Hydrothermal eruption jets: air entrainment and cooling

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HYDROTHERMAL ERUPTION JETS: AIR ENTRAINMENT AND COOLING

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SUMMARY – This paper proposes a quantitative model for the "fountain" of boiling water and rock particles ejected from a hydrothermal eruption. The model is based on principles of conservation of mass, momentum and energy. The main volumetric constituent is steam, which is diluted by entrained air as the jet rises. It is proposed that small rock particles and water droplets (from condensed steam) are carried up by the column while their weights are exceeded by the lift force of the rising gas mixture. They then exit from the column to be deposited. This paper primarily examines the form of the eruption jet when constrained by the conservation principles' requirements. A typical example is used to illustrate the model; calculated column shapes, fluid speed, phase compositions and particle exit heights are depicted and are discussed.

1. INTRODUCTION

Previous work on the modelling of hydrothermal eruptions has concentrated mainly on the flow below ground beneath the surface expression of the eruption (McKibbin, 1989; Smith, 2000; Smith and McKibbin, 1997, 1998, 1999, 2000). This paper moves the focus back onto the "fountain" of boiling water and rock particles which are ejected into the atmosphere as the eruption proceeds.

Conceptually, the eruption column is a vertical flow of steam and liquid water as well as entrained air. It is dominated volumetrically by the gas (water vapour and air) phase and it propels rock particles of various sizes from the ground, where they have been broken from the formation by the shear stress of the rapidly-passing fluid. As the column rises, it entrains some of the surrounding air; this leads to slowing and cooling of the flow.

It was suggested in earlier work (McKibbin, 1989) that the solid particles would form a significant volume fraction of the total flow. However, reconsideration of photographs of eruptions indicates that this is probably not the case, and that quantitative models should concentrate on the fluid flow, with the rock particles treated as occasional participants in the volume flux of the eruption column.

Consequently, the fluid flow model is worked out first. This is of a two-phase water mixture erupting, with air being entrained from the surrounding atmosphere.

The geometry of the flow model is that of a column of circular horizontal cross-section with a vertical axis (see Figure 1). The vertical fluxes are based on the average vertical speed of the fluid mixture. Where there are multiple components (liquid water + water vapour + air) it is assumed that their speeds are the same, i.e. they are well-mixed.

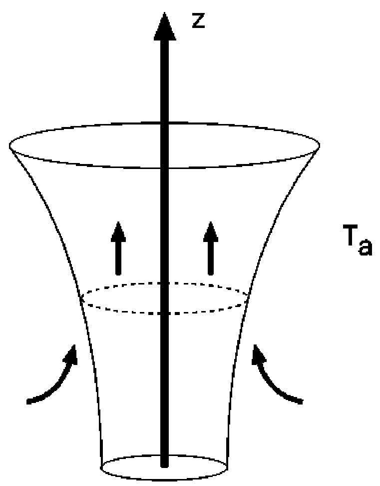

Figure 1. Schematic of the geometry of the hydrothermal eruption jet flow.

2. THE MODEL

The model is of a moving column (jet) of a steam-dominated mixture of water vapour and liquid water droplets which issues from the ground from a circular region of radius r_0. As it rises, it entrains air from the surrounding atmosphere, at ambient temperature T_{atm}.

The resulting jet rises vertically and grows in radius, $r(z)$, at height z above ground level (which is at $z = 0$). It is assumed that the flow has reached steady-state. As air is entrained into the flow, the total vertical mass flux M increases from $M(0) = M_0$ to $M(z) > M_0$ due to air entrainment.

The total water flow (liquid + vapour) remains constant (M_w) while the air flow (M_a) increases with z. The total mass flow is $M = M_w + M_a$.

The mass fraction of water per unit volume (X_w) decreases from $X_w(0) = 1$ as z increases. That for air ($X_a = 1 - X_w$) increases from $X_a(0) = 0$ as z increases.

The liquid mass fraction of the water flux M_w is denoted Y_ℓ and that of vapour is $Y_v = 1 - Y_\ell$. So $M_\ell = Y_\ell M_w = Y_\ell X_w M$ and $M_v = Y_v X_w M$. The total gas mass flow is then

$$M_g = M_v + M_a = Y_v X_w M + X_a M$$
$$= \left(Y_v + \frac{1 - X_w}{X_w} \right) M_w$$

where M_w is constant.

Mass conservation

Conservation of mass requires that the vertical rate of mass increase in the column is equal to the mass entrainment rate of air around the column surface:

$$\frac{dM}{dz} = \frac{dM_a}{dz} = 2\pi r \rho_{atm} E(w) \quad (1)$$

where the upwards mass flux within the column is

$$M = M_\ell + M_v + M_a = \rho \pi r^2 w \quad (2)$$

Here, $r(z)$ is the radius of the column (circular cross-section of the flow), and $w(z)$ is the mean vertical speed in the flow. It is assumed that the pressure within the column is the same as in the surrounding air. The density $\rho(z)$ of the mixture is given by $\rho = \rho_\ell + \rho_v + \rho_a$, with each component contributing a partial pressure to the total pressure of p_{atm} (the ambient pressure). Note that the density ρ_a of the air component of the stream is not the same as the atmospheric density ρ_{atm}, since the air component within the jet is at a reduced partial pressure.

$E(w)$ is the volume entrainment rate of air per unit surface area of the column, modelled here by the formula

$$E(w) = kw\left(w / w_0\right)^n \quad (3)$$

where k and n are constant dimensionless parameters. The fluid mass flux issuing from the ground is given by $M_0 = \rho_0 \pi r_0^2 w_0$ where the subscript 0 indicates values at $z = 0$.

Momentum conservation

The surrounding air that is being entrained is assumed to have zero initial vertical momentum. The momentum of the flow is reduced by gravitation and by the effective inertia of the entrained air; also, since it is assumed that the pressure in the column is uniform and the same as the surrounding atmosphere, there is no pressure work:

$$\frac{d}{dz}(Mw) = -\rho \pi r^2 g \quad (4)$$

Energy conservation

The energy equation takes into account changes in the kinetic and internal energy of the flow, and may be written in the form:

$$\frac{d}{dz}\left(\tfrac{1}{2} M w^2 + M_\ell u_\ell + M_v u_v + M_a u_a\right) = -\rho \pi r^2 g w + \frac{dM_a}{dz} u_{atm} \quad (5)$$

where the u_i are the specific internal energies of various components. Simplified formulae for some of the thermodynamic variables are given in the Appendix.

Since the water mass flux M_w is constant, it is useful to use X_w, the mass fraction of the water in the flow, defined by $M_w = X_w M$. This means that the air mass fraction is $M_a = X_a M = (1 - X_w)M$. Substitution into Equations (1) and (2) and some rearrangement gives a pair of differential equations for the system variables X_w and w:

$$\frac{dX_w}{dz} = -2\pi r \frac{X_w^2}{M_w} \rho_{atm} k w (w / w_0)^n \quad (6)$$

$$\frac{dw}{dz} = -\frac{X_w}{M_w}[\rho \pi r^2 g + k w^2 (w / w_0)^n] \quad (7)$$

Some simplification of Equation (5) involving use of the Appendix formulae gives

$$\frac{dT}{dz} = -\frac{\tfrac{1}{2} w^2 - c_a (T - T_{atm})}{[X_w c_w + (1 - X_w) c_a] X_v} \frac{dX_w}{dz} \quad (8)$$

3. RESULTS

Numerical integration of the three coupled ordinary differential equations (6 – 8) gives results with the following general features: the radius of the column increases with height as the flux increases due to entrainment, while the velocity decreases due to gravitational and inertial slowing. The radius diverges to an infinitely large value as the speed drops to zero. The (maximum) height of the column where this occurs is not known *ab initio*, and is found as part of the solution.

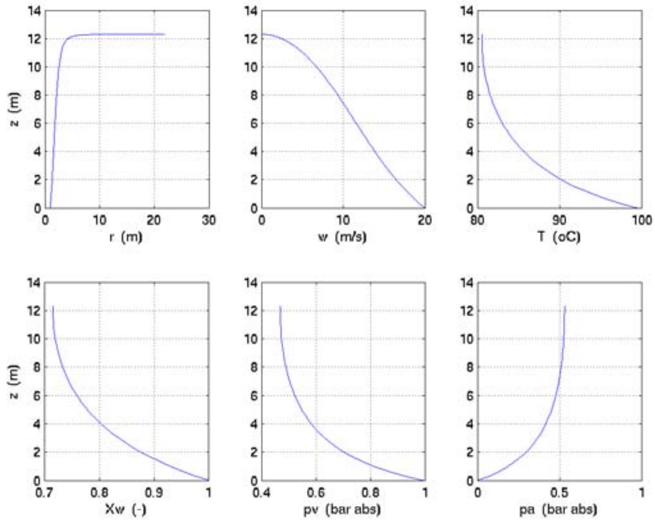

Figure 2(a). The radius, vertical speed, temperature, water mass fraction, partial pressure of the vapour and partial pressure of the air in the column as functions of height.

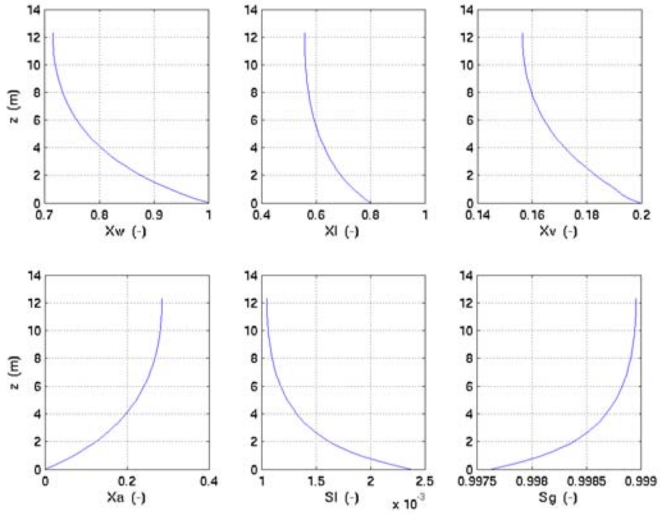

Figure 2(b). The total water, and the liquid and vapour mass fractions, the air mass fraction, and the liquid and gas saturations of the mixture, in the column as functions of height.

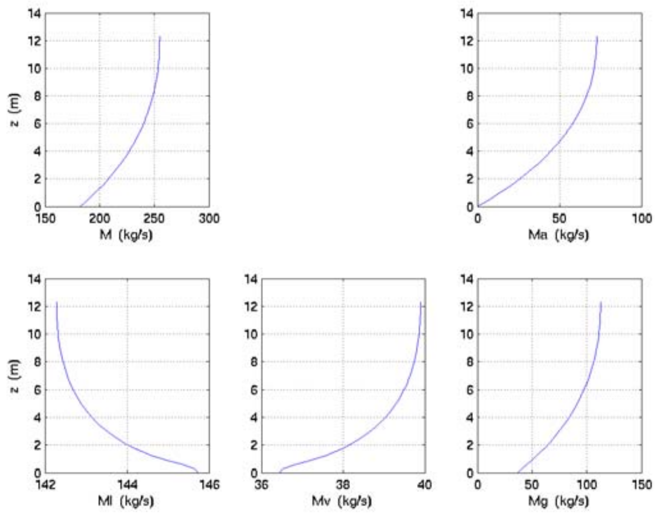

Figure 2(c). The total mass flux, the air flux, and the total liquid, vapour and gas fluxes, in the column as functions of height.

Using the parameters listed in Table 1 gives the results shown in Figure 2. [A non-zero but very small value of the air concentration at the ground is required in order that all components are available during the whole integration procedure.]

Table 1. Parameters for the example calculation shown in Figure 2.

r_0	1 m
w_0	20 m s^{-1}
T_0	99.6 ˚C
$Y_{\ell 0}$	0.8
X_{w0}	0.999999
$p=p_{atm}$	1 bar abs.
T_{atm}	20 ˚C
k	0.1
n	2

From Figure 1(a) it can be seen that the maximum column height for this example is about 12.2 m, where the calculated radius of the column becomes infinitely large and the vertical speed drops to zero. The temperature of the column reduces from the initial temperature to about 80 ˚C near the top. The water mass fraction reduces from 1 to about 0.7 as more and more air is entrained.

Figure 2(b) shows how the liquid and vapour mass fractions contribute to the total water mass fraction in the mixture. It also shows how the air mass fraction rises as entrainment takes place. Of some interest also is that the liquid saturation of the mixture reduces from its initial (small) value, showing that the mixture "dries" as it rises.

Figure 2(c) depicts the various component and phase mass fluxes *vs* height; these results follow from the those in Figures 2(a) and (b). The liquid mass flux M_ℓ reduces with height; this corresponds to some evaporation of the liquid water component. This is somewhat surprising, since it might be anticipated that the vapour would condense as it rises. Whether this is a feature of all examples can only be found by further testing.

4. RELEASE OF MATERIAL FROM THE COLUMN

Information about the fountain dynamics enables release heights of particular sized rock particles or water droplets to be found. It can be assumed that water droplets or particles will rise to a height in the column where their weight is exactly balanced by the upward force (drag) exerted by the hydrothermal eruption column. (Below this height, the lift will exceed the weight and the particles will continue to rise.)

Settling speed of particles

The settling speed S of a particular particle depends on its size, shape and weight. For a particle that is approximately spherical, with diameter d, the requirement that the particle's weight be exactly balanced by the drag exerted by the upward moving jet is expressed by

$$\rho_p \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{1}{2}\rho C_D \pi\left(\frac{d}{2}\right)^2 S^2 \qquad (9)$$

where ρ_p and ρ are the densities of the particle and gas respectively, and the drag coefficient C_D is a function of the Reynolds Number,

$$\mathrm{Re} = \frac{\rho d S}{\mu}, \qquad (10)$$

where μ is the dynamic viscosity of the gas mixture. Here, we assume the form (Perry *et al.*, 1984):

$$C_D = \frac{24}{\mathrm{Re}}(1 + 0.14\,\mathrm{Re}^{0.7}) \quad \text{for} \quad \mathrm{Re} < 1000$$

and (11)

$$C_D = 0.447 \quad \text{for} \quad \mathrm{Re} \geq 1000.$$

For a given particle size and density, Equations (9–11) can be solved to give S.

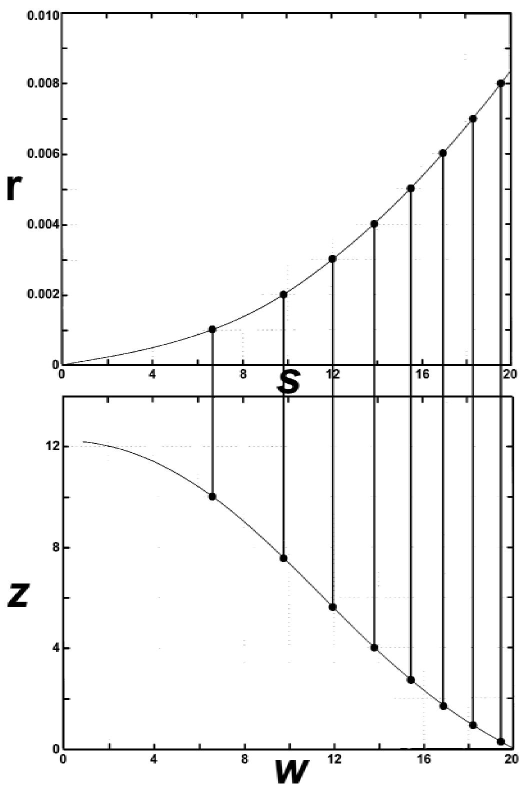

Figure 3. The top graph shows the settling speed of a spherical water droplet *vs* its release height. The bottom graph shows the speed of the eruption jet *vs* its height for the example given in Figure 2.

Release heights of particles

From the vertical speed of the fluid column, $w(z)$, and the settling speed of a particular particle, S, we can calculate the release height of that particle. For example, in Figure 3 a plot of a spherical water droplet's settling speed versus its radius is shown above the speed of a gas jet versus its height (from the example shown in Figure 2, calculated with the parameters given in Table 1).

By comparing the two graphs we can obtain a relationship between the water droplet's radius and its release height. The release heights for water droplets of particular radii are given in Table 2.

Table 2. Water droplet radius *vs* release height for the example using parameters shown in Table 1 and results shown in Figure 2.

r	z (m)
0	12.2
0.0005	11.4
0.001	10.0
0.002	7.6
0.003	5.6
0.004	4.0
0.005	2.7
0.006	1.7
0.007	0.8
0.008	0.2
0.0086	0

Once the heights to which rock particles are lifted is known, wind velocity and turbulence parameters allow calculation of the dispersion and deposit profiles of the aerosols and liquid droplets. Solutions of the advection-dispersion equation may be used to describe the airborne concentrations after release. 5Example mass deposit profiles are provided by McKibbin (2006, in these Proceedings).

5. SENSITIVITY ANALYSIS OF MODEL

The model is sensitive to the initial column radii and eruption speeds chosen. For a given initial jet speed, the initial radius must be greater than a particular value to produce reasonable results. In Figure 4, minimum initial radii are shown for given initial speeds for calculations using the parameters shown in Table 3. The initial mass fraction of water per unit volume X_{w0} and liquid mass fraction of the water flux is $Y_{\ell 0}$ are shown on the graph for each case.

Results show that the slower the initial speed of the jet, the larger the jet radius at the eruption vent is required for the model to produce reasonable results.

Table 3. Parameters for the calculation shown in Figure 4.

T_0	98.7 °C
$p=p_{atm}$	1 bar abs.
T_{atm}	20 °C
k	0.1
n	2

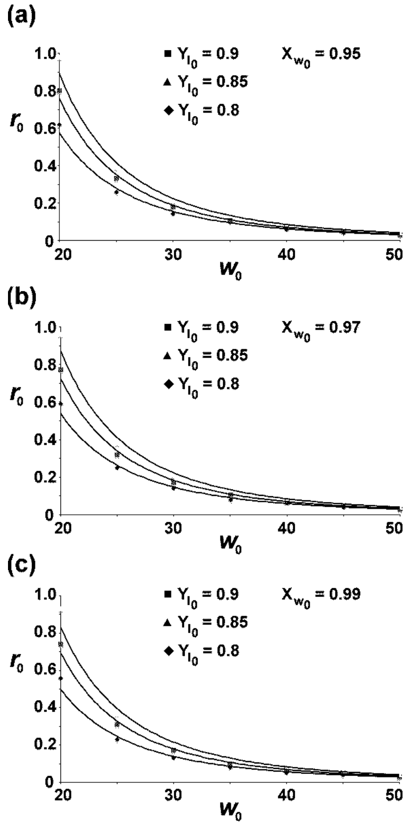

Figure 4. For the parameters shown in Table 3, minimum initial radii of the "eruption vent" are shown for a given initial jet speed.

For example, for initial jet speeds of 20 ms^{-1}, minimum jet radii at the vent of between 0.5 and 0.9 m are required, while for speeds of 30 ms^{-1}, radii of 0.1 – 0.3 m are required, and for speeds of 40 ms^{-1}, radii of only about 0.1 m are required.

SUMMARY

This work is a first attempt at modelling the fountain of boiling water and rock particles that are ejected in a hydrothermal eruption. Information about the jet dynamics will eventually help us to connect the flow below ground beneath the surface expression of the eruption (as previously studied in Smith, 2000; Smith and McKibbin, 1998, 1999, 2000) to the calculation of the dispersion and deposit profiles of the aerosols and liquid droplets [as modelled in McKibbin (2006, elsewhere in this Proceedings)].

A quantitative model for the boiling water jet and rock particles ejected from a hydrothermal eruption is proposed and heights at which the water droplets and rock particles exit from the column to be deposited are calculated. A typical example is used to illustrate calculated column shapes, fluid speed, phase compositions and particle exit heights.

General features of the results are that the radius of the column increases with height as the flux increases due to air entrainment, while the velocity decreases due to gravitational and inertial slowing, and the radius diverges to an infinitely large value as the speed drops to zero.

For the model to produce reasonable results, the initial jet radius must be greater than a certain amount for a given initial jet speed. Typical examples are provided to show the relationship between the minimum jet radius at the eruption vent and initial jet speeds.

REFERENCES

McKibbin, R. (1989) An attempt at modelling hydrothermal eruptions. *Proc. 11th New Zealand Geothermal Workshop 1989*, University of Auckland, pp. 267-273.

McKibbin, R. (2006). Modelling deposition of hydrothermal eruption ejecta. *Proc. 28th NZ Geothermal Workshop 2006*. University of Auckland. Elsewhere in this volume.

Perry, R.H., Green, D.W and Maloney, J.O. (Eds) (1984). *Perry's Chemical Engineers' Handbook.* McGraw-Hill, Singapore.

Smith. T.A. (2000). *Mathematical modelling of underground flow Processes in hydrothermal eruptions.* PhD Thesis in Mathematics, Massey University, Palmerston North, New Zealand.

Smith, T.A. and McKibbin, R. (1997) Modelling of hydrothermal eruptions: A review. *Proc. 19th NZ Geothermal Workshop 1997*, University of Auckland, pp.123-128.

Smith, T.A. and McKibbin, R. (1998) Towards a hydrothermal eruption flow model. *Proc. 20th NZ Geothermal Workshop 1998*, University of Auckland, pp. 387-392.

Smith, T.A. and McKibbin, R. (1999) An investigation of boiling processes in hydrothermal eruptions. *Proc. 21st NZ Geothermal Workshop 1999*, University of Auckland, pp. 229-236.

Smith, T.A. and McKibbin, R. (2000) An investigation of boiling processes in hydrothermal eruptions. In Eglesias, E., Blackwell, D., Hunt, D., Lund, J., Tamanyu, S. and Kimbora, K. (Eds.), *Proc. World Geothermal Congress 2000, International Geothermal Association*, pp. 699-704.

APPENDIX

Thermodynamic variables

Densities

$$\rho_v = p_v/461.527/T \ \text{kg m}^{-3}$$

$$\rho_a = p_a/287.099/T \ \text{kg m}^{-3}$$

Internal energies

$$u_v = 2375 \times 10^3 + c_v(\mathrm{T} - 273.15) \ \text{J kg}^{-1}$$

where $c_v = 1310 \ \text{J kg}^{-1}\ \text{K}^{-1}$

$$u_a = c_a(\mathrm{T} - 273.15) \ \text{J kg}^{-1}$$

where $c_a = 1010 \ \text{J kg}^{-1}\ \text{K}^{-1}$