Force, flight and fallout: Progress on modelling hydrothermal eruptions

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Total number of pages (Excluding Cover Page) = 7

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FORCE, FLIGHT AND FALLOUT: PROGRESS ON MODELLING HYDROTHERMAL ERUPTIONS

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SUMMARY – McKibbin (1989) began development of a mathematical model for hydrothermal eruptions. Early work concentrated on modelling the underground process, while lately some attempts have been made to model the eruption jet and the flight and deposit of ejected material. Conceptually, the model is that of a boiling and expanding two-phase fluid rising through porous rock near the ground surface, with a vertical high-speed jet, dominated volumetrically by the gas phase, ejecting rock particles that are then deposited on the ground near the eruption site. Reported field observations of eruptions in progress and experimental results from a laboratory-sized model have confirmed the conceptual model. The quantitative models for all parts of the process are based on the fundamental conservation equations of motion and thermodynamics, using a continuum approximation for each of the components. The three main zones of interest (underground flow, eruption jet and plume dispersion) may be connected to form an approximate, but complete, quantitative model of a hydrothermal eruption.

1. INTRODUCTION

Hydrothermal eruptions occur in many geothermal fields around the world. Besides the rare reported witnessing of an eruption actually taking place, most evidence is based upon breccia deposits on the surface or subsurface formations that record historical eruptions.

Hydrothermal eruptions are categorised as being natural or induced. Natural eruptions include prehistoric and unexploited-field eruptions. In New Zealand, prehistoric eruptions have occurred in many areas of the Taupo Volcanic Zone (TVZ). Waimangu has more recently been an area of interest for large natural eruptions, the latest occurrence being in 1973. Induced eruptions occur as a result of exploitation of geothermal fields. These have taken place in the Tauhara and "Craters of the Moon" areas of the Wairakei Geothermal Field (Bixley & Browne, 1988).

Such eruptions are violent and can last from a matter of minutes to several hours. Hydrothermal eruptions are distinct from geysers in that the former are non-cyclic in occurrence, take place without warning, and the ejected material is a slushy mixture of water and rock particles of all sizes. These are lifted from the ground by a high-speed jet of gas which is mostly steam, but which could contain non-condensible gases contained in the near-surface water. Most of the material is directed vertically upwards from vents that can be from 5 m to 500 m in diameter; it is then deposited nearby.

2. MODELLING HISTORY

The earliest attempt at modelling the mechanical processes of hydrothermal eruptions was made by McKibbin (1989). Previous speculation as to the initiating event had been based around the idea of a small seismic motion that disturbed latently-unstable conditions near the surface. The causes of the instability were supposed to be one of three types: the first was formation of a steam cap due to a drop in groundwater level with increased steam-flow to the surface; the second was hydraulic fracturing and brecciation allowing non-condensible gases to decrease water boiling pressures; the third was a reduction in lithostatic pressure caused by lifting of overburden material.

A discussion of these proposed mechanisms was given in McKibbin (1989); in particular, the type of eruptions classified as "hydrothermal" would preclude any large-scale "blast". The process is started near the surface; the greatest effect is when the fluid there is liquid, as that provides the largest potential for expansion and high fluid ejection speeds. A steam cap provides small expansion potential; that and the presence of non-condensible gases was explored in McKibbin (1996), where it was found that very large over-pressures of a gas such as CO_2 would be required to have any significant effect over that of pure water. Further, the potential of deeper overburden-lifting pore pressures to produce significant upward movement of large ground masses would be reduced by any escape of fluid which would immediately reduce the lifting force.

Conclusions from these and other considerations have led to the view that a hydrothermal eruption proceeds due to boiling of near-surface fluid. The expansion potential of such water can be expressed in terms of the "specific volume ratio" which compares the density of the fluid in the ground with that of the same mass when expanded to ambient atmospheric conditions (McKibbin, 1990; 1996). The greatest values are for liquid groundwater, while any steam fraction there considerably reduces the potential. The concept is based on mining by steam lifting, rather than by sudden explosions such as those caused by phreatomagmatic phenomena.

Observations of laboratory-scale eruptions were reported by Smith (2000) and Smith & McKibbin (2003); these provided qualitative confirmation of the conceptual model of eruptions beginning at the surface and progressing downwards into the matrix.

The lift caused by the motion of the boiling fluid escaping upwards to the atmosphere has to exceed the weight of rock particles at the ground surface as well as any cohesive forces that are binding them together. The latter were taken account of by McKibbin (1989, 1990) and Bercich & McKibbin (1992, 1993). An appropriate model for the movement of the boiling fluid was the subject of extensive investigation; this was reported by Smith & McKibbin (1997, 1998, 1999, 2000, 2003) and Smith (2000). The rapid process and high speed of the upwardly-moving fluid leads to the conclusion that there is little time for thermodynamic equilibration between the fluid and the rock matrix, or for dynamic separation of the two fluid phases. At most, the fluid state is controlled by the condition that the fluid is at saturated conditions and it flows as a two-phase boiling mixture, with negligible heat transfer between it and the solid matrix (i.e. adiabatic flow).

In the earliest modelling attempts, when the flow above the ground was less well-understood, it was supposed that conditions there were akin to a fluidized bed. Examination of reports of the (rare) field observations and photographs, as well as some of the volcanological literature, led to the concept that the rock particles were mere "passengers" within the rising jet of gas-dominated fluid, as indeed were the water droplets, and that the early model was incorrect. The rock particles and droplets form an extremely small volume fraction of the jet. While the water droplet mass may increase by condensation as the hot fluid mixture entrains cold air, the liquid phase still forms a very small volumetric fraction in the plume above the ground.

Investigations of the form of the jet based on considerations of these concepts were reported in Rynhart *et al.* (2000), McKibbin *et al.* (2005) and McKibbin & Smith (2006). The subsequent dispersion and deposition of the ejecta have been based on recently-developed models for particle transport by the atmosphere (McKibbin *et al.*, 2005; McKibbin, 2006). These models allow for agglomeration of wet particles, but do not allow specifically for evaporation of the water after release from the eruption jet. Models incorporating the latter phenomenon are under current investigation by the first author and other colleagues.

Quiescent conditions after an eruption has taken place were considered by Smith (2000), who modelled the subsequent groundwater flow near the event as the system "recovered" to a new pre-eruptive state. The almost incompressible liquid groundwater flow is controlled immediately by surrounding pressures, with conductive and/or convective heating following much more slowly.

This is a summary of the mathematical models used for the various parts of hydrothermal eruptions, using assumptions and simplifications as outlined above. Space precludes full mathematical descriptions of the various stages. The reader may find these together in McKibbin (2008) or severally in the references below.

3. BELOW GROUND

The main driving force of hydrothermal eruptions is supposed to be due to an increased pressure gradient immediately below the surface of the ground arising from the fluid near the surface being suddenly exposed to atmospheric conditions. In order for a particle of rock at the surface to be moved, the equilibrium of forces that govern the static particle rock matrix must be upset.

In summary, it is supposed that hot fluid, initially at rest in the rock matrix, begins to escape to the atmospheric conditions at the surface, flashing (boiling) and increasing in specific volume as the pressure decreases. The increasing speed of the two-phase fluid as it nears the surface provides lift to the rock particles. A hydrothermal eruption can take place when the fluid speed is great enough to eject the particles, i.e., the hydrodynamic lift is large enough to overcome the effective weight and cohesive stresses of the rock particles. The eroding surface and the flashing front in the formation are both considered to be moving boundaries.

Assumptions made in the model are as follows:

- the principles of conservation of mass, momentum and energy hold;
- the fluid below the flashing zone is motionless;
- the flowing fluid is a homogeneous mixture of liquid water and steam – it is assumed that, because boiling occurs very quickly, the two phases flow at the same speed;
- the moving fluid is at saturated (boiling) conditions;
- the process is quasi-steady, i.e., the eruption has already been initiated and is currently in progress, and is modelled as being, albeit for short time periods, nearly steady;

- the fluid lifts the rock particles at the surface, where conditions are atmospheric (p = 1 bar abs.);
- thermal conduction is ignored, i.e., there is negligible heat transfer between the fluid and rock in the flashing zone – the process is modelled as being adiabatic.

There are two moving boundaries to the flashing zone. The lower boundary, termed the "flashing front", propagates downward into the stationary fluid in the rock matrix at speed V. The boundary formed by the eroding ground surface above also propagates downward; its speed is V_{er}. The early models assumed that $V_{er} = V$ but this is not justifiable, nor is it necessary.

The flashing front moves downward at speed V into a fluid-filled medium of porosity ϕ and permeability k where the fluid has liquid saturation $S_{\ell d}$ (S_ℓ = volume fraction of liquid in the two-phase fluid), vapour saturation $S_{vd} = 1 - S_{\ell d}$, mixture density ρ_{fd}, and mixture specific enthalpy h_{fd}, all of which may vary with depth. The average fluid particle speed (pore-velocity) is V_f. Subcripts f, r, ℓ and v denote fluid, rock, liquid water and vapour (water gas) respectively. Parameter values pertaining to conditions at depth have a "d" subscript whilst those associated with conditions at the top (eroding) surface, where p = 1 bar abs., have a "t" subscript.

The technique is to refer quantities to a frame of reference that moves downward at the same speed V as the flashing front (see Figure 1). Because the motion is assumed to be quasi-steady, the frame of reference can be treated as inertial for the short periods of time considered. Within this frame, the flow is assumed steady.

Conservation of mass

The flashing front moves into a motionless fluid in the pores of the medium. The (constant) fluid mass flowrate per unit area m_f through this region and the flashing zone, relative to the moving frame of axes [see Figure 1(b)], is:

$$m_f = \phi\rho_f (V + V_f) \qquad (1)$$

but at the "flashing front", $V_f = 0$, and hence:

$$m_f = \phi\rho_f (V + V_f) = \phi\rho_{fd} V \qquad (2)$$

The fluid mixture density is given by fluid saturation-weighted combinations of the phase quantities.

Conservation of momentum

The fluid volume flowrate per unit area relative to the fixed frame of axes is given by using the simple Darcy's Law for the motion of the homogeneous two-phase fluid mixture:

$$w_f = \phi V_f = -\frac{k}{\mu_f}\left(\frac{dp}{dz} + \rho_f g\right) \qquad (3)$$

where the dynamic viscosity is here taken to be $\mu_f = \mu_\ell^{S_\ell}\mu_v^{S_v} = \mu_\ell^{S_\ell}\mu_v^{(1-S_\ell)}$, but other correlations are possible.

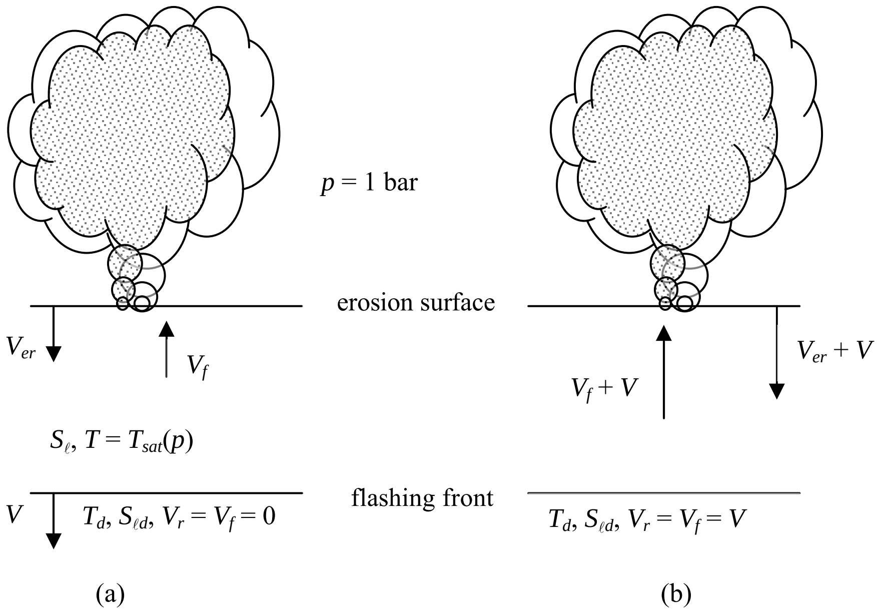

Figure 1. Schematic of flows: (a) relative to fixed axes; (b) relative to a frame of reference moving downwards with flashing front.

Assuming that the boiling fluid is moving as a homogeneous two-phase mixture, the corresponding liquid and vapour mass flowrates per unit area relative to the moving axes are $m_\ell = S_\ell \rho_\ell \phi (V + V_f)$, $m_v = (1 - S_\ell)\rho_v \phi (V + V_f)$ and the total mass flow per unit area is $m_f = m_\ell + m_v = \rho_f \phi (V + V_f)$. After use of Equation (2), Equation (3) can be rearranged to give an expression for the local pressure gradient:

$$\frac{dp}{dz} = -\rho_f g - \frac{\mu_f \phi V}{k}\left(\frac{\rho_{fd}}{\rho_f} - 1\right) \qquad (4)$$

From Equation (2) it can also be deduced, by applying the surface boundary conditions, that

$$V = \left(\frac{V_{ft}}{\frac{\rho_{fd}}{\rho_{ft}} - 1}\right) = \left(\frac{V_{ft}}{\frac{v_{ft}}{v_{fd}} - 1}\right) \qquad (5)$$

where $v = 1/\rho$ is the specific volume. Equation (5) relates the flashing front speed to the speed and density of the fluid as it reaches the surface. The overall volume expansion factor for the flashing fluid as it rises from the bottom to the surface is given by

$$v_{rat} = \left(\frac{v_{ft}}{v_{fd}}\right) = \left(\frac{\rho_{fd}}{\rho_{ft}}\right) \qquad (6)$$

Conservation of energy
The vertical energy flux per unit area associated with the fluid flow, relative to the fixed frame of axes, is given by $q_e = m_\ell h_\ell + m_v h_v = m_f h_f$, where

$$h_f = \frac{S_\ell \rho_\ell h_\ell + (1 - S_\ell)\rho_v h_v}{S_\ell \rho_\ell + (1 - S_\ell)\rho_v} \qquad (7)$$

is the so-called "flowing enthalpy" of the fluid. Boundary conditions at the flashing front give $q_e = m_f h_{fd}$. Because q_e is conserved and m_f is constant, $h_f = h_{fd}$ and the fluid flow is isenthalpic; the constant value of the specific enthalpy is given by that at the flashing front. Rearrangement of Equation (7) gives the liquid saturation in terms of the specific enthalpies and densities of the phases:

$$S_\ell = \frac{\rho_v (h_v - h_{fd})}{(\rho_\ell - \rho_v) h_{fd} + (\rho_v h_v - \rho_\ell h_\ell)} \qquad (8)$$

Equation of state
The fluid is assumed to be at saturated (boiling) conditions and the equation of state is:

$$p = p_{sat}(T) \qquad (9)$$

Lift condition
The condition that a rock particle is removed from the ground surface is related to the dynamic lift at the surface, given in terms of the relative speed of the fluid with respect to the rock (see McKibbin, 1989). The criterion that L_t, the dynamic lift there, exceeds the effective weight of the rock and cohesion (i.e. the net lift is positive) is given by

$$L_t > \phi(1 - \phi)(\rho_r - \rho_{ft}) g\, d + \text{cohesion} \qquad (10)$$

The most important parameter of interest is the fluid particle velocity at the ground surface. This is found by considering the lift condition (10). Given the temperature of the fluid at the flashing front, the bottom boundary conditions are determined using correlations for the pressure (at saturated conditions), densities, and enthalpies. The surface is assumed to be at atmospheric pressure. Equation (6) provides the volume expansion factor v_{rat} and the flashing front speed V may be found from Equation (5). Results (McKibbin, 1990) show that, for a reservoir temperature above 100 ˚C, the maximum fluid expansion occurs when $S_{\ell d} = 1$; the flashing front speed also appears smallest for $S_{\ell d} = 1$. One of the major benefits of this procedure was provision of an estimate of the thickness of the flashing zone. Some interesting qualitative information resulted for particular experimental cases. The reader is referred to McKibbin (1990) for these results.

Fluid properties
Correlations for the fluid thermodynamic properties for near sub-surface conditions can be found in Smith (2000). Since the temperatures of the fluid which is involved in an eruption are unlikely to exceed 150 ˚C, these are accurate enough for the modelling required here.

4. THE MOTION OF THE UPPER BOUNDARY

Bercich & McKibbin (1992, 1993) describe the process for finding the downward speed of the erosion surface. Given conditions at the flashing front, the pressure gradient equation (4) is integrated upwards until the lift criterion is satisfied.

In detail, given a value of V and a state point (T, $p_{sat}(T)$) in the flow, evaluation of the thermodynamic properties from correlations allows calculation of S_ℓ, μ_f and ρ_f and hence the local pressure gradient. Integration of Equation (4) from the flashing front upwards allows testing of the lift condition (10) at the point where $p = 1$ bar abs. (the ground surface). The value of flashing front speed V can be adjusted until the lift condition is exceeded; this will then give the flashing zone thickness.

A small increment in time allows the new position of the flashing front, and hence the thermodynamic conditions there, to be determined from the initial temperature and saturation profiles underground. The process is repeated; the positions of the flashing front and erosion surface are then found as functions of time.

The above analysis was initially based on a one-dimensional model, where the flashing and erosion surfaces were assumed to be horizontal planes. More recent work by Fullard (2007) has extended this to a 2-D configuration, where the eruption vent is assumed to be circular pit.

5. THE ERUPTION JET

A by-product of the above method is the time-dependent structure of the fluid stream that issues from the ground, as well as the volume erosion rate. The first of these may be used as surface boundary conditions for the fluid in the eruption jet. Given a particle size-distribution in the ground, the volume erosion rate allows the (uncoupled) problem of particle ejection to be calculated. This part of the process was formulated and described by McKibbin & Smith (2006) and McKibbin (2006).

The main features of this part of the model are the initial composition and speed of the emerging two-phase fluid stream and the entrainment of air into the jet. Conservation laws allow construction of a simple one-dimensional mathematical model for the jet, which is assumed to be approximately circular. It is also supposed that the pressure within the jet is close to atmospheric, with contributions of partial pressures from the entrained air and the water vapour. As the jet rises, it cools and some of the vapour condenses to satisfy the thermodynamic requirement of saturated conditions for the water component. Calculations show that the jet increases in diameter until it becomes infinitely wide at a certain height, the top of the jet, where the upward velocity becomes zero; the speed of the jet decreases approximately linearly with height above the ground. As the speed decreases with elevation, so too does its ability to lift the entrained particles, which are then released to fall and be deposited on the ground.

Because the jet slows with increasing elevation, any particle that leaves the surface must have been restrained from doing so by some cohesive force. If not, the lift criterion will not be exceeded immediately above the ground and the particle will not be elevated further. While small particles that are initially bound by cohesion become detached and accelerate upwards until the jet can not longer support them, larger clasts that are not cohesively attached, or which do not shed smaller fragments, might not leave the erosion surface but just remain close the ground. They would descend into the eruption crater as smaller particles are swept upwards past them – the eruption therefore may act as a size-sorting mechanism for the particles.

The geometry of the flow models is that of a column of circular horizontal cross-section with a vertical axis (see Figure 2). The vertical fluxes are based on the average vertical speed of the fluid. Where there are multiple components (liquid water + water vapour + air) it is assumed that their speeds are the same, i.e. they are well-mixed.

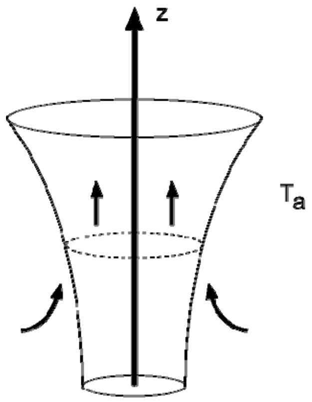

Figure 2: Diagram illustrating the geometry of the flow.

The model is of a moving column (jet) of a steam-dominated mixture of water vapour and liquid water droplets that issues from the ground from a circular region of radius r_0. As the fluid rises rises, it entrains air from the surrounding atmosphere, at ambient temperature T_a.

The resulting jet rises vertically and grows in radius, $r(z)$, where $r(0) = r_0$. It is assumed that the flow is at steady-state for a short period of time. As air is entrained into the flow, the total vertical mass flux $M(z)$ increases from $M(0) = M_0$ to $M(z) > M_0$ due to air entrainment. The total water flow (liquid + vapour) remains constant (M_w) while the air flow (M_a) increases with z. The total mass flow is $M = M_w + M_a$.

Mass conservation

Conservation of mass requires that the vertical rate of mass increase in the column is equal to the mass entrainment rate around the column surface:

$$\frac{dM}{dz} = \frac{dM_a}{dz} = 2\pi r \rho_{atm} E(w) \tag{11}$$

where the upwards mass flux within the column is $M = M_l + M_v + M_a = \rho \pi r^2 w$. Here, $w(z)$ is the mean vertical speed in the flow, $\rho(z)$ is the density of the fluid flow given by $\rho = \rho_\ell + \rho_v + \rho_a$ with each gas component contributing a partial pressure to the total pressure p_{atm}, the ambient pressure; it is assumed that the pressure within the column is the same as that of the surrounding air.

E(w) is the volume entrainment rate per unit surface area of the column, modelled here by $E(w) = kw\left(w / w_0\right)^n$, with constant dimension-less parameters *k* and *n*.

Momentum conservation

The surrounding air that is being entrained has zero vertical momentum initially. The momentum of the flow is reduced by gravitation and by the effective inertia of the entrained gas; since it is assumed that the pressure in the column is uniform and the same as the surrounding atmosphere, there is no pressure work.

Energy conservation

The energy equation takes into account changes in the kinetic and internal energy of the flow components.

Results

Numerical integration of the resulting three coupled ordinary differential equations gives results with the following general features: the radius of the column increases with height as the flux increases due to entrainment, while the jet speed decreases due to gravitational and inertial slowing; the radius diverges to an infinite value as the speed drops to zero. For further details and discussion, see McKibbin & Smith (2006).

6. FLIGHT OF THE EJECTA

In general, as discussed above, only small particles that have broken away from heavier clasts are able to be ejected by the emerging fluid stream. Heavier particles remain at ground level, jostled by the lower parts of the jet. The small particles are accelerated upwards until their weight equals the lift afforded by the decelerating column, when they are released into the surrounding air and are then moved by any cross-flow (wind) near the jet. The model for this part of the eruption is that of an advection-dispersion mechanism where the forces correspond to wind drag, gravitational settling and turbulence of the wind flow. The particles are released at different levels according to their size.

The components in this part of the model are small rock particles, water droplets and air. It is assumed here that the air-flow is not affected by the jet, and that the water droplets are neglected once they leave the upflow. The focus is therefore on the cohort of solid particles that is transported by the forces listed above, according to the advection-dispersion (sometimes called the convection-diffusion) equation. For simplicity, this is stated here for a mass Q_S of particles of a certain size with a corresponding free-air settling speed (terminal speed) *S*, released into a wind with horizontal speed *U*, and with turbulent dispersion coefficients $D_L = UL_L$ and $D_T = UL_T$ in the down- and cross-wind directions respectively, where L_L and L_T are the corresponding dominant length scales of the turbulence of the flow. The wind-flow parameters generally vary with elevation above the ground. The mass density (mass per unit volume of air) of the particles is denoted $c(x, y, z, t)$ in Cartesian coordinates, where the *x*-axis is aligned to the downwind direction. The particles are supposed to be released at point (0, 0, *H*) a distance *H* above the jet source, which is approximated by the point (0, 0, 0). The corresponding equation is:

$$\frac{\partial c}{\partial t} + U\frac{\partial c}{\partial x} - S\frac{\partial c}{\partial z} = D_L\frac{\partial^2 c}{\partial x^2} + D_T\frac{\partial^2 c}{\partial z^2} \qquad (13)$$
$$+ Q_S\,\delta(x)\delta(y)\delta(z-H)\delta(t)$$

Results from calculations where the atmosphere is modelled as a layered system were described in McKibbin (2006). Equation (13) applies in each layer where the wind speed and turbulence characteristics and the settling speed are considered constant, but possibly different from those in other layers. The transport of ejecta from a hydrothermal eruption is found by regarding the solution of Equation (13) as a basic building block. Different-sized particles have different settling speeds *S*, corresponding release heights *H* found from the eruption jet dynamics, and different associated cohort masses Q_S depending on the material mix that is ejected. The solutions for all sets of parameters may be found as a linear combination of the component cohorts. The solution may be found in closed form (McKibbin, 2006), so computation is straightforward.

While this equation may be used to estimate the flight and deposition of ejected particles for any given eruption, it relies on some knowledge about the size composition of the ejected material. Historical data from typical areas (such as Craters of the Moon where deposits have been analysed) could be used, but the role of liquid water in the joining of small particles into larger-sized agglomerates may be difficult to model.

7. SUMMARY AND CONCLUSIONS

The three main zones of interest (underground flow, eruption jet and plume dispersion) in hydrothermal eruptions, are able to be modelled to form an approximate, but complete, quantitative model of a hydrothermal eruption. An interesting question for further investigation arises: can this process be used to trace the motion forward through time and find any reason why it might eventually stop? Can one then go "back in time" and glean some information about the phenomenon's initiation?

NOMENCLATURE

Note: SI dimensions are given in square brackets; [–] denotes a dimensionless quantity

ϕ	porosity [–]
g	acceleration due to gravity [m s^{-2}]
h	specific enthalpy [kJ kg^{-1}]
k	vertical permeability [m^2]
L	dynamic lift [Pa m^{-1}]
μ	dynamic viscosity [kg m^{-1} s^{-1}]
m	mass flowrate per unit area [kg s^{-1} m^{-2}]
p	pressure [Pa, bar (1 bar = 10^5 Pa)]
q_e	energy flux/unit area [W m^{-2}]
ρ	density [kg m^{-3}]
S_ℓ	liquid saturation [–]
t	time [s]
T	temperature [˚C]
V	flashing front speed [m s^{-1}]
v	specific volume [m^3 kg^{-1}]
v_{rat}	volume expansion factor [–]
V_{er}	eroding surface speed [m s^{-1}]
V_f	average particle speed of fluid [m s^{-1}]
w_f	fluid volume flowrate per unit area [m s^{-1}]
z	vertical distance above flashing front [m]

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