TEMPERATURE RISE AROUND A WELLBORE FILLED WITH NUCLEAR WASTE

I.M. **KUTASOV** AND M. KAGAN

School of Petroleum Engineering, University of New South Wales, Sydney, Australia

SUMMARY – A simple empirical equation is used to approximate the rate of heat generation as a function of time for two types of nuclear spent fuel. This equation is utilized to determine the temperature rise around a wellbore filled with nuclear waste. It is assumed that (prior to placing the nuclear waste into wellbore) several insulating layers (casing) were used to protect the nuclear waste. An effective radius concept is introduced to evaluate the effect of casing's thermal resistance on temperature rise in the wellbore.

1. INTRODUCTION

At present storage of the nuclear waste in the subsurface cavities is the most effective technique to avoid the pollution of the man's environment. Several types of rock (rock salt, granite, shale, basalt) have been considered as candidates of repository rocks (Blesh et al., **1983;** Kappelmeyer and Haenel, **1974;** Mufti, **1971).** It seems that rock salt because of its impenetrability to water can be considered as an ideal repository rocks candidate. However, at high temperatures rock salt becomes very plastic and in addition rock salt has a low melting temperature (about **770°C).** For this reason, as was mentioned by Mufti **(1971),** any attempt to dispose nuclear waste (a powerful heat source) into a salt cavity must be preceded by an evaluation of the transient temperature field in an around the cavity. General mathematical solutions have been obtained to evaluate the temperature rise around cavities of spherical, cylindrical and other shapes (Mufti, **1971).** The solutions are expressed through complex integrals (with Bessel functions for a cylindrical cavity) and numerical evaluation of these integrals should be used. The major benefits of a numerical solution are the exibility provided for complex geological features and non-linear problems. At the same time the numerical analysis does not reveal basic relations between thermal properties of formations and the temperature field. An analytical approach can mitigate this disadvantage of a numerical solution and will allow estimates of variations in thermal conductivity, specific heat, and density of formations on temperature field. For example, the thermal conductivity of a generic salt at 50°C and at 300°C is 5.02 and **2.49** W/m °C (Blesh et al., **1983).** An analytical solution is obtained for a spherical cavity when the nuclear waste (which contains mainly long half-life isotopes) can be considered as a constant source of heat (Kappelmeyer and Haenel, **1974).**

Below we will consider a long cylindrical cavity (with a large length/diameter ratio) filled with nuclear waste. In this case the cavity (wellbore) can be considered as an infinite cylindrical source of heat.

Calculations conducted by Mufti **(1971)** revealed that for practical purposes a cylinder whose length is **5** times or more its diameter can be treated as an infinite cylinder.

For this simple case the temperature field in and around the wellbore is a function of time, radial distance, thermal diffiivity of formations, wellbore thermal resistance (skin), and time dependent heat production.

A simple empirical equation is used to approximate the rate of heat generation as function of time for two types of nuclear spent fuel. This equation is utilized to determine the temperature rise in and around a wellbore. It is assumed that (prior to placing the nuclear waste into wellbore) several insulating layers (casing) were used to protect the nuclear waste. An effective radius concept is introduced to evaluate the effect of casing's thermal resistance on temperature rise in the wellbore.

2. DIMENSIONLESS PARAMETERS

The dimensionless time and dimensionless distance are defined by:

$$t_D = \frac{\chi t}{r_i^2}; \qquad R = \frac{r}{r_i} \qquad (1)$$

where χ is the thermal diffusivity of the formation, r_i is the inside radius of the casing (insulating ring), t time, and r is the radial distance (at the well axis $r=0$).

The dimensionless temperature is defined by

$$T_D(R,t_D)=\frac{T(r,t)-T_i}{M}; \quad M=\frac{q_{ref}}{4\pi k}; \qquad (2)$$

where k is the thermal conductivity of the formation and T_i is the undisturbed formation temperature. The reference heat output per unit length per unit time, q_{ref}, is defined below.

3. APPROXIMATION OF THE HEAT SOURCE

In a nuclear waste repository the temperature rises are a result of the heat generation from radioactive decay. Usually the heat generation at the source plane is modelled with a sum of exponential decay terms (Blesch et al., 1983). This model is given by the expression

$$S=Q\sum_{n=1}^{N}E_n e^{-\lambda_n t} \qquad (3)$$

where S is the decay heat source, λ_n is the nuclide exponential decay constant (n =1, 2,..., N), E_n is the coefficient of decay heat source (n =1, 2,..., N), Q is the initial areal thermal loading of decay heat source, and t is time. Now we can define the reference heat output per unit length per unit time

$$q_{ref}=2\pi r_i Q \qquad (4)$$

Below we consider the CANDU and PWR spent fuels (Blesch et al., 1983). CANDU spent fuel can be represented with nine terms and PWR spent fuel can be presented with five terms. The fitted constants for both spent fuel types were obtained by Beyerlein and Claiborne (1980) and are presented in Table 1.

Equation 3 can be written in the form:

$$S=QF(t) \qquad (5)$$

$$F(t)=\sum_{n=1}^{N}E_n e^{\lambda_n t} \qquad (6)$$

We have found that the function $F(t)$ can by approximated by the following formula

$$F(t)=\frac{1}{a+bt} \qquad (7)$$

or

$$F(t)=\frac{1}{a+Bt_D}\; ; \quad B=\frac{br_i^2}{\chi} \qquad (8)$$

The values of b and a are listed in Table 2

Table 1. Constants of source terms for CANDU and PWR spent Fuel (Beyerlein and Claiborne, 1980).

Term	CANDU		PWR	
	λ_n 1/year	E_n	λ_n 1/year	E_n
1	$1.04\cdot10^{-4}$	$3.0716\cdot10^{-2}$	1.7958	$1.2746\cdot10^{-2}$
2	$2.80\cdot10^{-5}$	$2.4394\cdot10^{-2}$	$2.1303\cdot10^{-2}$	$7.0878\cdot10^{-3}$
3	$7.90\cdot10^{-3}$	$9.2520\cdot10^{-3}$	$1.7037\cdot10^{-3}$	$1.4163.10^{-2}$
4	$4.80\cdot10^{-2}$	$5.5072\cdot10^{-2}$	$8.4590\cdot10^{-2}$	$1.6229\cdot10^{-2}$
5	$1.60\cdot10^{-2}$	$8.9002\cdot10^{-2}$	$1.9005\cdot10^{-2}$	$5.9003\cdot10^{-2}$
6	$2.30\cdot10^{-2}$	$8.3982\cdot10^{-1}$		
7	6.93 ·	1.4988 10^{-2}		
8	3.47 ·	2.8097 ·		
9	8.88 · 1[illegible]	1.8804 · [illegible]		

4. THERMAL RESISTANCE OF THE WELLBORE

To take into account the effect of casing (the thermal resistance of the r_w-r_i ring) on the transient temperature inside the wellbore, we will use effective radius concept. This approach is widely used in transient pressure and flow well testing (Matthews and Russell, 1967) to evaluate the effect of formation damage (improvement) around the wellbore on the sandface pressure (at $r = r_w$). Firstly, we introduce skin factor (s) – a parameter which allows to determine the extent of well's thermal resistance.

$$s=\left(\frac{k}{k_{ef}}-1\right)\ln\frac{r_w}{r_i} \qquad (9)$$

$$r_{ia}=r_i\exp(-s) \qquad (10)$$

where k_{ef} is the effective thermal conductivity of casing and r_{ia} is the effective inside radius of the casing. The dimensionless time (for $r = r_i$) is based on the effective inside radius of the casing

$$t_{Da} = \frac{\chi t}{r_{ia}^{2}}; \qquad (11)$$

5. DIMENSIONLESS RADIAL TEMPERATURE

It is known that the solutions of the diffisivity equations for a cylindrical and linear source will converge at large dimensionless times (Ramey, 1962). In our case the dimensionless time is large ($t_D > 100$) and the cylindrical source (borehole) can be substituted by a line source. Thus the exponential integral solution (*Ei* – function) can be used. It is also well known that the superposition theorem (Duhamel's integral) can be utilized to derive solutions for time-dependent boundary conditions (Carslaw and Jaeger, 1959). In our case for $R > 1$:

$$T_D(R,t_D) = \int_0^{t_D} F(\tau)\frac{d\Psi_D(R,t_D-\tau)}{dt_D}\,d\tau \qquad (12)$$

$$\Psi_D(R,t_D) = -Ei\left(-\frac{R^2}{4t_D}\right) \qquad (13)$$

$$\frac{d\Psi_D(R,t_D-\tau)}{dt_D} = -\frac{\exp\left[-\frac{R^2}{4(t_D-\tau)}\right]}{t_D-\tau} \qquad (14)$$

where $\Psi_D(R,t_D)$ is the dimensionless radial temperature for a constant linear heat flow rate case.

Using the substitution

$$x = \exp\left[-\frac{R^2}{4(t_D-\tau)}\right] \qquad (15)$$

we obtained ($R > 1$),

$$T_D(R,t_D) = -\frac{\exp(-D)}{a+Bt_D}Ei\left(-\frac{R^2}{4t_D}+D\right) \qquad (16)$$

$$D = \frac{BR^2}{4(a+Bt_D)} \qquad (17)$$

It is not difficult to conduct integration (Equation 12) for time intervals of 40-100 and 100-200 years (spent fuel CANDU, Table 2). We also note that the software Maple V can be used for integration of Equation 12 and numerical calculations. To determine values of $T_D(1,t_D)$ in the last equations the dimensionless time t_{Da} should be used instead of t_D.

6. EXAMPLES OF CALCULATIONS

The computer program Maple **V was** used to calculate the radial dimensionless temperature for cavities filled with PWR and CANDU spent fuels. The results of calculations are presented in Tables 3 and 4 and Figures 1 and 2.

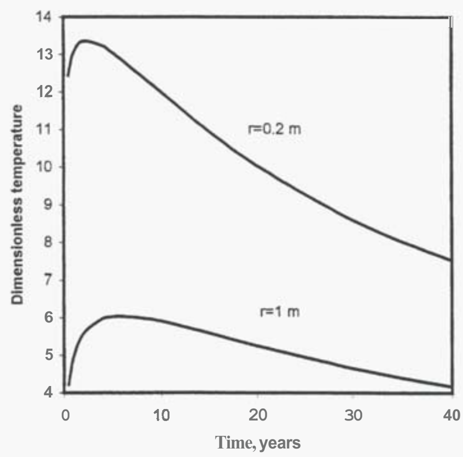

Figure 1. Dimensionless radial temperature. CANDU spent fuel, s=2.

We assumed that the inside radius of casing is 0.20 m and the thermal diffusivity of formations is 35.0 m^2/yr (0.0040 m^2/hr).

From Figures 1 and 2 one can see that the temperature rise sharply reduces with distance. For example, after 10 years the value of $T_D(R,t_D)$ for the distance of 0.8 m (Fig. 1) is reduced from 12.0 to 5.9, while approximately the same reduction in the dimensionless temperature can be observed for the distance of 39 m (R=5, R=200, Table 1). The thermal re-

Table 2. Coefficients *a* and *b* for CANDU and PWR Spent Fuel.

Spent Fuel	Time years	*b* 1/year	*a*	$\Delta F/F_{YO}$
PWR	0 - 200	0.03466	0.90583	3.8
CANDU	0-40	0.03231	0.86056	2.4
	40 - 100	0.06580	-0.58326	3.7
	100 - 200	0.08623	-2.26560	1.6

Table 3. Dimensionless radial temperature, CANDU spent fuel, s =2

Time	Dimensionless distance, R							
years	1	2	5	10	20	50	100	200
0.5	12.443	6.301	4.224	2.688	1.283	0.129	0.001	0.000
1	13.014	6.983	4.936	3.404	1.936	0.417	0.018	0.000
2	13.348	7.526	5.547	4.057	2.594	0.876	0.127	0.001
4	13.245	7.803	5.952	4.553	3.163	1.416	0.401	0.018
5	13.076	7.806	6.013	4.657	3.307	1.585	0.521	0.037
6	12.877	7.769	6.031	4.716	3.404	1.716	0.627	0.062
8	12.444	7.631	5.992	4.753	3.514	1.899	0.800	0.120
10	12.001	7.450	5.901	4.729	3.555	2.015	0.931	0.182
15	10.956	6.952	5.589	4.557	3.522	2.151	1.142	0.325
20	10.047	6.473	5.256	4.334	3.410	2.180	1.254	0.438
25	9.269	6.041	4.942	4.109	3.275	2.161	1.311	0.524
30	8.601	5.657	4.656	3.897	3.136	2.119	1.337	0.587
35	8.023	5.318	4.398	3.701	3.001	2.066	1.343	0.633
40	7.520	5.018	4.166	3.521	2.874	2.009	1.338	0.667

Table 4. Dimensionless radial temperature, PWR spent fuel.

Time	Dimensionless distance, R							
years	2	5	10	20	50	100	150	200
1	6.630	4.687	3.232	1.839	0.396	0.017	0.000	0.000
2	7.142	5.264	3.850	2.463	0.832	0.120	0.011	0.001
5	7.396	5.698	4.414	3.135	1.504	0.495	0.145	0.035
10	7.045	5.581	4.473	3.364	1.908	0.882	0.404	0.172
15	6.564	5.278	4.304	3.328	2.034	1.081	0.587	0.308
20	6.104	4.958	4.089	3.218	2.059	1.185	0.707	0.415
30	5.326	4.384	3.670	2.954	1.997	1.261	0.837	0.554
40	4.718	3.918	3.312	2.705	1.892	1.261	0.887	0.629
50	4.236	3.541	3.015	2.487	1.782	1.231	0.901	0.667
80	3.257	2.758	2.381	2.003	1.497	1.102	0.861	0.685
100	2.831	2.412	2.094	1.776	1.351	1.019	0.816	0.666
120	2.509	2.147	1.873	1.598	1.232	0.946	0.771	0.641
150	2.148	1.848	1.621	1.393	1.090	0.853	0.709	0.601
200	1.742	1.508	1.331	1.154	0.918	0.735	0.623	0.540

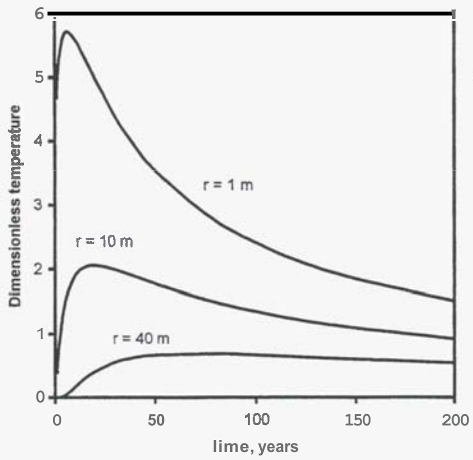

Figure 2. Dimensionless radial temperature, PWR spent fbel.

sistance of casing (expressed through the skin factor) significantly increases the temperature inside the cavity. (Fig. 1, r=0.2 m). At each radial distance the dimensionless temperature has a maximum. For a cavity filled with CANDU spent fbel this maximum at R=2 (r=0.4 m) occurs after approximately 5 years (Table 3). From Tables 3 and 4 follows that the temperature field around the cavity changes very slowly with time. Thus the conclusion that the temperature rise resulting from the deposition of radioactive waste underground is limited to small zones around cavities (Mufti, 1971; Kappelmeyer and Haenel, 1974) is confirmed by our calculations.

7. CONCLUSIONS

An empirical equation is suggested to approximate the rate of heat generation as a function of time for two types of nuclear spent fuel. This equation is used to determine the dimensionless temperature distribution around a cylindrical cavity (wellbore) filled with nuclear waste. An effective radius concept is introduced to evaluate the effect of casing's thermal resistance on temperature rise in the wellbore. It is shown that the temperature rise resulting fiom the deposition of radioactive waste underground is limited to small zones around the cavity.

8. ACKNOWLEDGMENTS

We would like to thank Mr. J. (Nino) Zajaczkowski for assistance in preparing this paper.

9. REFERENCES

Beyerlein, S.M. and Claibome, H.C., 1980. The Possibility of Multiple Temperature Maxima in Geological Repositories for Spent Fuel fiom Nuclear Reactors. ORNL/TM-7024, *Oak* Ridge National Laboratory, *Oak* Ridge, TN.

Blesch C.J., Kulacki F.A., and Christensen R.N., 1983. Application of Integral Methods to Prediction of Heat Transfer fiom a Nuclear Waste Repository. Technical Report ONWI-495.

Carslaw, H.S. and Jaeger, J.C., 1959. ***Conduction of Heat in Solids.*** Second Edition, Oxford University Press, London.

Matthews, *C.S.* and Russel, D.G., 1967. ***Pressure Buildup and Flow*** *Tests* ***in Wells,*** Monograph Series, SPE, 1, Richardson, TX.

Mufti, I.R., 1971. Geothermal Aspects of Radioactive Waste Disposal into the Subsurface", *J. Geophis. Res.,* Vol. 76, 8568-8365.

Kappelmeyer, O. and Haenel, R., 1974. ***Geothermic with Special Reference to Application.*** Gebruder Bomtraeger, Berlin, Stuttgart, 1-240