WAVE PROPAGATION IN SEMI-INFINITE POROUS MEDIA

A. GORIN, A. VAKHGUELT & SARIM N. AL-ZUBAIDY

School of Engineering & Science, Curtin University of Technology, Sarawak Campus, Malaysia

SUMMARY – The propagation and structure of a finite amplitude wave is studied in this paper. The characteristics of the soil structure are used for consideration of pressure (density, velocity, or temperature) disturbances in a volume containing porous medium. The exchange processes are considered for heat transfer in an integral form and for momentum exchange in the Darcy form. Because of the high interaction between the skeleton and internal gas the finite amplitude wave is largely dissipative. It is shown that during the wave propagation the wave amplitude will decrease with the distance by exponential curve. The analysis provides a basis for estimating the capabilities and efficiency of the implementation of the finite amplitude wave for diagnostic of geothermal reservoirs.

1. INTRODUCTION

The necessity of considering wave propagation in porous media stems from the fact that this phenomenon occurs in a large number of geophysical situations and engineering applications. Of special interest is the harnessing of the wave propagation for diagnostics of geothermal reservoirs and enhancement of transport phenomena with intent of increasing thermal energy that can be drawn from this zone.

The key problem is in obtaining a wave equation for kinematic characteristics taking into account main properties of a porous medium, wave-porous-medium interaction, and an inherent feature of these type processes, namely heat and momentum transfer between phases.

Propagation and structure of a finite amplitude wave employed for underground diagnostics could be modelled by the finite amplitude wave propagation in material with the porosity close to underground massive porosity.

The relationships governing propagation of acoustic waves in porous media have been used by Nikolaevskiy et al. (1970) while Marble (1970), Davidson (1975, 1976), Popov (1968), Rytov and Vladimirsky (1938) investigated the propagation of weak (primarily acoustic) pulses in gases containing liquid droplets or solid particles.

The porous media could be modelled by an assembly of the spherical particles, with population m in a given unit volume considered constant, regularly packed with the space between them filled with air.

Any disturbances, which can happen in the area next to this granular material, will propagate through the media. The signal will go through the rigid phase and also through the gas phase of the porous media. Consideration of the interphase processes is vital for studying of processes in these media. In all above studies the phase interactions were treated by applying the Newton-Richman law for description of heat transfer and the Stokes law for description of mechanical interaction. However, the application of these laws is valid only in the case of quasi-steady processes, whereas for unsteady conditions, which are rather frequently encountered in practice, it is incorrect to represent heat transfer and friction between phases in this form.

In the present work we shall concentrate on the derivation and analysis of equations for description of the evolution of finite amplitude perturbation, allowing for relaxation processes induced by unsteady heat and momentum transfer between the phases. Only right calculation of the exchange processes will allow obtaining appropriate data for dissipation.

2. THEORY

Let us consider the propagation of the non-steep pressure waves with finite amplitude through the granular material. This wave propagates through porous part of the media. Two dimensional, finite amplitude perturbations propagate through a fill made up of spherical particles of uniform radius δ and density ρ_p. We assume that a sufficient number of particles lie along a wavelength, so that the porous mixture can be regarded as a continuum.

It is assumed that the solid skeleton of the above porous media is absolutely rigid, then the equations of conservation for the mass, momentum and energy, and the equation of state need to be formulated only for the gaseous phase.

Let finite-amplitude perturbation propagates through an infinite granular material, modelled by the absolutely rigid solid skeleton (porosity M = const). When considering the porous media it is possible to model this media as an infinite volume regularly packed by spherical particles of the uniform radius δ with interparticle space occupied by gas, Figure1.

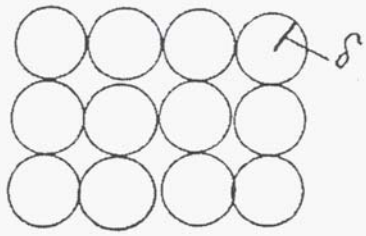

Figure 1: Model **of** porous medium

For this case we can assume that only part of the surface of those particles will participate in the interphase heat and momentum exchange, then the equations of conservation for the mass, momentum and energy, and the equation of state need to be formulated only for the gaseous phase with correction for the thermal and momentum interaction between phases.

Neglecting viscosity and thermal conductivity, in the conservation equations, we obtain the following set of equations, analogous to that given by Nikolayevskiy et al. (1970), for describing the evolutions of waves in granular material with allowance for thermal and force interaction:

$$M\rho_t + M(\rho u)_x = 0$$

$$M\rho u_t + M\rho u u_x + MR(\rho v)_x =$$
$$6(1-M)\pi\rho\delta\nu u + \frac{9}{2}(1-M)\frac{\rho}{\delta}\sqrt{\frac{\nu}{\pi}}\int_0^t \frac{u_y dy}{\sqrt{t-y}}$$

(1)

$$M\rho c_v v_t + M\rho c_v u v_x + M\rho R v u_x =$$
$$-6(1-M)\frac{a}{\delta^2}\rho_p c_p \times$$
$$\frac{\partial}{\partial t}\int_0^t v\{\vartheta_3[0, \exp(-\frac{\pi^2 a}{\delta^2}(t-y))]-1\}dy +$$
$$\frac{9}{2}(1-M)\frac{\rho}{\delta}\sqrt{\frac{\nu}{\pi}}u\int_0^t \frac{u_y dy}{\sqrt{t-y}}$$

where ρ and u are the density and velocity of the gas; R is the specific gas constant, c_v is the specific heat capacity at constant volume, a is the thermal diffusivity of the particle and ν is kinematic viscosity of the gas and ϑ_3 is theta function (Gradshtein & Ryzhik, 2000).

Here and later subscripts refers to the derivative with respect to the x - coordinate in the direction of wave propagation and with respect to time t .

In the above set, Equations (1) are equations of continuity, motion and energy for an ideal gas. The interactions between phases are accounted through unsteady conditions of the exchange processes between phases. The heat exchange is obtained from solution of the general conduction equation for the spherical particle

$$\theta_t - a\theta_{rr} - \frac{2a}{r}\theta_r = 0 \qquad (2)$$

where θ is the relative particle's temperature, and r is its instantaneous radius.

The last equation is solved in spherical coordinates under the following initial and boundary conditions:

$$\theta = \theta_0 = 0 \qquad \text{at} \quad t = 0$$

$$\theta_{|r=\delta} = \theta_\delta = \vartheta(t) \qquad \text{at} \quad t > 0 \qquad (3)$$

$$\frac{\partial\theta}{\partial r}_{|r=0} = 0 \qquad \text{at} \quad t > 0$$

where $\vartheta(t)$ is a temperature in the gas phase.

The solution of Equation (2) under initial and boundary conditions (3) is (Carslaw & Jaeger, 1989)

$$\theta = \sum_{n=1}^{\infty}\frac{2\delta}{rn\pi}(-1)^{n+1}\sin\left(n\pi\frac{r}{\delta}\right)\times$$

$$\frac{\partial}{\partial t}\int_0^t \vartheta(y)\exp\left[-n^2\pi^2\frac{a}{\delta^2}(t-y)\right]dy$$

The heat flux to the single particle is

$$q_p = \lambda_p(\frac{\partial\theta}{\partial r})_{r=\delta} =$$
$$-\frac{2\lambda_p}{\delta}\sum_{n=1}^{\infty}\frac{\partial}{\partial t}\int_0^t \vartheta(y)\exp[-n^2\pi^2\frac{a}{\delta^2}(t-y)]dy$$

(4)

where λ_p is the thermal conductivity of the particle material.

For the force of interaction between a spherical particle and the gas surrounding it is, in general case (Landau & Lifshits, 1987)

$$F = 2\pi\rho\delta^3(\frac{1}{3}u_t + \frac{3\nu}{\delta^2}u + \frac{3}{\delta}\sqrt{\frac{\nu}{\pi}}\int_0^t u_y \frac{dy}{\sqrt{t-y}}) \quad (5)$$

The first term in relation *(5)* allows consideration of the force responsible for the induced mass and is significant in considering the motion of gas bubbles in a liquid, and can be neglected in our case of relative motion between solid skeleton and a gas in porous area as insignificant.

The second term in Eq. *(5)* is the Stokesian frictional drag, which is much smaller than the Basset force (third term of Eq. *(5)* in the case of unsteady flow over the particles.

We can assume now that we are able to use the equation of state for ideal gas and also we can neglect by the first term of the relation for the particle's drag force, because it is responsible for the induced mass.

The term describing heat transfer between phases in this energy equation can have another form if the medium is modelled let us say, by cylinders or plates, instead of having it be modelled by ensemble of spherical particles.

Let us consider the propagation of small perturbations in two-phase medium consisting of the gas and rigid particles.

We shall assume that deviations of density, velocity and temperature $(\rho-\rho_0)/\rho_0$, u/C, and $(\vartheta-\vartheta_0)/\vartheta_0$ from equilibrium are first-order infinitesimals (here $C = \sqrt{p/\rho}$ is the speed of sound; ρ_0 and ϑ_0 are the unperturbed density and temperature, while ρ, u, and ϑ are the instantaneous values of the density, velocity of the perturbation and temperature).

Substituting $\rho = \rho_0 + \rho'$, $u = u'$, and $\vartheta = \vartheta_0 + \vartheta'$ (ρ', ϑ', and u' are the values of density, temperature and velocity perturbations in Eqs (1) and dropping infinitesimals of an order higher than two, we will obtain, by following the procedure presented by Rudenko and Soluyan (1975), the next wave equation

$$u_x - \frac{\kappa-1}{2\kappa}u_t - \frac{\kappa+1}{2}uu_\tau - 3\frac{1-M}{M}\times \frac{\pi\delta\nu}{\kappa}u(1+u) - \frac{9}{2}\frac{1-M}{Ma}\sqrt{T\frac{\nu}{\pi}}\int_0^\tau \frac{u_y}{\sqrt{\tau-y}} - 9\frac{1-M}{M}\frac{\kappa}{\delta}\sqrt{T\frac{\nu}{\pi}}u\int_0^\tau \frac{u_y dy}{\sqrt{\tau-y}} + 6\frac{1-M}{M}\frac{T}{\tau_p}\frac{\rho_p}{\rho_0}\frac{\partial}{\partial\tau}\int_0^\tau [1+(\kappa-1)u]\times \{\vartheta_3[0,\exp(-\frac{\pi^2 a}{\delta^2}(\tau-y))]-1\}dy = 0 \quad (6)$$

where κ is the ratio of the specific heat capacity at a constant pressure to the specific heat capacity at constant volume and T is the period of perturbation.

The inverse substitution $u'/C = \rho'/\rho_0$ and

$$\vartheta'/\vartheta_0 = (\kappa-1)\rho'/\rho_0$$

or

$$u'/C = \vartheta'/[(\kappa-1)\vartheta_0]$$

and

$$\rho'/\rho_0 = \vartheta'/[(\kappa-1)\vartheta_0]$$

yields the similar equations for density increment ρ' or temperature increment ϑ'.

Equation **(6)** is valid **for** arbitrary relationships between characteristic particle heating time τ_p and the period T of the applied perturbation.

It is thus seen that the propagation of waves in a bulk granular material can be described by a single, nonlinear evolution equation.

It is evident that the principal contribution to the decay of waves is made by the unsteady interaction between phases. These effects make a much greater contribution to decay of waves than viscosity and heat conduction.

3. ANALYSIS

The obtained Eq *(6)* describing the propagation of the perturbations through porous part of the granular material belongs to the category of the Burgers equation which was extensively used in studies **of** the wave propagation in two-phase medium with low gas content and with the dissipation related to the heat and momentum exchange between phases.

The Burgers equation is well known and it has an analytical solution in the limiting case of strong dissipation in the single-phase medium and weak dispersion.

The evolution Equation **(6)** allows for an easier way to consider the propagation of the perturbations in porous media, as compared to the initial systems of the equations (1).

Figure 2 shows the evolution of the wave front in the granular material consisting of the polyethylene granules with uniform radius $\delta = 1$ mm, with air filling **all** the internal volume of this media, porosity $M = 0.42 m^3 / m^3$, $T = 10^{-5}$ sec.

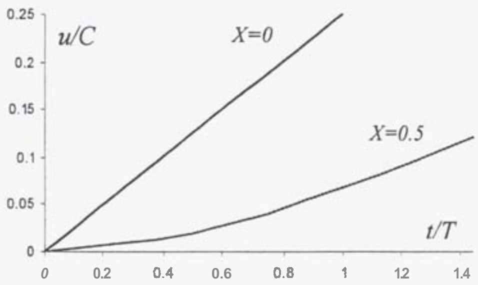

Figure 2. Propagation of the perturbation in the granular material. (Dimensionless amplitude of perturbs-tion u/C versus dimensionless time t/T for different distances from the entrance to the system)

The amplitude of the perturbation in the granular material is decreasing with the high rate. This graph is obtained by numerical solution of Equation **(6).**

The dissipation of the pressure wave is shown in Figure **3.**

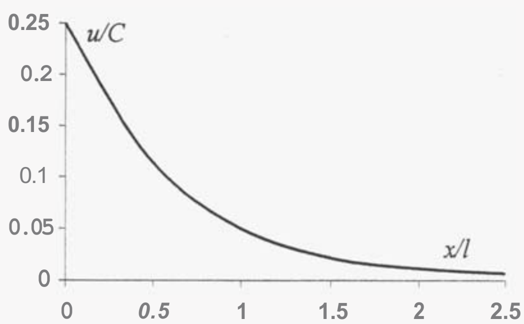

Figure **3.** The dissipation of pressure wave in the porous medium represented in Figure 1. (Dimensionless amplitude of the wave u/C versus dimensionless distance x/l)

This pressure wave is dissipating with the rate in inverse proportion to the length of the granular material.

4. CONCLUSION

The derived dimensionless Equation **(6)** allows consideration of the pressure pulses dissipation in porous media with different phase properties. This equation also enables the calculation of the wave profile evolution and defines the effects of the interphase exchange processes on the propagation of the perturbations. The derived equation facilitates the analysis and evaluation of the effects of interphase exchange processes on the dissipation of the pressure pulses in any porous media.

The analysis of the propagation of the pressure pulses in porous media provides a basis for estimating the capabilities and efficiency of the implementation of the finite amplitude wave for diagnostic **of** the geothermal reservoirs.

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