FUTURE PERFORMANCE PREDICTIONS BY LUMPED PARAMETER MODELS: A FIELD APPLICATION

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ABSTRACT

This paper deals with the application of lumped parameter models (1-tank, 2-tank open/closed, 3-tank open/closed) to a field case. The models are used to match the long-term observed pressure behavior of Hofsstadir geothermal field which is a typical low-temperature liquid dominated geothermal system, in West Iceland. Once the parameters of the models are determined by history matching, future performance predictions are made under given production/reinjection scenarios by using the Randomized Maximum Likelihood method.

1. INTRODUCTION

Managing geothermal fields efficiently requires reliable prediction of geothermal reservoir potential. This can only be achieved by using the appropriate reservoir model which describes the change in reservoir pressure (or water level) as a function of time or cumulative fluid production.

Simple analytical models as well as complex numerical models can be used to simulate geothermal system potential. A simpler approach is known as lumped parameter modeling. Lumped parameter models provide an attractive alternative to numerical modeling of geothermal reservoirs with fewer modeling parameters. Therefore, in this study lumped parameter models have been discussed to simulate reservoir behavior.

In the lumped models considered in this work, the geothermal system is assumed to be composed of mainly three parts; the reservoir, the aquifer, and the recharge source, which are represented by different tanks having different properties. The models are used to match the long-term observed water level or pressure response of a field to a given production history. For history matching purposes, an optimization algorithm based on the Levenberg-Marquardt method is used to minimize an objective function based on weighted least-squares, to estimate relevant aquifer/reservoir parameters. In addition, the parameters are constrained during the nonlinear minimization process to keep them within physically meaningful limits and compute statistics (e.g., standard 95% confidence intervals) to assess uncertainty in the estimated parameters. Moreover, the quality of the matches are evaluated through the Root Mean Square errors (RMS).

Once the parameters of the model are determined by history matching, the future performance of the reservoir is predicted for different production/reinjection scenarios to optimize the management of a given low-temperature geothermal field.

To provide sustainable production, it is very important to reflect the uncertainties that arises from errors (i.e. modeling and measurement errors, etc.) to the future predictions. Hence instead of dealing with a single deterministic response, one can analyze various possible outcomes of the future predictions. Therefore, in this study the Randomized Maximum Likelihood (RML) is used for predicting the uncertainty in future flow behavior predicted by lumped parameter models.

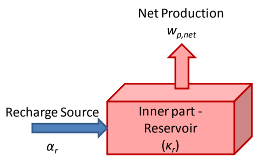
2. LUMPED PARAMETER MODELING

Lumped parameter models which have been reported in the literature (Whiting and Ramey, 1969; Castanier, Sanyal, and Brigham, 1980; Brigham and Ramey, 1981; Grant, 1983; Gudmundsson and Olsen, 1987; Axelsson, 1989; Sarak et al., 2003a and 2003b; Axellson et al., 2005; Sarak et al., 2005) have been used extensively for predicting pressure (or water level) changes in low-temperature geothermal systems in Iceland, Turkey, The Philippines, China, Mexico, and other countries.

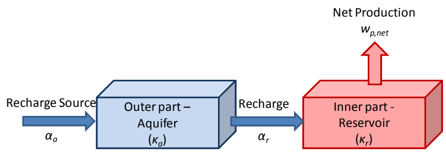
Generally, in all lumped parameter models, a geothermal system is represented by only a few homogeneous tanks and is visualized as consisting of mainly three parts: (1) the central part of the geothermal system-reservoir; (2) outer parts of the geothermal system-aquifer, and (3) the recharge source. The first two are treated as series of homogeneous tanks with average properties. The recharge (or constant pressure) source can be connected to the other parts of the reservoir or directly to the central part of the reservoir and is treated as a "point source" that recharges the system. If there is no connection to the recharge source, the model would be closed, otherwise would be open. Three different open lumped-parameter models are depicted in Figure 1.

In one-tank open lumped parameter model sketched in Fig.1 (a), geothermal system is considered to be composed of a reservoir and a recharge source. The reservoir is produced at a net mass rate ($w_{p,net}$) which is defined as the difference between production and reinjection rate, and the recharge source at a constant pressure of p_i supplies water.

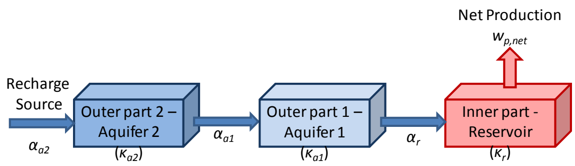
The model shown in Fig. 1(b) represents a two-tank open lumped parameter model, where the first tank, in which production/reinjection occurs, represents the innermost part of the geothermal system (reservoir). The changes in pressure in this part are monitored and production/reinjection rates are recorded. In the second tank, representing the outer part of the system (aquifer) that is connected to the recharge source, there is neither production nor reinjection and it recharges the reservoir. Fluid production causes the pressure in the reservoir to decline, which results in water influx from the outer (aquifer) to the inner part of the system (reservoir). The recharge source represents the outermost part of the geothermal system.



(a) one-tank open lumped parameter model



(b) two-tank open lumped parameter model



(c) three-tank open lumped parameter model

Figure 1: Illustration of three different lumped parameter models.

In the three-tank open model (Fig. 1-c) the innermost part of the system is considered as a single reservoir tank and the outermost part of the system is considered as two interconnected aquifer tanks. The outer aquifer is connected to a recharge source at a constant pressure of p_i. Thus the system is called open three-tank model.

When using the lumped parameter models considered in this work (Fig. 1), the simulated model (output) response represents pressure or water level changes for an observation well for a given net production history (input). The number of model parameters increases as the number of tanks or the complexity of the lumped model increases.

The lumped parameter models considered here are based on the conservation of mass only and hence are valid for low-temperature liquid reservoirs under the assumption that variations in temperature within the system can be neglected (i.e. the simulated systems are assumed to be isothermal).

Here and throughout, α represents the recharge constant between the tanks in kg/(bar-s), κ the storage capacity (or coefficient) of a tank in kg/bar, and p_i the initial pressure of the recharge source in bar. The geothermal system is assumed to be in hydrodynamic equilibrium initially; i.e., the initial pressure, p_i, is uniform in the system. In cases for which the initial system pressure (or initial water level), p_i, is known, p_i can be eliminated from the unknown set of model parameter vector.

Further details about the lumped-parameter models used in this study can be found in Sarak et al. (2003a, 2003b, and 2005).

3. PARAMETER ESTIMATION

After a geothermal reservoir has been produced for a period of time, a lumped parameter model can be matched to observed pressure (or water level) data with the available production and reinjection rate history to obtain optimum parameters of a particular lumped model. As more data become available, more information can be obtained about the reservoir and the system.

Fitting model parameters to the observed data requires accurate and fast approaches. The method of least squares fitting is a convenient one to apply. As is well known, the traditional (unweighted) least squares estimation is often unsatisfactory when some observations are less reliable than others and/or various measurements having disparate orders of magnitude are simultaneously used in estimation. In the former case, it is required that the parameter estimates will be more influenced by the more reliable observations than by the less reliable ones. In the latter case, it would like to be sure that any information contained in the data with small magnitudes is not lost because of summing together squares of numbers of such disparate orders of magnitude.

Therefore, in this work, weighted least-squares fitting is considered so that the above mentioned disadvantages associated with the standard least squares fitting can be overcome. The weighted least-squares objective function (Eq. 1) is used for the parameter estimation.

$$O(\mathbf{m}) = \sum_{j=1}^{N_d} \left[\frac{\mathbf{y}_{obs,j} - \mathbf{f}_i(\mathbf{m})}{\sigma_{d,j}} \right]^2 \quad (1)$$

Here, **y** refers to the vector of measured or observed pressure change data, and contains all N_d pressure change measurements that will be used for estimating the model parameters by nonlinear regression. Here **f** refers to the N_d-dimensional vector of computed pressure-change data from a considered lumped model, for a given **m** . **m** represents the total number of unknown model parameters. $\sigma_{d,j}$ represents the error variance for each observed data.

The lumped-parameter model responses are nonlinear with respect to the model parameters. Thus, Eq. 1 calls for nonlinear minimization techniques. The Levenberg-Marquardt method which is a gradient based algorithm is used to minimize objective function (Eq.1).

The details of optimization algorithm used in this study are given by Tureyen *et al*. (2010).

When history matching problem is viewed, one can attach statistical measures to quantify the quality of a match as well as the uncertainty of the model parameters estimated. The standard statistical measures used for assessing the quality of a match and the reliability of estimated parameters are the root-mean-square error (RMS) and confidence (usually 95% percent) intervals. The value of RMS defined by Eq. 2 shows the quality of fit quantitatively.

$$RMS = \sqrt{\frac{1}{N_d} \sum_{j=1}^{N_d} \left[\mathbf{y}_{obs,j} - \mathbf{f}_j\left(\mathbf{m}^*\right) \right]^2} \quad (2)$$

where $\mathbf{m}^*$ represents the optimized parameter vector. The lower the RMS value, the better the fit between field and computed data. This does not necessarily mean that the lumped model giving the smallest RMS value be the most appropriate model for the history-matched data and should give the most reliable predictions. While it is important to improve the overall match of available data, it is equally or even more important that the history-matched model be able to predict reliably the uncertainty (from a statistical point of view) in predictions due to the fact that a certain amount of error (i.e., modeling and measurement errors, etc) will always be introduced into the estimated parameters from the history-matching process.

Statistical confidence intervals are known as a useful tool to give a quantitative evaluation of model discrimination and assessment of uncertainty in the estimated parameters. In general, the larger the confidence interval, the higher the uncertainty in the estimated model parameters.

There is a relationship between the confidence intervals and the RMS. This relationship could be complicated in the models having large number of model parameters and when the parameters show correlation among them. One may expect the uncertainty as reflected by the confidence intervals for some parameter estimates to increase with the increasing complexity of a model, while the value of RMS improves. However, as long as the lumped model selected is appropriate and there are sufficient observed data available to support the model, all parameters should have "acceptable" confidence interval ranges and the RMS value should be close to the standard deviation of measurement errors in observed pressure data. Then, one can accept the model. Otherwise, one rejects the model because confidence intervals do not support the model from a statistical point of view. In short, the best fitting lumped model is the one providing not only the smallest possible acceptable confidence intervals for all parameters but also the smallest possible RMS value among the lumped models used for history matching (Onur and Tureyen, 2006).

4. PREDICTION OF FUTURE PERFORMANCE

The ultimate goal in any geothermal reservoir study is to predict future performance and even more important to predict the uncertainty in future predictions under different management options. This is necessary to determine the production/reinjection practices that will provide sustainable exploitation of the geothermal system in consideration. Uncertainty in all future predictions of pressure changes is inherent due to (i) measurement errors or noise in observed data, (ii) modeling errors, (iii) span of the available observed data (pressure change data and production history), and (iv) nonlinear relationship between model parameters and observed response.

It is very important to propagate these uncertainties (mentioned above) on the future performance predictions to provide sustainable production of geothermal energy from the system. In this study, to incorporate uncertainties both in the model and observed data to future performance predictions the Randomized Likelihood Method (RML) is performed.

This method has been shown to be quite efficient for the assessment of uncertainty in performance predictions for nonlinear problems. Onur and Tureyen (2006) have shown its detailed application regarding lumped parameter modeling on synthetic examples.

5. FIELD APPLICATION:HOFSSTADIR FIELD

The Hofsstadir Geothermal Field, located in Iceland discussed in this paper is a typical liquid-dominated low-temperature field. The hot water from Hofsstadir geothermal field is mainly used for district heating system.

Two main feed zones are determined from results of cutting analysis and well logs, one is located at depth of 819m and other is at about 171-175m. The water temperature is between 86-87^{o}C.

In 1996, a production well, HO-01, was drilled to a depth of 855 m. The average yearly production of the single production well (HO-01) since that time has been of the order of 18 kg/s. Since a continuous water level drawdown has been observed in the field, the reinjection was started in early 2007 by injecting the return water from heating system into reinjection well HO-02.

The Hofsstadir geothermal field is discussed in literature by Gaoxuan (2008) and more recently by Axelsson *et al.* (2010). Axelsson (2010) used this field data to simulate the pressure response of the field and to estimate its production capacity.

5.1 Available data of the field

A continuous water level record (about 11 years) was available from HO-01. In addition to this, the production rate was also monitored. Unfortunately, the reinjection rate was not monitored properly during the first four months of reinjection started. Reinjection is started on 22.04.2007 and reinjection flow rate have been recorded since 29.08.2007.

All observed data available for this field application is obtained from Gaoxuan (2008) and Axelsson *et al.* (2010). For the period of 19.03.1997 - 14.12.2007, the production and reinjection rate history of the field and observed water level history of well HO-01 are presented in Figures 2 and 3, respectively.

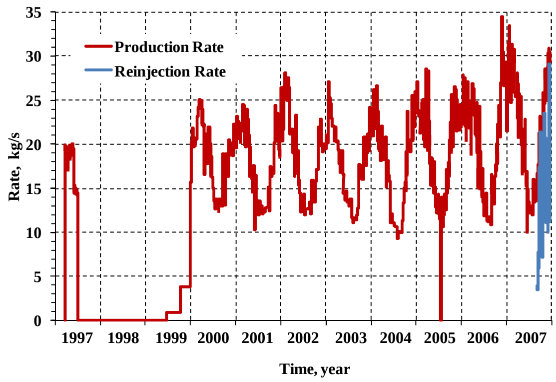


Figure 2: Production and reinjection rate history of Hofsstadir Geothermal Field.

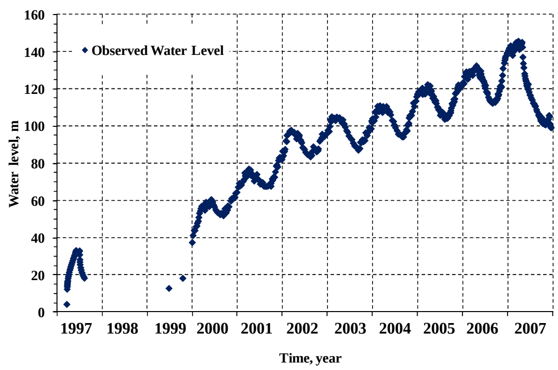


Figure 3: Observed water level in well HO-01.

For the field application discussed below, all observed data obtained from literature were given in terms of water levels. Since lumped parameter models used in this study are derived in terms of pressure, all the observed water level data first converted to pressure equivalence and then used in regression algorithm. Thus, all parameter estimates are given in pressure units. However, all graphical results are presented in terms of water levels to be consistent with the published field data.

5.2 Modeling of Hofsstadir Geothermal Field by Lumped Parameter Models

In modeling studies, one-, two- and three-tank lumped parameter models are performed and results are compared with Axelsson *et al.* (2010) results. The RMS value for the match and 95% confidence intervals for the model parameters are used to determine the best appropriate model. Firstly, the flow rate and water level data until reinjection started (period of 19.03.1997-22.04.2007) are used in modeling studies. Then whole data (period of 19.03.1997-14.12.2007) are used with some assumptions for the first four months of reinjection data that are missing.

5.2.1 Modeling results for the period of 19.03.1997-22.04.2007

The modeling results of one-tank, two-tank open/closed, and three-tank open/closed models for the period of 19.03.1997-22.04.2007 are shown in Figures 4, 5, and 6, respectively.

Axelsson *et al.* (2010) described 3-tank closed model as a pessimistic model. Therefore, Axelsson's match for 3-tank closed model is compared with our 3-tank closed model result. Axelsson's match and our match look almost identical (Figure 6).

Next, nonlinear regression analysis based on 1-, 2- and 3-tank models are performed to estimate the model parameters. The best fit was obtained with the parameters given in Table 1. Here and throughout, the numbers given in parentheses represent the 95% confidence interval for the relevant parameters.

A comparison of the results given in Table 1 and Figs. 4, 5 and 6 indicates that the RMS value for 1-tank model (0.921 bar) is the highest for all the models tried. A higher RMS value is the result of a greater deviation between the model and observed water level data. Therefore, 1-tank model is rejected because its RMS value is larger than that of other models tried, although it has acceptable confidence intervals for the parameters.

Based on the definition an estimate of a parameter is acceptable if its confidence interval range is less than 95% of the estimated value itself, all the confidence intervals computed for all the parameters for 2-tank open/closed and 3-tank open/closed models are acceptable (Table 1). Therefore, the RMS values will probably be the discriminating measure for the best model that represents the actual system. Since the 3-tank open model has the smallest RMS value (0.292 bar), it can be stated that 3-tank open model is the best appropriate model to represent the Hofsstadir geothermal field.

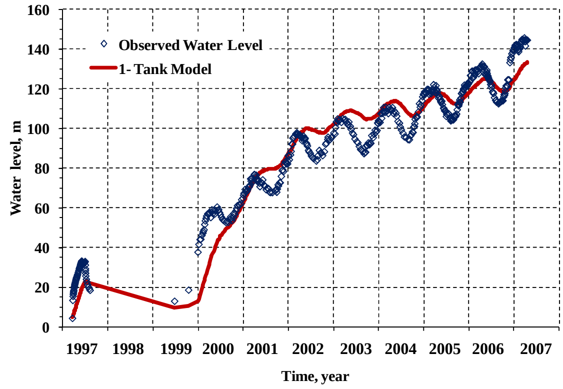


Figure 4: Simulation result of 1-tank open model.

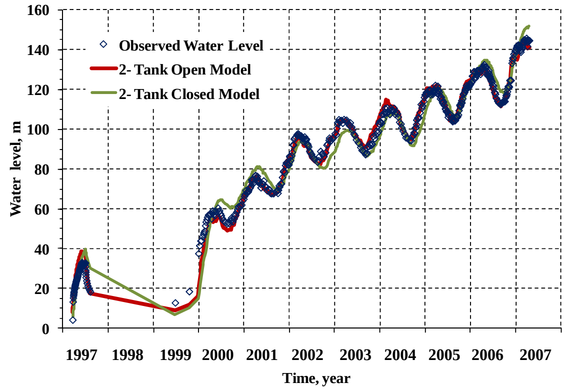


Figure 5: Simulation results of 2-tank models.

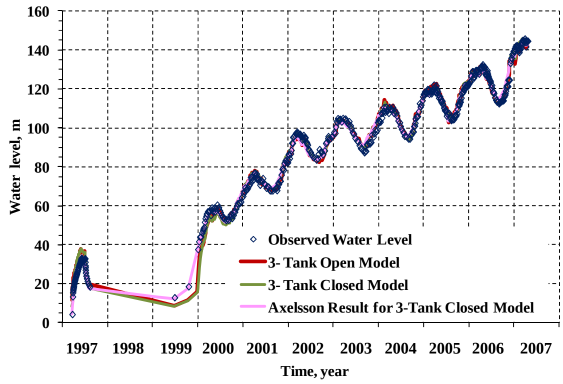


Figure 6: Simulation results of 3-tank models and Axelsson result for 3-tank closed model.

5.2.2 Modeling results for the period of 19.03.1997-14.12.2007

As it is mentioned above, the reinjection rate was not monitored properly during the first four months of reinjection started (22.04.2007-29.08.2007). Therefore, some modeling studies are performed to figure out the missing reinjection data. As a result of these modeling studies, the best match between the observed and simulated water level data is obtained for the cases of;

Case 1: reinjection rate is fixed at 7.74 kg/s (≈ 8l/s),
Case 2: reinjection is varied as 40% of the production rate,

during the period of reinjection data (22.04.2007-29.08.2007, Figure 7). The 3-tank open model is performed for the history matching purpose. The best parameters obtained are given in Table 2 and the history matching is plotted in Figure 8.

Table 1: Parameters of the best fitting lumped parameters (19.03.1997-22.04.2007).

	1-Tank	2-Tank Closed	2-Tank Open	3-Tank Closed	3-Tank Open
α_{a2}, kg/bar-s	--	--	--	--	2.279 (±0.107)
κ_{a2}, kg/bar	--	--	--	1.02x10^{9} (±1.42x10^{8})	2.10x10^{8} (±1.93x10^{7})
α_{a1} (or α_a), kg/bar-s	--	--	2.029 (±0.034)	2.767 (±0.167)	6.683 (±0.812)
κ_{a1} (or κ_a), kg/bar	--	6.45x10^{8} (±2.41x10^{7})	1.89x10^{8} (±7.60x10^{6})	1.42x10^{8} (±1.10x10^{7})	5.19x10^{7} (±1.31x10^{7})
α_r, kg/bar-s	1.676 (±0.025)	3.137 (±0.092)	6.047 (±0.215)	6.764 (±0.316)	10.496 (±1.825)
κ_r, kg/bar	8.43x10^{7} (±4.28x10^{6})	3.17x10^{7} (±1.85x10^{6})	1.36x10^{7} (±1.10x10^{6})	1.19x10^{7} (±1.18x10^{6})	6.34x10^{6} (±1.85x10^{6})
RMS, bar	0.921	0.564	0.315	0.297	0.292

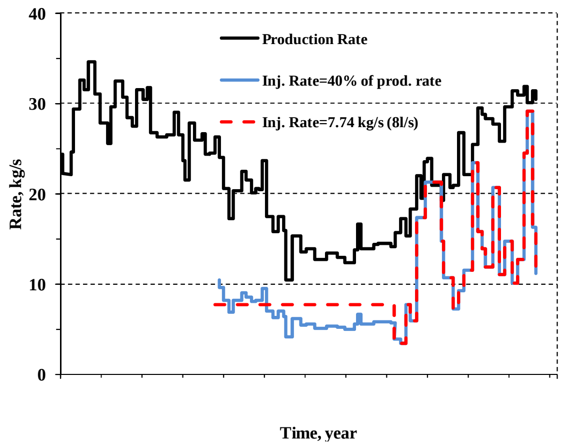


Figure 7: Rate history used in modeling.

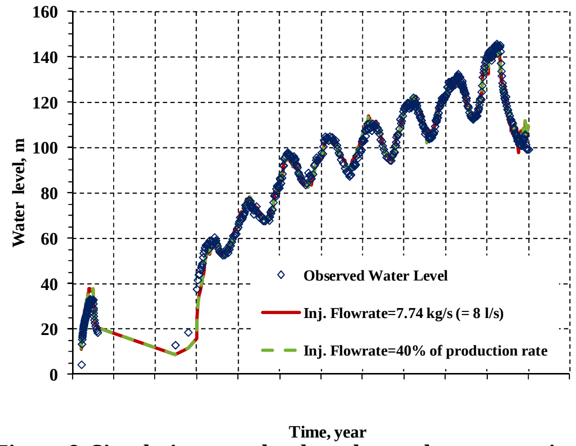


Figure 8: Simulation results based on the assumption that the reinjection rate is fixed at a constant value and the reinjection is varied as proportional to production rate.

Table 2: Parameters of the best fitting lumped parameters (19.03.1997-14.12.2007).

	3-Tank Open	
	Case 1	Case 2
α_{a2}, kg/bar-s	2.313 (±0.101)	2.286 (±0.009)
κ_{a2}, kg/bar	2.12x10^{8} (±1.82x10^{7})	2.07x10^{8} (±1.55x10^{7})
α_{a1} (or α_a), kg/bar-s	6.518 (±0.691)	6.809 (±0.707)
κ_{a1} (or κ_a), kg/bar	4.79x10^{7} (±1.08x10^{7})	3.99x10^{7} (±1.02x10^{7})
α_r, kg/bar-s	10.856 (±1.899)	11.866 (±2.629)
κ_r, kg/bar	6.61x10^{6} (±1.79x10^{6})	6.07x10^{6} (±1.93x10^{6})
RMS, bar	0.300	0.305

Since, the smallest RMS value is obtained in Case 1, it can be stated that the best match is obtained in the case of reinjection fixed at a constant value of 7.74 kg/s (Table 2) for the period of missing reinjection.

5.3 Future Performance Predictions of Hofsstadir Geothermal Field by RML

Future performances of the Hofsstadir geothermal field are predicted by using 3-tank open model for the next 30 years. Three different production/reinjection scenarios are generated to predict the future performance:

(I) Production rate is maintained as 19.34 kg/s (20 l/s) for the next 30 years without reinjection (Figure 9).

(II) Production rate is maintained as 19.34 kg/s (20 l/s) for the next 30 years and 45% of production is injected in summer and 70% in winter period (Figure 10).

(III) Production and reinjection rate used in Scenario II are increased by 5% each years for the next 30 years (Figure 11).

The RML method has been performed on the production/reinjection rate data given in scenarios described above by using the 3-tank open model. For each model 100 realizations are generated for each scenario and the results are compared.

The realizations of the observed data is obtained by adding random noise from a $N(0,0.085264)$ distribution. Once the history matching is completed, future predictions are performed with the optimal model parameters for rate history given in Figs 9-11.

Figures 12, 13 and 14 illustrate the history matching period and the future 30 year predictions for 3-tank open model for three different scenarios given above.

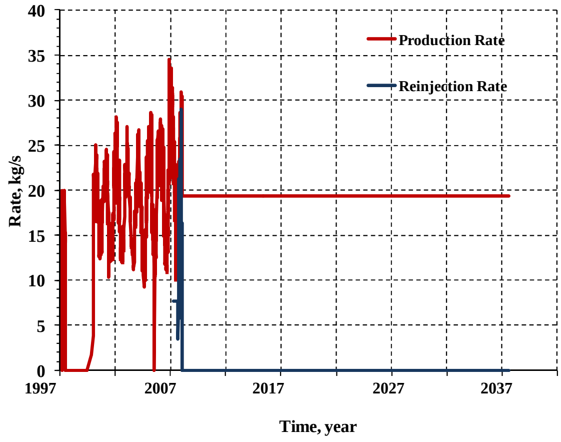


Figure 9: Rate history for Scenario I.

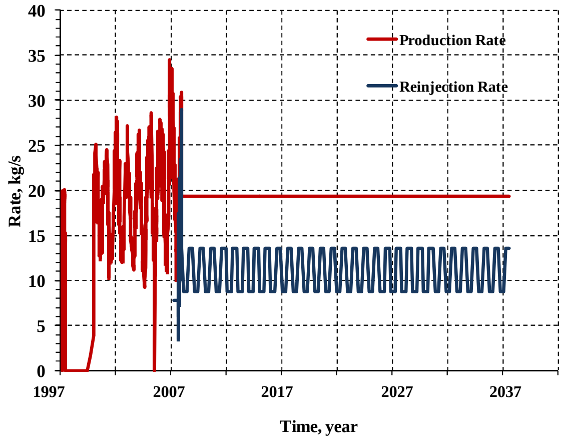


Figure 10: Rate history for Scenario II.

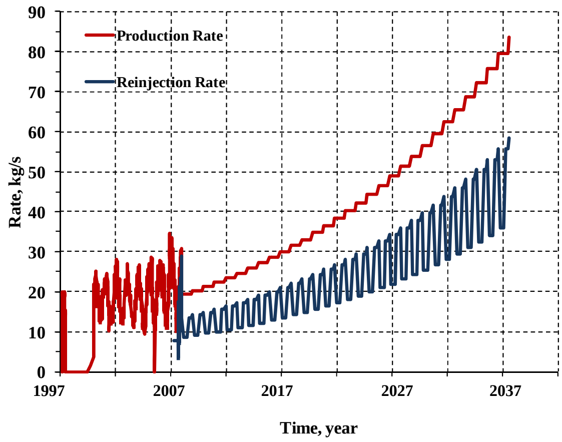


Figure 11: Rate history for Scenario III.

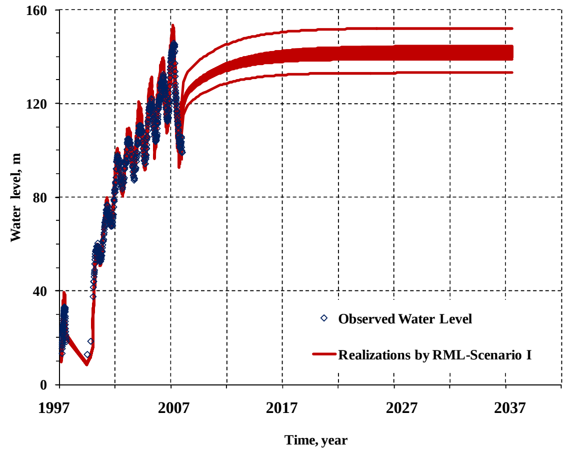


Figure 12: Realizations of predicted water level generated by RML for Scenario I.

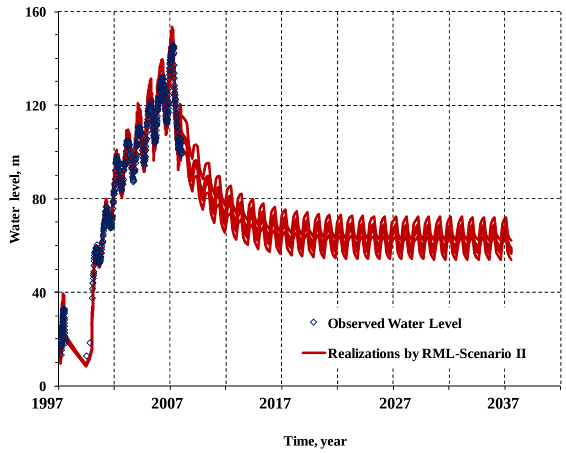


Figure 13: Realizations of predicted water level generated by RML for Scenario II.

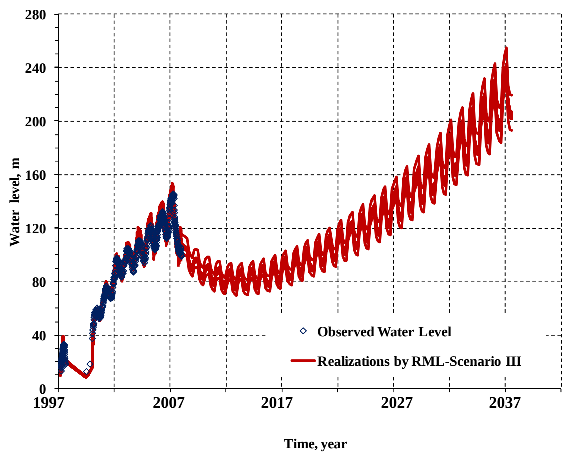


Figure 14: Realizations of predicted water level generated by RML for Scenario III.

In the case of Scenario I (production rate is maintained as 19.34 kg/s without reinjection), the water level is predicted in between 130m and 150m (Figure 12) at the end of 2037.

For Scenario II (production rate is maintained as 19.34 kg/s and reinjection rate is 45% of production in summer and 70% in winter) , the water level is predicted in between 55m and 70 m (Figure 13).

In the case of that production/reinjection rates are increased by 5% in each year (Scenario III), the water level drops continuously for next 30 years. The maximum drawdown in water level is predicted in between 195 m and 255 m (Figure 14).

6. CONCLUSIONS

On the basis of this study, the following specific conclusions can be stated:

(i) The use of the RMS value for the fit and the confidence intervals for all model parameters are the important statistical measures to discriminate the best appropriate model to represent the field behavior.

(ii) Due to its lowest RMS value the 3-tank open model seems to be the best appropriate model to represent the Hofsstadir geothermal field.

(iii) The RML method is quite efficient method for the assessment of uncertainty in performance predictions for nonlinear problems. The main objective of this paper was to demonstrate RML application to a field data. The RML method has successfully been implemented to the Hofsstadir geothermal field data.

(iv) The reinjection has a significant effect to maintain the pressure in the reservoir.

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