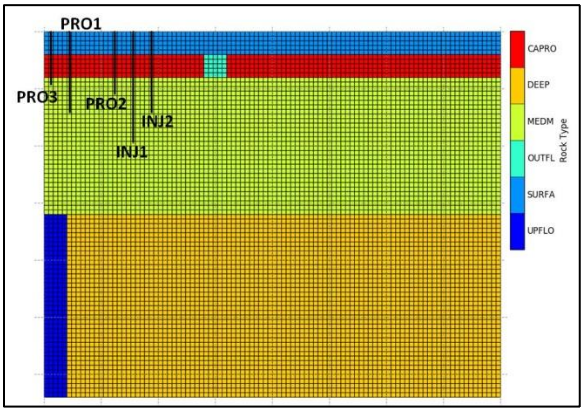

Figure 1. Model setup and well locations.

Table 1. Rock-types with corresponding permeability and porosity values for the true system.

Rock-type	Description	kx (mD)	kz (mD)	porosity (%)
SURFA	Surface above caprock	50.00	10.00	0.10
CAPRO	Caprock	0.50	0.10	0.10
OUTFL	Spring/outflow location	1.00	5.00	0.10
MEDM	Productive reservoir layer	70.00	7.00	0.10
DEEP	Basement rock	0.50	0.50	0.10
UPFLO	Upflow area below the reservoir	0.10	10.50	0.10

The adjustable parameters that were of interest for this study include the vertical and horizontal permeability of all the rock-types, rock porosity, mass flow rate and enthalpy in the upflow blocks and heat flux. In actual application, uncertainty analysis should be preceded by a calibration step. We have chosen to set the parameters to their correct values. This is to eliminate the need to carry out an actual calibration.

4. TEST FOR NONLINEARITY

Linear analysis relies heavily on the assumption that the model with which it is applied to is linear and is dependent on the Jacobian matrix. Since geothermal models are known to be nonlinear, the Jacobian matrix is therefore dependent on the parameter values used when this matrix is evaluated.

We first investigated how the sensitivity of the common observations varied with the parameters. To achieve this, we run the model by varying one parameter at a time over a wide range of parameter values and noting how some of the important simulated data changed in response to that change in parameter value. The data that we chose to focus on were the natural state temperature and pressure at the bottom of each of the production wells and the transient pressure and enthalpy values of the production wells after 5 years of operation. We also limited the parameters to look at to the permeability and porosity of the production layer, upflow mass flow and upflow enthalpy.

Figure 2 - 4 show how the abovementioned simulated quantities varied with varying parameter values. The nonlinearity of the model was manifested by the change in the slope on each curve. It is clear from the plots that the simulated observations do not vary linearly with the parameters. It is important to note that the horizontal axis of each plot is log scaled. The nonlinear behaviour gets worse when the parameters are not log transformed although the results for that case were not shown here.

In Figure 2, the natural state data (both temperature and pressure) and transient pressure are near linear with respect to the permeability parameters (horizontal and vertical) except for the seemingly anomalous trend at extremely low permeability values. The enthalpy observation appears linear for high permeability values but the gradient show rapid change at low permeability values. This drastic change in slope corresponds to the point where boiling starts.

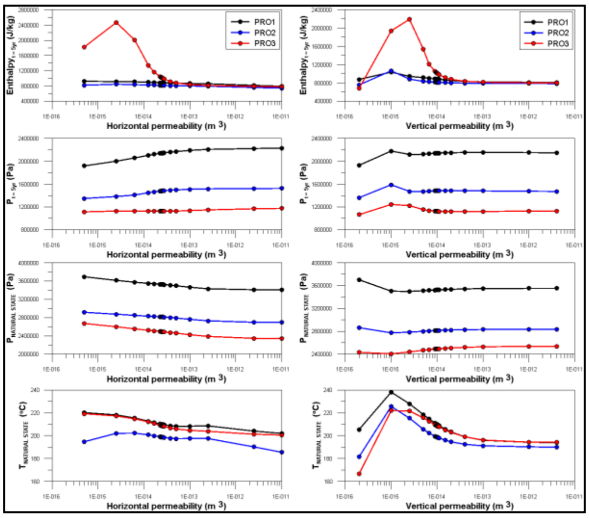

Figure 2. Variation of the natural state (temperature and pressure) and transient (pressure and enthalpy) data in the production wells with varying horizontal and vertical permeability.

This same behaviour is also seen in Figure 3 where the slope of the enthalpy curves with respect to the upflow parameters suddenly change (and even reverse sign) at high upflow mass and enthalpy values. Such change again corresponds to the point when the boiling happens. Boiling also seems to affect the slope of the natural state data. With boiling, the curve for the natural state temperature flattens out suggesting a reduction in sensitivity with respect to the upflow parameters. The opposite happens with the transient pressure wherein the slope of the curve increases at high enthalpy.

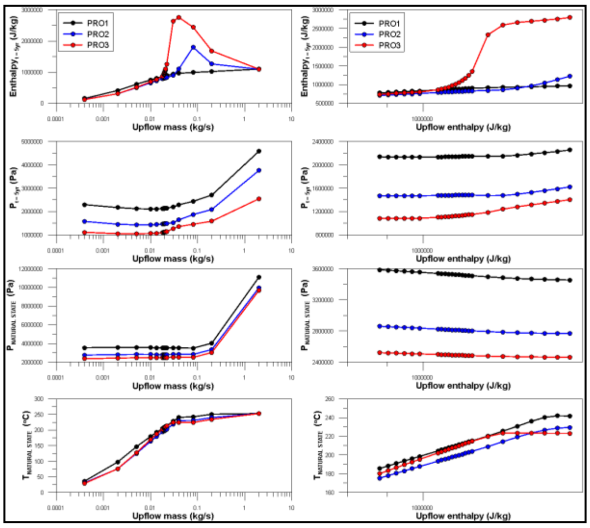

Figure 3. Variation of the natural state (temperature and pressure) and transient (pressure and enthalpy) data in the production wells with varying upflow mass flow and enthalpy.

Figure 4 shows that the natural state temperature and pressure are unresponsive to the changes in the porosity. The same can be said with the production history pressure data. Production enthalpy is also generally insensitive with the porosity value except for well PRO3 where the well is slightly boiling.

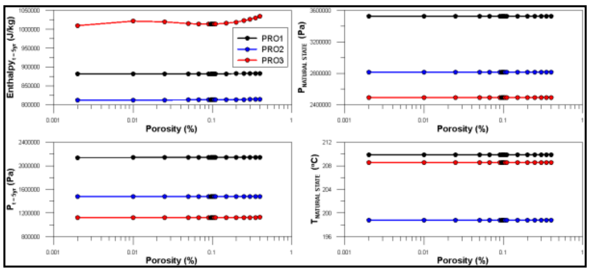

Figure 4. Variation of the natural state (temperature and pressure) and transient (pressure and enthalpy) data in the production wells with varying porosity.

The results of this exercise suggest a few important points when using linear uncertainty analysis in a geothermal model. First, it is better to use the log transform of the parameters (for both linear analysis and gradient based automated calibration techniques) because it makes the model response more linearly related with respect to the parameters. Second, even when the parameters were log-transformed, the model was still nonlinear therefore the results when applying the equation (5) must be treated as local characterization of the parameter uncertainty. Lastly, there is a huge difference in the gradient values when the model remains single phase as compared to when the model is boiling.

Doherty and Hunt (2009) argued that linear analysis is applicable even in cases where the observation data or the parameter values are unknown suggesting that linear analysis can be applied in a pre-calibration settings. Admittedly, performing the analysis in this manner can potentially provide valuable information to guide the modeller during the calibration process. We however recommend caution when using it in making major decisions regarding field operations or data gathering activities especially if the model was still not properly calibrated. For example, using an uncalibrated model that shows non-boiling response to assess a field that is highly two-phase will possibly have misleading results.

5. PARAMETER UNCERTAINTY

This section summarizes the results when equations (5) and (6) were applied to assess the post-calibration parameter uncertainty. As mentioned earlier, there was no actual model calibration done in this investigation. Instead, the parameters were set to their correct values to eliminate the need for calibration (i.e. the model is assumed to already be well calibrated). On the basis of the results in Section 4, we looked at 2 different models. In the first model, the upflow enthalpy was set to a low value (1100 kJ/kg) to maintain liquid enthalpy discharge for the production wells. The second model pertains to the model described in Section 3 where the wells are flowing with two-phase enthalpy. For each of the two models, we also looked into the case in which the production history data was missing (i.e. only natural state data was used) during the calibration and the case in which the production history data was available.

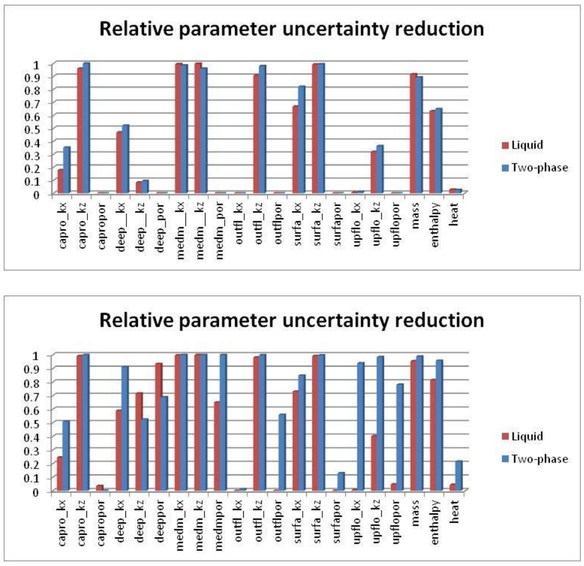

Figure 5. Relative parameter uncertainty reduction bar chart for the different model parameters. The figure on top corresponds to the case where only the natural state temperature data is available while the figure in the bottom corresponds to the case where transient pressure and enthalpy are included.

The finite difference method in PEST (Doherty, 1994) was used to evaluate the Jacobian matrix. A diagonal matrix (i.e. off-diagonal elements set to zero) was used to serve as the covariance matrix of the measurement noise ($C_{y_{obs}}$) and parameter prior ($C_{x_{prior}}$). The standard deviation for the natural state temperature, transient pressure and transient enthalpy data (square root of the diagonal elements of $C_{y_{obs}}$) were set to 1.5 °C, 0.5 bar and 15 kJ/kg, respectively. The prior standard deviation (square root of the diagonal elements of $C_{x_{prior}}$) for the log of permeability, porosity, upflow mass flow, upflow enthalpy and heat flux were set to 1.00, 0.60, 0.20, 0.05 and 0.30, respectively.

Figure 5 shows the calculated relative parameter uncertainty reduction for all of the parameters defined in the model using equation (6). The cases in which the well discharge remains liquid and the discharge turned two-phase were investigated. We also looked at how the parameter uncertainty reduction differs if only the natural state temperatures are used in the calibration against the case wherein transient pressure and enthalpy data are included as calibration constraint. The following are the major observations from the figure:

1. There is a general increase in "relative parameter uncertainty reduction" when transient data was included as calibration data.
2. The vertical permeability of the caprock and outflow rock-types was more constrained than its horizontal permeability.
3. The production layer rock-type permeability (medm) was highly constrained.
4. The natural data was not able constrain the rock porosity.
5. The values of the caprock and surface rock porosity are still poorly constrained even in the presence of production history calibration data.
6. There are parameters that have almost zero reduction in standard deviation when the well discharge remains liquid but became well constrained when the wells are flowing at two-phase enthalpy. The most notable of these are the outflow rock porosity, upflow rock porosity and upflow rock horizontal permeability.
7. The heat flux parameter has low relative parameter uncertainty reduction. The value slightly improved when the wells are flowing at two-phase enthalpy but still low compared to other parameters.

6. CORRELATION BETWEEN PARAMETERS

This section takes a look at the off-diagonal elements of the posterior parameter covariance matrix given by Equation (5). These off-diagonal elements signify the degree of correlation between two pairing parameters. In model calibration context, parameter correlation arises because there is insufficient data to enable unique estimation of the parameter values. For example, if two parameters are positively correlated, then the value of both parameters can be increased by an appropriate amount to produce the same response. If they are negatively correlated, one parameter can be increase while other can be reduced to get the same simulation outcome.

The most direct indicator of parameter correlation is the correlation coefficient which is obtained by dividing the each elements of C_x with the standard deviations of the pairing parameters (i.e. $corr_{ij} = \frac{C_{x_{ij}}}{\sqrt{C_{x_{ii}} * C_{x_{jj}}}}$). This process converts the covariance matrix into a correlation matrix. Correlation coefficient has an absolute values ranging from 0.0 (uncorrelated) to 1.0 (strongly correlated).

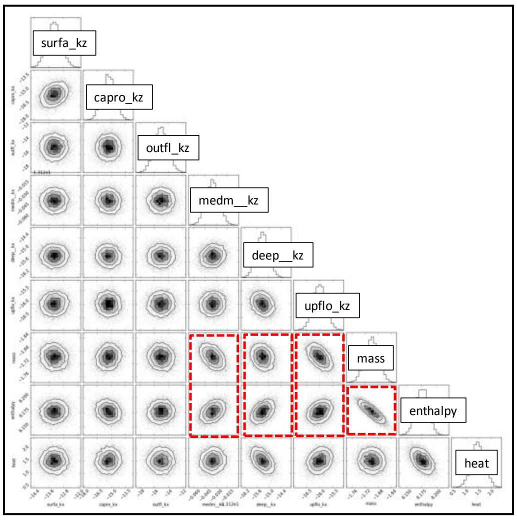

Figure 6. Correlation structure between the horizontal permeability and upflow parameters.

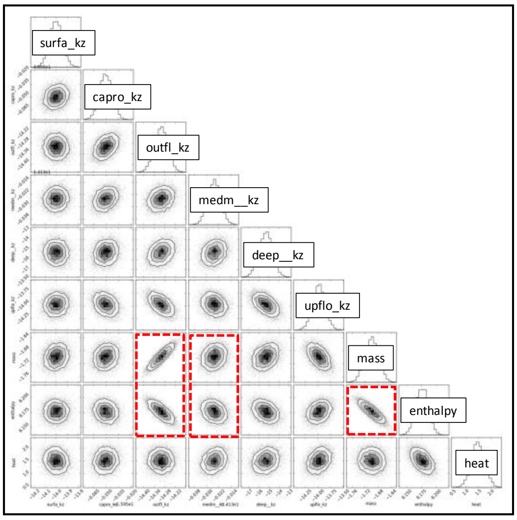

Figure 7. Correlation structure between the vertical permeability and upflow parameters.

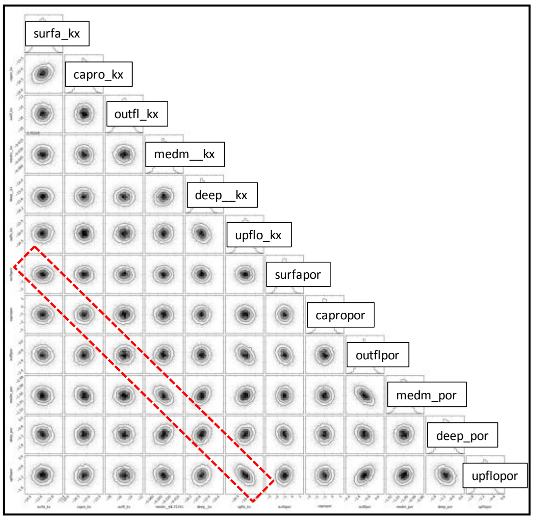

Figure 8. Correlation structure between the horizontal permeability and porosity parameters.

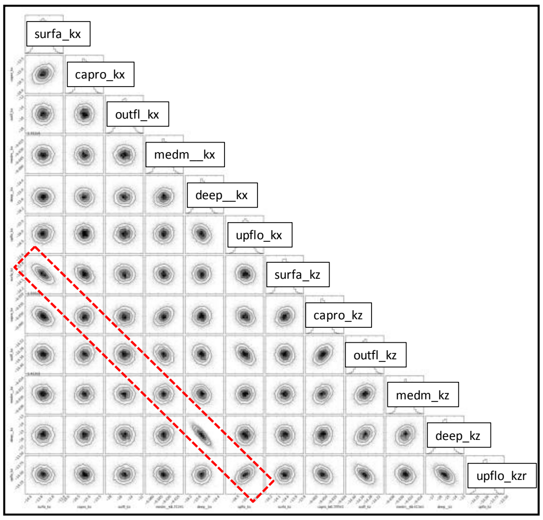

Figure 9. Correlation structure between the horizontal and vertical permeability parameters.

Figure 6-9 show the visual equivalent of the correlation matrix (in the form of a corner plot) for the parameters in the working model. These corner plots correspond to the model in which the both natural state and production history data are included as calibration data with two-phase discharge from the wells.

It is important to note that Figure 6-9 do not show the full structure of the correlation matrix for all the parameters in the model. More importantly, the correlation structure is expected to be dependent on how the model parameters were defined (e.g. how the rock-types were distributed). Therefore some of the results may not necessarily agree with what would be obtained when used with other models. Nonetheless, there were a few features in the resulting correlation matrix that we believe might be true for most models or are worth mentioning:

1. There is a strong negative correlation between the upflow mass flow and enthalpy (see Figure 6/Figure 7). The upflow enthalpy is also correlated with the heat flux at the bottom of the model.
2. The upflow mass flow and enthalpy is strongly correlated with the vertical permeability of the outflow rock-type (Figure 7). They also appear to be slightly correlated with the production layer's horizontal and vertical permeability (see Figure 6 and Figure 7). The same correlation (although weak) is seen with the upflow and basement rock-type permeabilities (see Figure 6 and Figure 7).
3. Figure 8 indicates that there is a negative correlation between the horizontal permeability and porosity of the production reservoir and upflow rock-types.
4. The horizontal and vertical permeabilities of the production layer rock-type appear to be independent of each other. However, strong negative correlation is seen between the surface and basement rock-types' vertical and horizontal permeabilities (see Figure 9).

7. PARAMETER UNCERTAINTY AS A FUNCTION OF DISTANCE FROM THE WELLS

Considering that there were only a few observation wells in the model, it is reasonable to expect that the permeability and porosity values of the blocks far away from the wells were poorly constrained. In this section, we look at the spatial variation of the relative parameter uncertainty reduction in the permeability and porosity parameters. To achieve this, we redefined the rock-type distribution in the model by increasing the number of rock-types from 6 to 360.

A separate rock-type is assigned to every 5x5 grid blocks thus allowing the perturbation of the permeability and porosity in smaller regions during the finite difference computation of the Jacobian matrix. Despite using more rock-types, the permeability and porosity values for the blocks were maintained at their original values so that the model response is still the same as those obtained from the previous sections. The results presented here also correspond to the model where the wells were producing at two-phase enthalpy with both the natural state and production history data included.

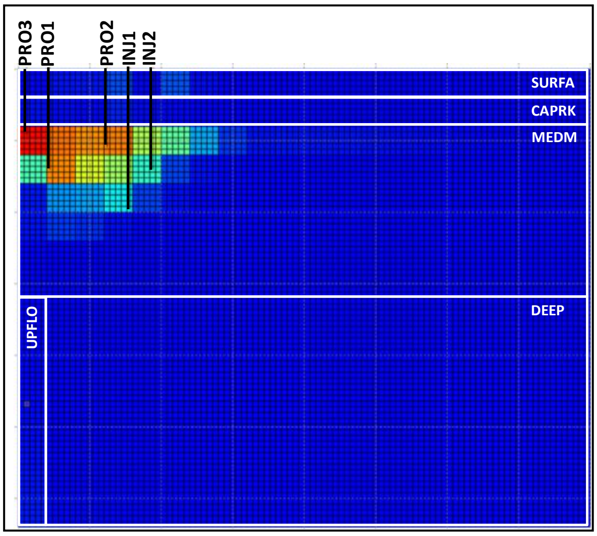

Figure 10. Relative parameter uncertainty reduction plot for the horizontal permeability using the model with 320 rock-types.

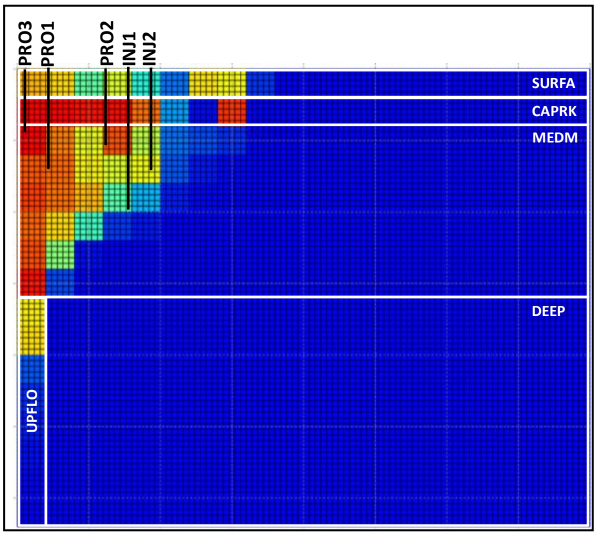

Figure 11. Relative parameter uncertainty reduction plot for the vertical permeability using the model with 320 rock-types.

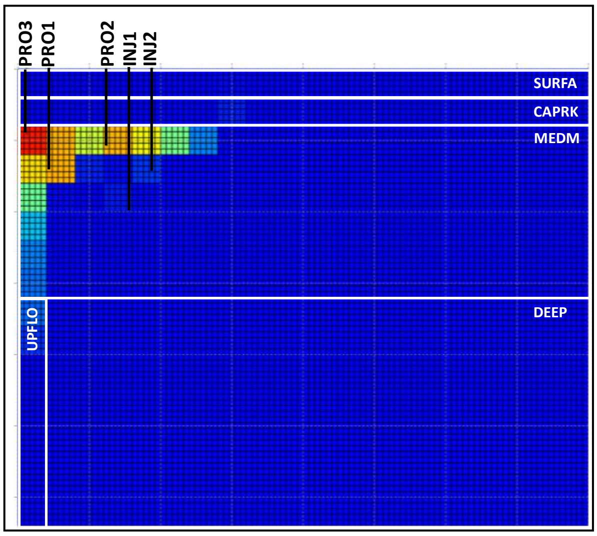

Figure 12. Relative parameter uncertainty reduction plot for the porosity using the model with 320 rock-types.

The results of this exercise are shown in Figure 10-12. In these figures, the blue colour corresponds to zero or very low relative parameter uncertainty reduction while the red colour corresponds to values close to 1.0. Thus, the blue regions in the model are areas in which the observations have little to no information about their permeability or porosity values.

Figure 10, suggests that we are only able infer the horizontal permeabilities of the blocks that are about 100m to 300m away from the wells. The plot also suggests that there was no information available to uniquely estimate the values of the horizontal permeability of the caprock layer. The non-blue region in Figure 11 is bigger than in Figure 10 implying that the extent at which the natural state and transient data informs about the vertical permeability is wider than that of the horizontal permeability. In addition, Figure 11 also shows a significant area of the caprock and surface layers that have relatively high uncertainty reduction. The non-blue region in Figure 12 is mostly confined to the shallow region of the production layer possibly coinciding with boiling zone in the reservoir.

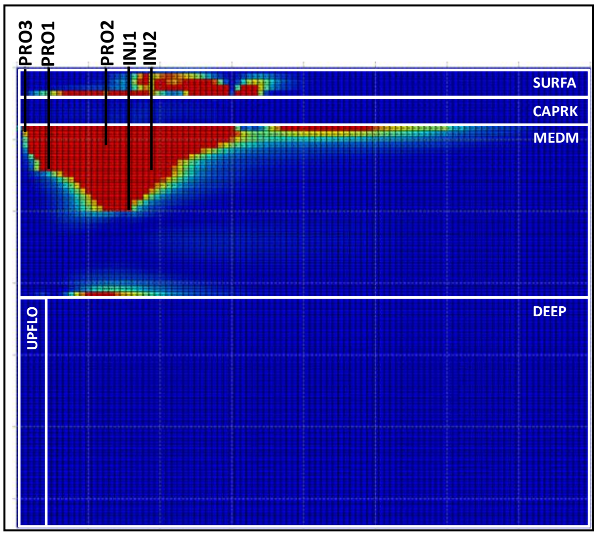

Figure 13. Relative parameter uncertainty reduction plot for the horizontal permeability of each blocks in the model.

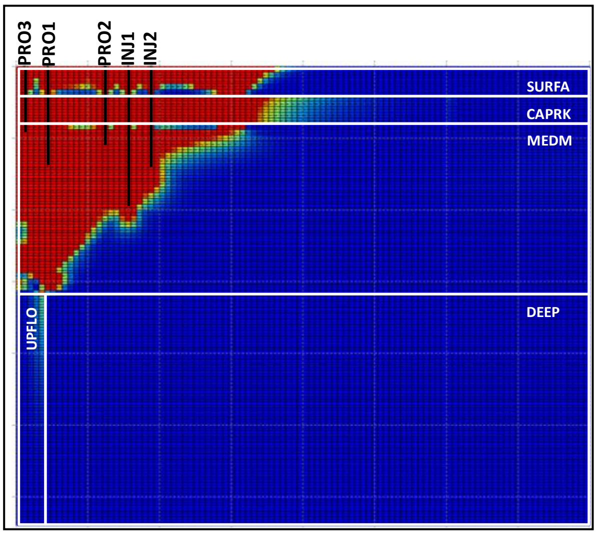

Figure 14. Relative parameter uncertainty reduction plot for the vertical permeability of each blocks in the model.

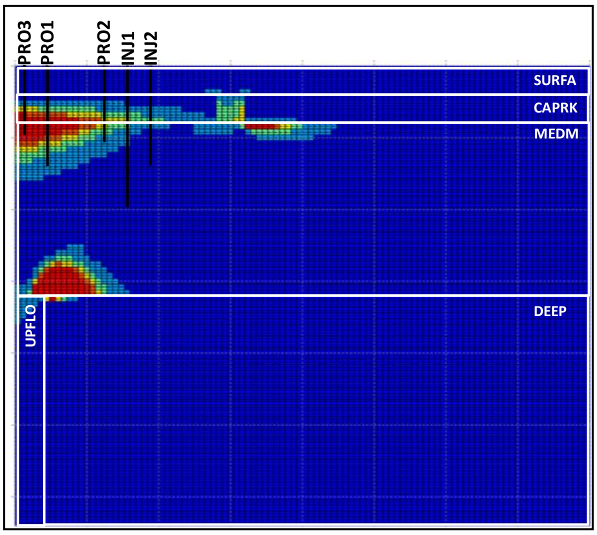

Figure 15. Relative parameter uncertainty reduction plot for the porosity of each blocks in the model.

We repeated the analysis on a model in which each block has its own rock-types thus treating the permeability and porosity of each block as adjustable parameters. This setup results in 24000 adjustable parameters. We calculated the Jacobian matrix for this model using the adjoint method described by Bjarkason, *et al.* (2016b).

The calculated relative parameter uncertainty reduction of the individual blocks is shown in Figure 13-15. The result are generally similar to those in Figure 10-12. The main difference however is that the values here are very small (only less than 0.1). These small values imply that uniquely estimating the individual block permeabilities and porosities is impossible. Calibrating the model with this parameterization scheme would require supplementing the calibration data with soft data to regularize contraints.

8. SUMMARY AND CONCLUSION

There is much to be gained in undertaking linear uncertainty analysis in conjunction with real world geothermal models. When properly implemented, lessons learned through such exploration may aid modellers during the model development and calibration. The work presented here demonstrated how linear analysis can be used to assess the uncertainty associated with the parameter values that came out from the calibration process. This has been carried out using an assumed to be well calibrated synthetic model but the procedure can be repeated using any calibrated model of a real geothermal reservoir.

An important thing to note when using linear analysis is the nonlinearity of geothermal models. Log-transforming the parameters seem to reduce the degree of nonlinearity. But even with that, it was shown with this simple example that the sensitivity of the most commonly used data for model calibration with respect to the permeability, porosity and upflow parameters changes depending on the value of the parameters. It was seen that there was a significant difference with the derivative between a model with liquid discharge and a model two-phase discharge from the wells. Therefore, modelers need to be extra cautious when using linear analysis on an uncalibrated model.

For the idealized model in Sections 5 and 6, most of the parameters appears to be well constrained by the calibration data. The natural state temperature data was able to constrain most of the permeability and upflow parameters. The relative parameter uncertainty reduction generally increases (as to be expected) when production history data was used in calibrating the model. Data from fields with two-phase discharge also appear to contain more information about the permeability, porosity and upflow parameters than those coming from field with single phase liquid discharge. We were also able to demonstrate that parameter correlation is present even with this very simple parameterization scheme.

Lastly, we were able to locate the regions in the model where the uncertainty in the permeability and porosity values has been reduced. As expected, the results that we obtained suggests that the data that we have chosen for calibration is only able to inform about the permeability and porosity parameters in blocks that are not too far from the wells. By using linear analysis, the region which is poorly constrained by the calibration data has been clearly delineated. This makes it easier to infer whether or not a certain forecast from the model would be reliable. Prediction in areas where the relative parameter uncertainty reductions are low needs to be given lower confidence levels and this needs to be properly communicated to the decision makers that rely on the model results.

ACKNOWLEDGEMENTS

The authors would like to express their appreciation to the financial support from Energy Development Corporation, the New Zealand Ministry of Business, Innovation and Employment, and the Energy Education Trust of New Zealand.

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