FORCED CONVECTION AND EFFECTIVE TRANSPORT PROPERTIES IN PACKED BEDS

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SUMMARY – This paper reports the results of experimental investigation of thermally developed forced convective heat transfer to fluid flowing through vertical electrically heated tube of diameter D =52 mm packed with spherical glass particles. The experiments were carried out for water and 47%-glycerine aqueous solution as a working fluid and ratio of the particle diameter to the tube diameter d of 0.017, 0.062 and 0.17. Radial temperature profiles inside the bed have been measured at axial position x/D of 8.01 and 9.03. A two-dimensional quasi-homogeneous model was employed to obtain overall heat transfer coefficients, as well as effective thermal conductivities and apparent wall heat transfer coefficients. New correlations for the thermal conductivity at high Peclet numbers and the heat transfer coefficients at medium Reynolds numbers (inertial flow regime) are developed.

1. INTRODUCTION

There are many areas of engineering and science for which understanding of convective heat transfer through a fluid-saturated porous medium is important. Geothermal systems is an important example where knowledge of transport properties in porous media makes it possible to estimate an efficiency of geothermal energy extraction and to propose specific lines of attack on the problem. These properties is the subject of investigation when devising thermal.techniques for secondary oil recovery and effective heat exchanges with solid matrix consisting of packed spheres or porous medium, in the catalytic chemical reactors design.

Forced convection in packed channels has been a subject of intensive study in engineering literature for more than seven decades (see Wakao & Kaguei (1982); Cheng et al. (1991); Nield & Bejan (1998): Kaviany (2001) for a review of literature).

When generalising experimental data on heat transfer the main efforts of investigators were directed to obtaining more or less universal relationships in a representative range of the flow and geometrical parameters.

By analogy with forced convection in empty channels these data are often represented in the form

$$Nu = c Re^n Pr^m f(d/D)$$

where $Nu=hL/\lambda_f$, $Re=uL/\nu$, $Pr=\nu/a$ are the Nusselt, Reynolds and Prandtl number respectively; h is the heat transfer coefficient; u is the flow velocity in packed bed; ν, λ_f, $a=\lambda_f/\rho c_p$, ρ and c_p are the kinematic viscosity, thermal conductivity, thermal diffusivity, density and specific heat capacity of fluid respectively; L is the length scale; d is the characteristic size of the packed bed (as a rule, it is the diameter of balls composing the packed bed); D is the tube diameter (or the hydraulic diameter of a channel).

The exponents n and m are to be chosen from the best fit to the experimental data, and, according to the results obtained by different authors, they vary from 0.5 to 1.5.

The most discrepant picture is observed in the inertial flow regime (moderate Reynolds numbers) preceding transition and turbulent filtration (see for details Gorin et al., 1996).

It is apparent that the available generalising empirical correlations hold only for experimental conditions under which they were obtained and they can also be of more complicated form.

The resistance to heat transfer was found to increase greatly near the wall. The additive effect in the wall vicinity is generally expressed as the wall heat transfer coefficient.

For the description of the mechanism of heat transfer in packed beds the concept of "effective thermal conductivity" is used. The knowledge of the thermal conductivity and the heat transfer coefficient are crucial when analysing experimental data on both average heat transfer characteristics and on the structure of thermal fields and when devising and testing theoretical models.

In general this effective thermal conductivity λ_{ef} is not a property in the true thermodynamic sense but is a function of a process. It is a common practice to correlate the effective conductivity as the sum of the stagnant conductivity λ_{st} of the porous medium saturated with a stagnant fluid and the dispersive component due to filtration flow that is a linear function of the Reynolds number

$$\lambda_{ef} = \lambda_{st} + b\rho c_p u d$$

where b is an empirical constant.

According to the data by various authors ***b*** can take different values depending on the working liquid-solid pair, the packed bed geometry, the form of balls and the ratio of tube-to-particle diameter *D/d*. Examples are found in the next experimental values of *b:*

- air-glass:
  $b = 0.1\,1$ (Yagi &Wakao, 1959);
  $b = 0.07 \div 0.115$ (Aerov et al., 1979);
- water-glass:
  $b = 0.55$ (Kunii & Suzuki, 1966);
  $b = 0.075 \pm 0.009$ (Kharitonov et al., 1997)

and

$b^{-1} = 8[2 - (1-2d/D)^2]$ (Schlunder (1966).

As this take place, it should be borne in mind that in the general case the values of the effective thermal conductivity can vary within the bed due to the velocity distribution nonuniformity across the packed bed, and in the immediate vicinity of surfaces embedded in a porous medium they are far less than those for the bulk flow and may be close to the molecular property.

The wide scatter of the experimental values of the effective thermal conductivity and the heat transfer coefficient can be attributed to their dependence on the length of packed bed (Li & Finlyason, 1977), to the effect of the ratio of fluid-to-particle thermal conductivity, the ratio of particle-to-tube diameter, and to the fluid Prandtl number $Pr = \nu/a$ (Dixon & Creswell, 1979).

These discrepancies in published data on the macroscopic transport coefficients do not allow a conclusion to be drawn in favour of one of the empirical relationships.
As a result we are unable to accurately estimate the contribution of the near-wall heat transfer in the overall thermal resistance. This is also true for the verification of theoretical models with the empirical or semiempirical transport properties – they must be very accurate and reliable representatives of the data.

The currently available information does not allow any unique conclusions on the behaviour of the effective thermal conductivity as well as heat transfer coefficients to be made. Powerful and universal correlations are unlikely.
Undoubtedly, there is a need for additional data under various test conditions.

For this reason, the purpose of the work reported in this paper was to determine the effective thermal conductivity for the cases of water ($Pr \cong 5.0 \div 7.0$) and 47%-glycerine aqueous solution (with Pr up to 50), and to develop correlations for the heat transfer coefficients at moderate Reynolds numbers (inertial flow regime). Our study is confined to packing of glass spheres of three particle-to-tube diameter ratios d/D = 0.017, 0.062, and 0.17.

2. EXPERIMENTAL PROCEDURE

2.1 Analysis

Heat transfer of fluid flow through a cylindrical packed tube heated from the wall is considered under the assumptions that the system is at steady state and filly developed, fluid and solid are in local thermal equilibrium (the single-temperature approximation) and the physical properties of fluid are independent of temperature variations, and the mass velocity of fluid is uniform across the tube diameter (the plug flow model).

By using the concept of the effective thermal conductivity with a constant radial component over the tube cross-section and the negligibly small axial component and by introducing additional (apparent) wall heat transfer coefficient to account for the decrease in the effective conductivity near the wall the following differential equation for the temperature profile may be written

$$u\frac{\partial T}{\partial x} = \frac{\lambda_{ef}}{\rho c_p}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \qquad (1)$$

The inlet and boundary conditions of this equation are

$$T = T_0 \ \text{at} \ x=0; \quad \partial T/\partial r = q/\lambda_{ef} \ \text{at} \ r=R; \quad \partial T/\partial r = 0 \ \text{at} \ r=0 \qquad (2)$$

The solution of **Eq** (1) for the thermally fully developed pipe flow with these boundary conditions is well known

$$T(r,x) = T_0 + \frac{2q}{\rho c_p u}\frac{x}{R} - \frac{qR}{4\lambda_{ef}}\left(1 - \frac{2r^2}{R^2}\right) \qquad (3)$$

The averaged-over-the-tube cross-section temperature is thus seen to be

$$\bar{T} = \frac{2}{R^2}\int_0^R rT\,dr = T_0 + \frac{2q}{\rho c_p u}\frac{x}{R}$$

Let us ascribe an additional wall thermal resistance to the thin near-wall zone of thickness $\delta_T \ll R$. Then the temperature T_s at the boundary $R - \delta_T \approx R$ as it immediately follows from Eq (3) is

$$T_s = T_0 + \frac{2q}{\rho c_p u}\frac{x}{R} + \frac{qR}{4\lambda_{ef}} = \bar{T} + \frac{qR}{4\lambda_{ef}} \qquad (5)$$

With the definition of heat transfer coefficient the following equation may be written

$$\frac{1}{h} = \frac{T_w - \bar{T}}{q} = \frac{T, - T_s}{q} + \frac{T_s - \bar{T}}{4}$$

from whence taking into account **Eq** (5) the following relation for the heat transfer coefficient comes

$$\frac{1}{h} = \frac{1}{h_w} + \frac{R}{4\lambda_{ef}} \qquad (6)$$

where $h, = q/(T_w - T_s)$ is the near-wall heat transfer coefficient.

2.2. Procedure

The effective thermal conductivity is determined as follows (Quinton & Storrow, 1956).
From Eq (3) it follows that

$$\lambda_{ef} = q \Big/ \left(2R\frac{\partial T}{\partial r^2}\right) \qquad (7)$$

The experimental temperature profiles are approximated with a parabola by the method of least squares after which the effective thermal conductivity can be determined from Eq (7).
Next the temperature T_s is calculated from Eq (5).
By knowing the temperature T_s one can determine the wall heat transfer coefficient h_s.

2.3. Experimental apparatus

The experimental setup used in this study consists of a closed circulation loop. The working fluid (water or 47% -glycerine aqueous solution) was pumped sequentially from a reservoir by a rotary pump into the working section, mixer section, flow-meter section, shell-tube heat exchanger and than it returned into the reservoir.
The working section was a copper tube with inner diameter D= 52 mm, wall thickness of 1.6 mm and length of 566 mm. Twelve nichrome-constantan thermocouples were caulked in the tube wall. A thin layer of electrical (mica) insulation was applied to the outer tube wall, and over it a nichrome tape of 0.5x10mm cross-section was wound. The outside of the test section was covered by a layer of thermal insulation. Together with the wall temperature, the fluid-temperature at both the working section inlet and outlet, the heater temperature and the pressure drop across the working section were all measured. As a grainy medium, glass balls with diameter d = 0.9, 3.2, and 8.9 mm were used.
Temperature profiles over the packed tube cross-section were measured by a comb-type probe of nine regularly spaced thermocouples. The thermocouples were made from nichrome and constantan wires of 0.1mm diameter.
The thermocouples next to the tube wall were positioned at distances of 6mm. (It should be pointed out that Quinton & Storrow (1956) measured the temperature profiles directly at the outlet of the packing).

3. RESULTS AND DISCUSSION

Experiments were conducted over a heat flux range from 0.11 to 3.2 Kw/m^2 and at the moderate temperature differences (3÷5°C) between the wall and working fluid so as to eliminate the effect of natural convection.

Experimental data for various flow rates and heat input have been analysed to give h, h, and λ_{ef}.

According to the procedure of determining λ_{ef} the temperature profiles were measured in the thermally developed region where the heat transfer coefficient and the normalized temperature $(T_w-T)/(T_w-T_0)$ become independent of the axial position.
Fig.1 shows a typical example of the variation of the heat transfer coefficient expressed by the Nusselt number Nu_d based on the ball diameter d as a length scale along the working section for the case of 47%-glycerine aqueous solution and [illegible].

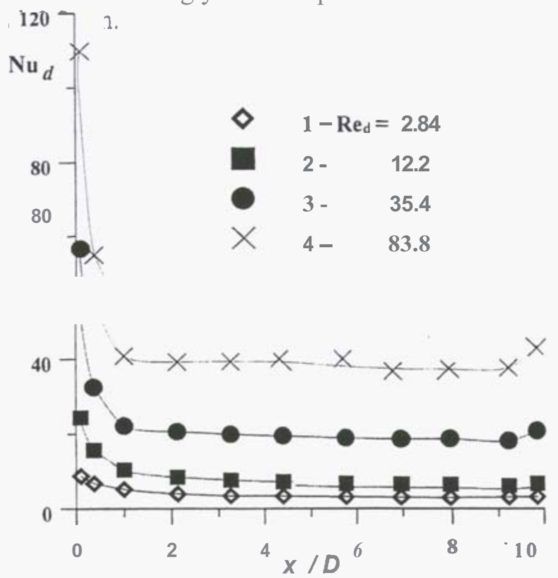

Figure 1. Local heat transfer coefficient along tube length with 47%-glycerine aqueous solution as a working liquid and d=3.2 mm.

3.3 Temperature profiles and effective thermal conductivity

For experiments with spheres of diameter 3.2 mm and 0.9 mm thermocouples for temperature profile measurements were placed at x= 467 mm, or x/D = 9.03. In the case of d = 8.9 mm the radial temperature distributions were measured at x/D= 8.01.

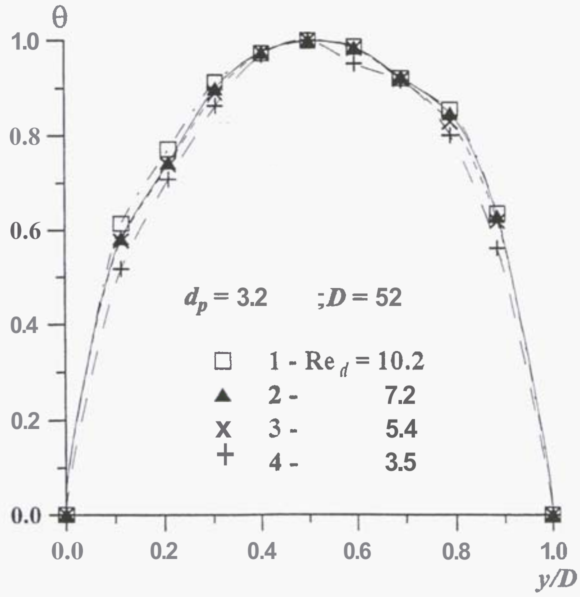

Figure 2. Radial temperature profiles of glycerine aqueous solution flow in tube packed with spheres of d = 3.2 mm at small Reynolds numbers.

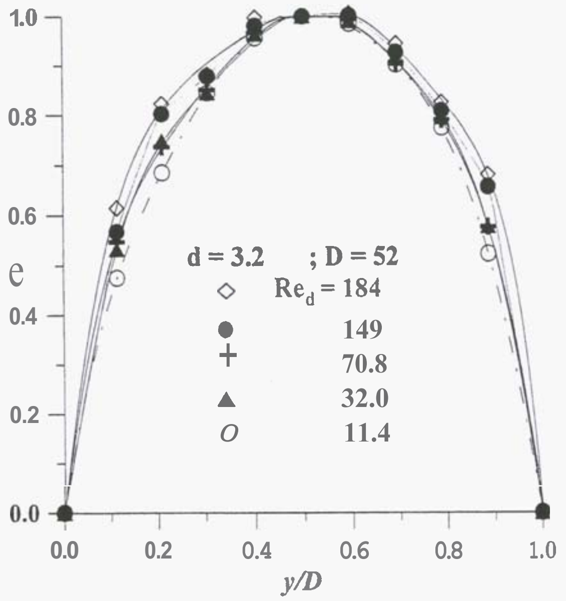

Figure 3. Radial temperature profiles of water flow in the tube packed with spheres of d = 3.2 mm in inertial and transition flow regimes

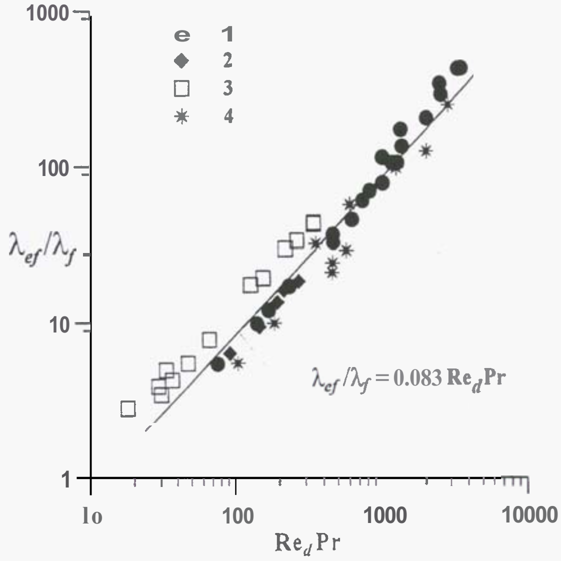

Figure 5. Effective radial thermal conductivity λ_{ef} vs. Pe = $Re_d Pr$
1 - d = 3.2 mm (water); **2** - d = 3.2 mm (aqueous glycerine solution); **3** - d = 0.9 mm (water): **4** - d = 8.9 mm (water);

As can be seen fiom Figure 4 the measured temperature values for the comparatively close Reynolds numbers for the cases of the 47%-glycerine aqueous solution and water for the sphere diameter of 3.2 mm and 8.9 mm and theoretical parabolic profile are in good a ment.

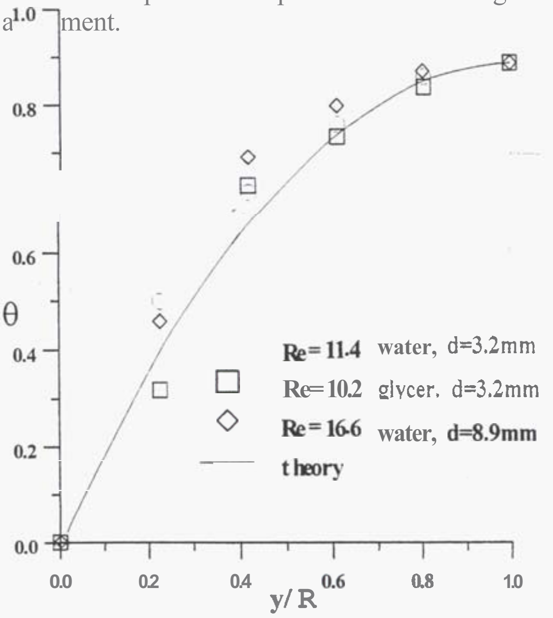

Figure 4. Comparison between measured and calculated parabolic temperature profiles

Experiments showed the temperature profiles became more complete with further increase in the Reynolds number. A sharp drop in fluid temperature in the vicinity of the wall at high Reynolds numbers points to the relative importance of the wall heat transfer resistance.

The measured radial temperature profiles of working fluid flow and heat fluxes have been used in calculating λ_{ef}/λ following the foregoing procedure. The results for the effective thermal conductivity are shown in Fig. 5.

The correlation equation for λ_{ef}/λ_f obtained from the present measurements for large Peclet numbers Pe = $Re_d Pr$ is

$$\lambda_{ef}/\lambda_f = 0.083\, Re_d Pr \qquad (8)$$

The contribution of the stagnant conductivity to the total effective thermal conductivity was found to be an insignificant.

3.2 Wall and overall heat transfer coefficients

On the basis of the measured overall heat transfer h (the maximum experimental error in the heat transfer coefficient is estimated to be 6 %) and the effective thermal conductivity one can estimate the near-wall heat transfer coefficient h_w.

The table below presents estimations of the proportional contribution of the near-wall thermal resistance to the overall thermal resistance for water filtration through the tube packed with the spheres of diameter d = 3.2 mm:

Re_d	388	207	170	71	33.6	11.4
h_w^{-1}/h^{-1}	0.90	0.83	0.77	0.74	0.69	0.65

As could be seen from the Table the wall heat transfer resistance becomes the more dominant the bigger the Reynolds number. The thermal resistance of packed beds in turbulent filtration actually is determined by turbulent separated flow in the near-wall zone (Gorin et al., 1996).

The wall heat transfer coefficient is usually expressed by Nu,,,, based on the thermal conductivity of the fluid. The plot of Nu_w versus Red is presented in Fig. 6.

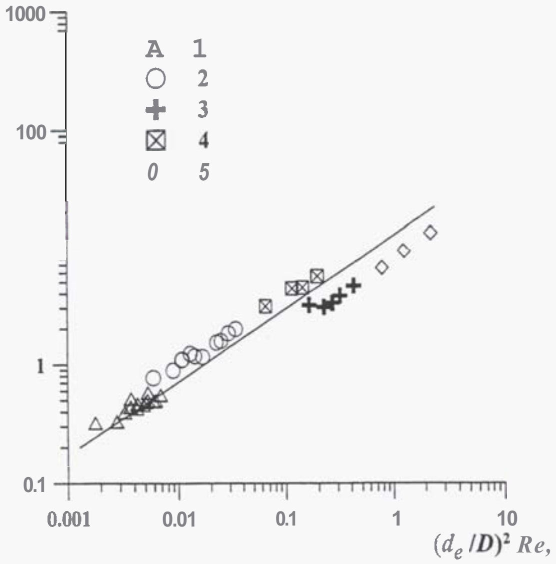

Figure *6*. Near-wall heat transfer at moderate Reynolds numbers
1 - *d/D* = 0.062, 47%-glycerine aqueous solution;
2 - *d/D* = 0.062, water (our data);
3 - *d/D* = 0.09, air (Dixon et al., 1984);
4 - *d/D* = 0.0722 , air; **5** - *d/D* = 0.167, air (Yagi & Wakao, 1959).
Solid line corresponds to Eq (9).

Fig. 6 represents also the values of Nu, obtained by other authors for the case of air filtration. These data are in a good agreement with our data and confirm the correlation equation **(9).**

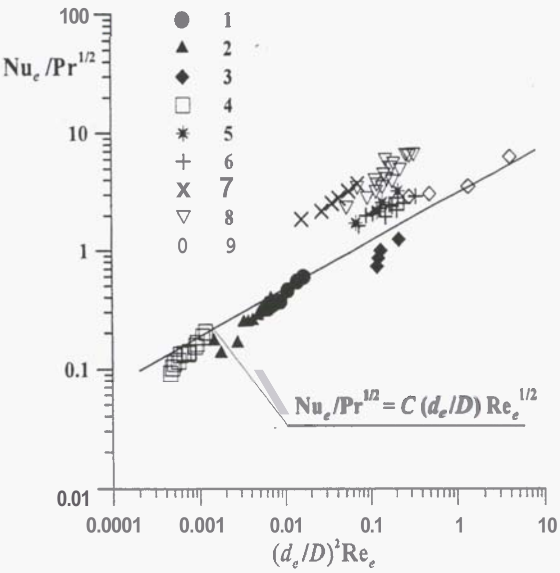

Figure 7. Overall heat transfer coefficient in inertial flow regime
1 – *d/D* = 0.062, water; **2** – *d/D* = 0.062, aqueous glycerine solution; **3** – *d/D* = 0.17, water; **4** – *d/D* = 0.017, water (this paper);
5 – *d/D* = 0.07, air; **6** – *d/D* = 0.11, air (Yagi & Kunii, 1961);
7 – *d/D* = 0.04, air; **8** – *d/D* = *0.07,* air (Verschoor & Schuit, 1950);
9 – *d/D* = 0.098, water (Niles & Martin, 1990);

The correlation equation for Nu_w based on the present experimental results is obtained as

$$Nu_w = 7.5\left(\frac{d_e}{D}\right)Re_e^{1/2}Pr^{1/2} \qquad (9)$$

Experimental values of the overall heat transfer coefficient **are** presented in Fig. 7. The correlation equation completely corresponds in structure to the wall heat transfer correlation (9).

The figure also represents experimental data of some other authors. They confirm the trend $Nu \propto Re^{1/2}$ for the inertial flow regime. It has not been possible to obtain quantitative agreement due to lack of information about conditions of experiments and other parameters that are needed to convert data.
(The authors call the reader's attention to the fact that heat transfer data in Figs 6 **and** 7 were processed in terms of equivalent characteristic length scale $d_e = \frac{2}{3}\frac{\varepsilon}{1-E}d$).
It is significant that the correlations for the overall and near-wall heat transfer coefficients are identical in their structure. What this means is heat transfer mechanisms in the near-wall zone and the core flow have much in common.

It is remarkable also that heat transfer in annular packed beds was demonstrated in our paper (Dekhtyar' et al., 2002) to follow a correlation almost identical with equation (9) both for one- and few-layered packing. **A** simple model **was** proposed in that report to provide one possible explanation for correlation (9). The model, in essence, is this: the velocity field in a packed bed is considered to consist of an array of eddies and jet-type flows adjacent to them (space-periodic cellular velocity field). If convective heat dispersion is determined by heat transfer through these cells then the effective thermal conductivity will be $\lambda_{ef}/\lambda_f = \gamma Pe^{1/2}$, where γ is the coefficient depending on packed bed geometry and flow conditions. In view of the fact that $Nu = hD/\lambda_{ef} = const$ at thermally developed region we can obtain Correlation (9).

4.CONCLUSIONS

Direct measurements of the radial temperature profiles accompanied by the determination of the effective thermal conductivity made possible reasonably accurate quantitative estimation of the apparent wall thermal resistance and its proportional contribution in overall heat transfer in the thermally developed region.

At moderate Reynolds numbers, i.e. for the inertial regime preceding the transition and turbulent ones, the correlations for the overall and apparent heat transfer coefficients are identical in their structure. The employment of the homogeneous model leads to the trend $\lambda_{ef} / \lambda_f \propto Pe^{1/2}$ rather than the linear one that is the most commonly used and appears to be justified for higher Reynolds numbers. More experimental data will be necessary to elucidate heat transfer mechanisms at moderate Reynolds numbers.

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