TWO PHASE FLOW GOVERNING EQUATIONS FOR TRANSIENT GEOTHERMAL WELL AND PIPELINE SIMULATION

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SUMMARY – Governing equations for transient gas-liquid flow as a result of the sequential spatial-temporal averaging procedure are presented. This set of equations forms the basis for the elaboration of the friction losses calculation technique and one-dimensional models for transient two-phase flow in geothermal wellbores and pipelines for bubbly, slug and churn-turbulent flow patterns. These models comprise two transient partial differential mass conservation equations for each phase and a momentum equation for gas-liquid mixture. The algebraic slip velocity and the perfect gas state equation are added to complete the model. The friction losses are represented by using the concept of turbulent boundary layer with vanishing viscosity with only a single additional empirical coefficient.

1. INTRODUCTION

The equations governing the transient flow of two-phase fluids, the mass continuity, momentum and energy equations, are complex in their general form. Transient simulation of gas-liquid flows requires not only major computing efforts but is crucially dependent upon the development of physically adequate models with minimum empirical relationships and coefficients. One may establish that the development of the topic is taking two paths. The first one meets the nuclear industry needs and the second approach is quite adequate for the oil and gas industry requirements.

In the nuclear industry, these investigations have developed for the study of Loss-of-Coolant Accidents (LOCAs) so that fast transients are of major interest (TRAC-PF1, 1981), while in the oil and gas industry, the realm of research lies in relatively slow transients conditioned by the transport velocities of the two-phase mixture (Black et al., 1990; Bendiksen et al., 1991); Taitel & Barnea, 1997; Masella et al., 1998).

It is worth noting that both of these situations can be realised in geothermal wells and pipelines.

A geothermal well may be considered as a vertical column with liquid flow in the lower part, which flashes in the upper part due to the reduction of the static pressure. Vapour generation persists downstream of the flashing point at a lower rate as a result of bubble nucleation and growth of existing bubbles, with a consequent increase of mixture velocity as the two-phase liquid gradually rises up the well. Geothermal two-phase flow parameters are commonly calculated from steady-state models based on the momentum equation for two-phase mixture with a various body of empirical data (Ambastha & Gudmundsson, 1986; Ansari et al., 1994; Karaalioglu & Watson, (1999).

In the general case the evaluation of the effects of various phenomena occurring during upward flow of the mixture can be carried out with the equations for two-phase flow in pipes and heat and mass transfer between phases.

Gas (vapour)-liquid flows have peculiar features that are inherent in these media.

First at all, gas-liquid flows are characterised by the presence of various forms of motion depending on the mass flow rates and external conditions: ranging from its simplest form of stratified flow with continuous interface, to foaming flow with its complex multiply connected interfaces that are time-varying.

These peculiarities are manifested through the forces of phase interaction (Basset force, virtual mass force, Joukowski force etc.) and as various inertial effects that are embodied in a generalised turbulent shear stress tensor.

Clearly it is impossible to take into account all effects.

A comprehensive academic simulator for transient two-phase flow that includes detailed description of flow patterns, taking into consideration geometrical and regime parameters, phase interaction, thermodynamic properties including metastable state and heterogeneous nucleation and very many other factors would be very complex. This approach would become empirical because of the very large number of coefficients that would have to be proposed.

The computing time of such simulators would be too large for the majority of engineering applications.

Also the effects of channel geometry and gas entry into the channel are strong and it is improbable that empirical correlations will provide the required accuracy. Such an approach may be used to model conditions of designs that differ only a little from those used as the basis of the model and coefficients, but it is unlikely to be suitable for the investigation of new and different problems. For this, quite simple models are needed, as free as possible of empirical coefficients.

The potential for such models is still far from exhausted.

The aim of this study is the elucidation of the more important factors, the development of physically proper qualitative patterns of phenomenon on the basis of simplified governing equations and to allow the possibility of devising an engineering technique for calculation of gas-liquid flow parameters in geothermal wells and pipelines.

The general set of equations for transient gas-liquid flows is proposed.

The case of high Reynolds number is used for the calculation of friction loses on the basis of a homogeneous model justified by the estimation of relative phase velocity and virtual mass. In distinction to the calculative or exquisitely empirical techniques mentioned above, a single additional empirical coefficient is required for the fkiction calculation.

Simplified one-dimensional transient models are formulated on the base of three partial differential equations, two of which are the mass conservation equations for gas and liquid phases, and the third one is the momentum equation for gas-liquid mixture. Phase and mixture velocities are connected through the slip relationship (drift **flux** model).

An additional equation that required completing the model is the equation of state that relates the gas phase density and the pressure. We assume that the gas behaves **as** a perfect gas, i. e. the gas is compressible whereas the liquid is considered incompressible.

These models can be used for prediction of two-phase gas (vapour) ₓ liquid flow in geothermal wells and pipelines.

Some algebraic manipulation will be omitted for reasons of report space and we will give final results only except in situations being crucial importance for understanding of physical model and assumptions

2. THEORY OF METHOD

2.1. General equations

The problem of derivation of averaged equations for the description of transfer phenomena in multiphase flows has attracted the attention of researchers for many years.

Frankl (1953) was among the first to propose the general set of differential equations for turbulent flow of incompressible liquid with suspended solid particles and with no phase change. The rule of averaging of a certain function Φ_n over volume Ω_n occupied by the n-th phase for the time Δt he wrote in the form

$$\overline{\Phi}_n = \frac{\overline{\Phi'_n \varphi'_n}}{\varphi_n}$$

where Φ_n are the functions characterising the n-th component, such **as** the density ρ, the velocity v, the external force X and the molecular stress tensor Π, with the concentration φ_n in the region Ω.

But Frankl's approach did not take into account the fluctuations of concentration and their correlations with the kinematic characteristics of the mixture.

To eliminate this limitation Djunin (1961) proposed a sequential spatial-temporal averaging procedure that is expressed as:

$$\overline{\Phi}_n = \frac{1}{\Omega\varphi}\int_{\Omega}\Phi_n d\Omega; \quad \overline{\overline{\Phi}}_n = \frac{1}{\Delta t}\int_{\Delta t}\overline{\Phi}_n dt = \overline{\left(\frac{\Phi_n}{\varphi_n}\right)}$$

The sequential spatial-temporal averaging of Φ_n yields the following relation

$$\overline{a,\varphi_n} = a\overline{\varphi_n} + \overline{\Phi'_n\varphi'_n}$$

Here prime denotes the fluctuations.

Averaged continuity **and** momentum equations are

$$\frac{\partial \rho_n \varphi_n}{\partial t} + \frac{\partial}{\partial x_k}\rho_n\left(\varphi_n v_{nk} + \overline{\varphi'_n v'_k}\right) = 0 \qquad (1)$$

$$\frac{\partial}{\partial t}\rho_n\left(\varphi_n v_{ni} + \overline{\varphi'_n v'_n}\right) + \frac{\partial}{\partial x_k}\rho_n\varphi_n v_{nk} v_{ni} = \rho_n\left(\varphi_n X_{ni} + \overline{\varphi'_n X'_{ni}}\right) - \frac{\partial}{\partial x_k}\tau_{nnik} \qquad (2)$$

Hereafter, the summation signs Σ over n and overscribed **bar** denoting the sequential spatial-temporal averaging of functions are omitted (but not for the correlations); $i, k = 1,2,3$.

It should be noted that with no phase change equation (1) is an analog of Schmidt's diffusion equation (Schmidt, 1925).

τ_{nnik} is the component of the generalised averaged tensor of the molecular and turbulent shear stresses of the n-th phase:

$$\tau_{nnik} = \Pi_{nik}\varphi_n + \overline{\Pi'_{nik}\varphi'_n} + \rho_n\left(v_{ni}\overline{v'_{nk}\varphi'_n} + \varphi_n\overline{v'_{ni}v'_{nk}} + v_{nk}\overline{v'_{ni}\varphi'_n} + \overline{v'_{ni}v'_{nk}\varphi'_n}\right) \qquad (3)$$

The set of equations (1) and **(2)** with the shear stress tensor (3) is most general view of governing equations.

The employment of these equations is possible on a basis of simplified physical models of transfer processes in one phase or another, phase interaction, taking into consideration flow patterns and regimes and initial and boundary conditions. **As** an example we refer to a very simplified version of these equations that was implemented by Djunin (1963) for the particular case of a gas-solid particle flow, namely, the snowstorm mechanics.

Let us take up some potentials of the sequential spatial-temporal averaging procedure as applied to gas (vapour) - liquid flows.

For the case of the gas-liquid flow with void fraction φ in terms of the mass flow rates G we will have the next general set of equations (again with no phase change):

$$\frac{\partial \rho_l(1-\varphi)}{\partial t}+\frac{\partial \overline{G}_{lk}}{\partial x_k}=0$$

$$\frac{\partial \rho_g \varphi}{\partial t}+\frac{\partial \overline{G}_{gk}}{\partial x_k}=0$$

$$\frac{\partial G_{li}}{\partial t}+\frac{\partial G_{lk}v_{li}}{\partial x_k}=\rho_l\left[(1-\varphi)X_{li}+\overline{(1-\varphi)'X'_{li}}\right]-\frac{\partial \tau_{llik}}{\partial x_k} \quad (4)$$

$$\frac{\partial G_{gi}}{\partial t}+\frac{\partial G_{gk}v_{gi}}{\partial x_k}=\rho_l\left[\varphi X_{gi}+\overline{\varphi' X'_{gi}}\right]-\frac{\partial \tau_{ggik}}{\partial x_k}$$

where suffixes l and g denote liquid and gas phases respectively;

$$\vec{G}_l=\rho_l(1-\varphi)\vec{v}_l, \quad \vec{G}_g=\rho_g\varphi\vec{v}_g$$

$$\vec{G}_n=\rho_n\varphi_n\vec{v}_a+\rho_n\overline{\varphi'_n\vec{v}'_n}$$

This set of Eqns **(4)** will be used as the unified methodological basis for the following simulation of transient gas-liquid flows.

2.2. Friction losses

Let us take a look at the turbulent shear stress tensor for two-dimensional flow. Assuming that

$$u'_l\approx u'_g=u', \quad v'_l\approx v'_g\approx v', \quad u_g=u_l+u_B$$

where u_B is the bubble-rise velocity (the terminal rise velocity of a single bubble in stagnant liquid). Disregarding triple correlations such as $\overline{u'v'\varphi'_n}$ and using the definition (3), **we** have:

$$\tau=\rho_l\left[u_l\overline{v'(1-\varphi)'}+(1-\varphi)\overline{u'v'}+v\overline{u'(1-\varphi)'}\right]+ \quad (5)$$
$$+\rho_g\left[u_l\overline{v'\varphi'}+\varphi\overline{u'v'}+v\overline{u'\varphi'}\right]$$

But $\rho_g u_B\overline{v'\varphi'}<<\rho_l u_l\overline{v'(1-\varphi)'}$ because $\rho_g<<\rho_l$

For the gas-liquid mixture of the velocity u and the density of $\rho=\rho_l(1-\varphi)+\rho_g\varphi$ the turbulent tensor then takes the form

$$\tau=\mathbf{u}_t\overline{v'\rho'}+\rho\overline{u'v'}+v\overline{u'\rho'} \quad (6)$$

For a **flow** in a channel $\mathbf{v}=0$ we can write

$$\tau=\rho\overline{u'v'}+u\overline{\rho'v'} \quad (7)$$

Notice that this equation was appeared in the article by Levy **(1963).** But this fact **has** to be regarded as a coincidence because the analogy with compressible liquid and the correct temporal averaging on the basis of the rules used by Levy does not give **an** additional contribution like $u_l\overline{v'\rho'}$ into the turbulent shear stress.

The relation **(7)** can be used as a basis for the calculation of friction loses in gas-liquid mixtures. Assuming the turbulent momentum and mass transfer to be the same, we define the fluctuation components according to mixing length theory.

So we can write

$$\tau=l^2\frac{du}{dy}\frac{d(\rho u)}{dy} \quad (8)$$

where l is the mixing length.

The relation between ρ and u can be found using the assumption of similarity for the fields of the velocity and mass concentration. Further using various laws for the shear stress variation over the boundary layer (channel) thickness and for the mixing length l one can solve Eq **(8)** with respect to the distribution of velocity u. This is the conventional approach used by the majority of authors. But in this way some problems arise, which are related to the application of the turbulence models, i.e. the closure schemes, and empirical constants from the single-phase theory of turbulence to multiphase flow.

One can avoid some difficulties intrinsic to the conventional approach by using the theory of turbulent boundary layers with vanishing viscosity, which was developed by Kutateladze & Leont'ev **(1964, 1972)** for the calculation of single-phase turbulent boundary layers. The physical essence of the theory of turbulent boundary layers with vanishing viscosity lies in the fact that in considering some ideal turbulent boundary layer, the integral characteristics of heat and mass transfer are mainly determined by the properties of the conservative part of the turbulent core, and their relative changes under the complex conditions of nonisothermality, fluid injection, physical and chemical transformations, etc. do not depend on empirical constants and are not related to any kinds of the semiempirical theories.

The concept was used in our previous studies (Gorin, **1978, 1982;** Nakoryakov & Gorin, **1994)** for the calculation of the wall shear stress τ_W in steady-state two-phase turbulent boundary layers and two-phase flow in tubes and on a permeable plate and where the limiting (Re→ ∞) relative friction law for homogeneous two-phase flow with variable density was formulated as:

$$\Psi_\infty^{1/2}=\left(\frac{\rho_l}{\rho_s}\right)^{1/2}\int_0^1\frac{\tilde{l}}{\tilde{l}_0}\left(\frac{\tilde{\tau}}{\tilde{\tau}_o}\right)^{1/2}\frac{d\tilde{u}}{1-(1-\rho_l/\rho_s)\tilde{u}}$$

where $\Psi_\infty=(5/\varsigma_0)_{\mathrm{Re}\to\infty}$ is the relative resistance.

We have used here the next notations:

$\tilde{u}=u/u_s$; $\tilde{\tau}=\tau/\tau_W$; $\tilde{l}=l/\delta$; $\tilde{y}=y/\delta$;

$\varsigma=2\tau_W/(\rho_s u_s^2)$.

The Reynolds number is defined by the mixture velocity and the liquid viscosity.

The **flow** of an incompressible liquid with the density ρ_s and the velocity u_s of mixture at the outer edge of the boundary layer (tube axis) corresponding to those of a homogeneous mixture is taken as a "standard" flow. These parameters

are marked by the index "0". For the case of a tube, instead of the boundary layer thickness δ one should use the tube radius R and the friction factor is then defined by the mixture velocity averaged . over . the tube cross section $\varsigma = 2\tau_W / \left(\rho_s \langle u \rangle_s^2\right)$.

Making some simplifying assumptions one can calculate key parameters of the two-phase mixture:

- the mixture velocity profile

$$\frac{u}{u_s} = \left(\tilde{y}^n - \frac{\rho_s}{\rho_l}\right) \Big/ \left(1 - \frac{\rho_s}{\rho_l}\right)$$

- the void fraction profile

$$\frac{\varphi}{\varphi_s} = \left(1 - \tilde{y}^{-n}\,\frac{\rho_s}{\rho_l}\right) \Big/ \left(1 - \frac{\rho_s}{\rho_l}\right)$$

- the density of mixture distribution

$$\rho / \rho_s = \tilde{y}^{-n}$$

The final algorithm for the calculation of the basic parameters of steady-state gas-liquid flow in a tube (without derivation and details) is performed as follows.

At first, find the average parameters of the flow.

1. An average density of mixture is defined as ($\tilde{y} = y / R$)

$$\langle \rho \rangle = 2\int_0^1 (1-\tilde{y})\rho d\tilde{y} = \frac{2\rho_s}{(1-n)(2-n)} \qquad (9)$$

2. The mixture velocity averaged over the cross-section has the form

$$\langle u \rangle = \frac{u_s}{1-\rho_l/\rho_s}\left[1 - \frac{\rho_l}{\rho_s}\frac{2}{(1+n)(2+n)}\right] \qquad (10)$$

3. The total mass flow rate per unit area is

$$G = \frac{\rho_l u_s}{1-\rho_l/\rho_s}\left[\frac{\langle \rho \rangle}{\rho_l} - 1\right] \qquad (11)$$

4. The quality is

$$x = \frac{G_g}{G} = \frac{u_s / \langle u \rangle}{\rho_l/\rho_g - 1}\,\frac{1-\rho_l/\rho_s}{\langle \rho \rangle/\rho_l - 1} \qquad (12)$$

Here $n = -\frac{h}{2\kappa}\left(\frac{\varsigma_0}{2}\right)^{1/2} ln\frac{\rho_s}{\rho_l}$; κ is the Karman constant.

The deformation coefficient h for the velocity profile in a single-phase flow can be calculated fiom the well-known power velocity profiles for the respective range of Reynolds number (standard data).

And now we can find the Martinelly factor $X_{L0}^2 = \tau_W / \tau_0$:

$$X_{L0}^2 \varsigma_0 = 8k^2 n^2 \kappa^2 \left[1 - \frac{\rho_s}{\rho_l}\right] \frac{2}{(1-n)(2-n)} \qquad (13)$$

As was shown by Gorin and Nakoryakov & Gorin (ibid) through the comparison of the direct measurements of wall shear stress for two-phase flow with the calculated results from the simple algebraic formulae (10) – (13), the single additional empirical coefficient k can be taken to be equal to 1.17 for the majority of examined experimental data in bubbly, slug and churn-turbulent regimes.

2.3, One-dimensional models

The difficulties of correct and detailed accounting for external forces and phase interactions, plus mathematical difficulties make the full solution of the general set of transient transport equations impracticable.

The only rational way to proceed is physically justified simplification.

The one-dimensional models are the first option, and the potential of these models is still far fiom exhausted.

Mass conservation equations for the gas and liquid phases can be written as (by neglecting the diffusion transport along longitudinal z-axis)

$$\frac{a}{\partial t}\rho_g \varphi + \frac{a}{\partial z}\rho_g \varphi u_g = 0 \qquad (14)$$

$$\frac{\partial(1-\varphi)}{\partial t} + \frac{\partial}{\partial z}(1-\varphi)u_l = 0 \qquad (15)$$

The liquid phase is considered to be incompressible.

Hereinafter the corner brackets denoting the cross-sectional averaging are omitted because of the cumbersome arrangement of formulae.

The momentum equation for a two-phase mixture results from the summation of those for the liquid and gas phases in a channel with the circumference U and the cross-section S (for circular tubes $U/S = 2/R$):

$$\frac{\partial G}{\partial t} + \frac{\partial}{\partial z}Gu = -\frac{dp}{dz} - \frac{\tau_W U}{S} - \rho_m g \mathrm{Sin}\,\vartheta \qquad (16a)$$

θ is the angle of inclination of the pipe with respect to the horizontal.

The flow momentum Gu is composed of that of the gas and that of the liquid according to

$$Gu = G_g u_g + G_l u_l$$

Together with the definitions of the phase mass flow rates one can also write for the flow momentum

$$Gu = G^2\left[\frac{x^2}{\varphi\rho_g} + \frac{(1-x)^2}{(1-\varphi)\rho_l}\right],$$

so that the flow momentum equation takes the form

$$\frac{\partial G}{\partial t} = -\frac{dp}{dz} - \frac{\tau_W U}{S} - G^2 \frac{d}{dz}\left[\frac{x^2}{\varphi\rho_g} + \frac{(1-x)^2}{(1-\varphi)\rho_l}\right] - \rho_m g \mathrm{Sin}\,\vartheta \tag{16b}$$

where the three last terms are the friction losses, gravitational gradient and pressure drop due to acceleration.

The momentum equation can also be used in the form

$$\frac{\partial G}{\partial t} = -\frac{dp}{dz} - \frac{2\tau_W}{R} - G\frac{du}{dz} - \rho_m g \mathrm{Sin}\,\vartheta \tag{16c}$$

The gas phase and mixture velocities can be connected through the slip relation (drift-flux model):

$$u_g = Cu + u_r$$

In the bubbly and the slug flow regimes, for the local drift velocity u_r one can use either (Zuber & Findlay, 1965):

$$u_r = u_B(1-\varphi)^m \tag{17a}$$

or (Wallis, 1969)

$$u_r = u_B\varphi(1-\varphi)^m \tag{17b}$$

where the exponent changes from $m = 0$ to $m = 3$ depending on the bubble size and u_B is the terminal rise velocity of a single bubble in an infinite medium.

The value of the rise velocity can be calculated by the following formulae:

$$u_r = 1.53\left[\frac{\sigma g(\rho_l - \rho_g)}{\rho_l^2}\right]^{1/4} \tag{17c}$$

for the chum-turbulent bubbly flow regime, and

$$u_r = 0.35\left[\frac{g(\rho_l - \rho_g)d}{\rho_l}\right]^{1/2} \tag{17d}$$

for the slug flow.

The constant C characterises the relationship between the void fraction and flowing volumetric quality, and the value of this coefficient is usually determined by experiments. In our case c can be calculated from the profiles of the mixture velocity, the void fraction and the density of mixture above (Sect. 2.2) by analogy with Bankoff (1960) or Zuber & Findlay (1965).

An additional equation required to complete the model is the state equation that relates the gas phase density to the pressure. One can take that the gas phase behaves as a perfect gas ($\Re$ is the gas constant):

$$P = \rho_g \Re T \tag{18}$$

For isothermal flow, the temperature T is constant.

For *slow transient processes* (they are characterised by small perturbations of low frequencies with large wavelength compared to dimensions of structural parameters of the flow pattern considered) the momentum equation takes the form (quasi-equilibrium momentum balance):

$$-\frac{dp}{dx} = \frac{2\tau_W}{R} + G\frac{du}{dz} + \rho_m g \mathrm{Sin}\,\vartheta \tag{19}$$

In case of *large tube diameter* one can neglect by the friction losses. ***So,*** we will have

$$-\frac{dp}{dz} = G\frac{du}{dz} + \rho_m g \mathrm{Sin}\,\vartheta \tag{20}$$

So, equations (9) – (13), (14) – (16), (17) and (18) under given boundary and initial conditions complete the problem of simulation of *transient two-phase **flow***. Eqs (19) or ***(20)*** can be used instead of **Eq** (1***6)*** depending on the character of the problem.

Notice also that the set of Eqs (19) (or (20)), (9)-(13), (17) and (18) is, in fact, the calculation method for *steady-state two-phase flows*.

To use one or other formulae (17) for the local drift velocity the flow regime needs to be recognised. Current state of art does not allow circumventing the use of one of the empirical regime maps in spite of the fact that they were obtained for particular cases and their use for other cases must be assumed approximate.

The particular type of the flow momentum equation used, i.e. (16a), (16b) or (16c) depends on purpose of the task and the mathematical technique to be used.

3. CONCLUSIONS

The main purpose of this report is to present the ideas and governing equations derived on the basis of sequential spatial-temporal averaging procedure for transient two-phase flow and its simplified one-dimensional versions (models) for geothermal wellbores and pipelines with bubbly, slug and chum-turbulent flow regimes. The governing equations have provided also the basis for the friction loss calculation. The slip velocity relation, the perfect gas state equation and the assumption of liquid incompressibility have been used.

The evaluation test of models is in progress now.

One should bear in the mind that in real two-phase systems, the gas generally does not behave **as** a perfect gas and the liquid phase properties can vary with temperature and pressure.

Geothermal fluids are mixtures of water, salts and noncondensable gases. In principle, correctives to account for these effects can be easily added to the present models.

It should be noted also that certain of the formulae cited above (for example, these for the relative phase velocities) are not final recommendations but simply reasonable suggestions. This is also

true for the estimation of transient flow pattern transitions. The problems mentioned are opened for many variations and different approaches. Subsequent extension of the developed general approach and one-dimensional models depends on the applications and particular versions should be built to suit requirements and having regard to the peculiarities of the problem.

4. ACKNOWLEDGEMENTS

The author acknowledges the value of discussions with his colleagues, Xinli Lu and Arnold Watson, in the development of this paper, and for editorial assistance.

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