DETERMINATION OF THE FORMATION PERMEABILITY AND SKIN FACTOR FROM A VARIABLE FLOW RATE DRAWDOWN TEST

I.M. KUTASOV

School of Petroleum Engineering, University of New South Wales, Sydney, Australia

SUMMARY - At present, the flow rate variation during a drawdown test is usually treated as a sequence of discrete constant rates and the principle of superposition is used. Below a more general method of analyzing data of variable flow rate drawdown (VFRD) tests is presented. The flow rate q(t) is considered as a continuous function of time (t). It is assumed that the function q(t) can be approximated by a polynomial of some degree. The pressure response for the VFRD test is obtained by the direct integration of the Duhamel integral. A Cartesian plot and a linear regression computer program are used to process test data and to provide an estimate of the values of skin factor and formation permeability. An example of calculations for a field test is presented.

1. INTRODUCTION

A drawdown test at a constant flow rate is often used to determine formation permeability and the extent of the formation damage (or improvement) around production or injection wells. A control of the wellhead pressure is required to maintain a constant flow rate. In many drawdown tests the sandface flow rate is changing with time. Below we present a technique of processing data of VFRD tests. It is assumed that the flow of a single-phase fluid in an infinite reservoir is described with a diffusivity equation in cylindrical coordinates:

$$\frac{\partial^2 p}{\partial r^2}+\frac{1}{r}\frac{\partial p}{\partial r}=\frac{\phi\mu c_t}{k}\frac{\partial p}{\partial t} \qquad (1)$$

The use of this equation implies a number of conventional assumptions: isothermal flow of fluids of small and constant total compressibility, (c_t); constant porosity, (ϕ); permeability, (k), and fluid viscosity, (μ); and the neglect of gravity forces. It is assumed that the initial reservoir pressure (p_i) is constant throughout the reservoir.

Let it be assumed also that during a variable flow rate drawdown (VFRD) test, the sandface flow rate is arbitrary an continuous function of time.

$$q(t)=q_{ref}\,q_D(t)\neq 0 \qquad (2)$$

$$p(r,0)=p_i; \quad r>0 \qquad (3)$$

Where q_{ref} the reference bottomhole flow rate. To determine the bottomhole flowing pressure during the VFRD test it is necessary to obtain the solution of Equation (1) with the following initial and boundary conditions:

$$q(t)=\frac{2\pi kh}{\mu}\left(r\frac{\partial p}{\partial r}\right)_{r_w}; \quad p(\infty,t)=p_i \qquad (4)$$

where h is the reservoir thickness and r_w is the wellbore radius.

2. DIMENSIONLESS VARIABLES

The dimensionless time is defined by:

$$t_D=\frac{kt}{\phi\mu c_t r_w^{\,2}} \qquad (5)$$

The dimensionless time is very large and in practice the cylindrical source (borehole) is substituted by a line source and the exponential integral solution (Ei – function) can be used.

The dimensionless wellbore pressure drop is defined **by:**

$$p_{wD} = \frac{p_i - p_{wf}(t)}{M}; \quad M = \frac{q_{ref}\ \mu}{4\pi kh} \qquad (6)$$

Let us assume that dimensionless sandface flow rate $q_D(t)$ can be approximated **by** a polynomial **of** some degree n

$$q_D(t_D) = \sum_{i=0}^{n} a_i t^i \qquad (7)$$

$$q_D(t_D) = \sum_{i=0}^{n} b_i t^i \qquad (8)$$

$$b_i = \sum_{i=0}^{n} a_i \left(\frac{t}{t_D}\right)^i \qquad (9)$$

where a_0, a_1, a_2, ...a_n are constant coefficients

3. DUHAMEL'S INTEGRAL

It is well known that the superposition theorem (Duhamel's integral) can be **used** to derive solutions for time-dependent boundary conditions (Carslaw and Jaeger, 1959). In our case :

$$p_{wD}(t_D) = \int_0^{t_D} q_D(\tau) p_D'(t_D - \tau)\, d\tau \qquad (10)$$

where $p_D(t_D)$ is the dimensionless sandface pressure for the constant-rate case without the skin effect.

$$p_D'(t_D) = \frac{dp_D(t_D)}{dt_D} \qquad (11)$$

$$p_D(t_D) = -Ei\left(-\frac{1}{4t_D}\right) \qquad (12)$$

$$p_D'(t_D) = \frac{1}{t_D}\exp\left(-\frac{1}{4t_D}\right) \qquad (13)$$

The skin effect can be included into Equation **(10) by** adding the dimensionless pressure **drop** **2s**q_D and considering Equation **(13)** one obtains:

$$p_{wD}(t_D) = \qquad (14)$$

$$\int_0^{t_D} q_D(\tau)\exp\left[-\frac{1}{4(t_D-\tau)} t_D\right]\frac{d\tau}{t_D-\tau} + 2sq_D(t_D)$$

4. WORKING FORMULAS

Taking into account the conventional assumption that

$$t_D >> 1; \quad \exp\left(-\frac{1}{4t_D}\right) \approx 1 \qquad (15)$$

and using the logarithmic approximation **of** the Ei function we **obtained** (Kutasov, **1987):**

$$\frac{p_i - p_{wf}}{q_D(t)} = M \qquad (16)$$

$$\left[\ln t - \frac{F(t,n)}{q_D(t)} + \ln\frac{4k}{\phi\mu c_t r_w^2} - 0.57722 + 2s\right]$$

where **0.57722** is the Euler's constant and

$$F(t,n) = \sum_{i=1}^{n} a_i t^i \sum_{j=1}^{i} \frac{1}{j} \qquad (17)$$

From a linear plot of
$X = \ln t - F(t,n)/q_D(t)$ versus
$Y = (p_i - p_{wf})/q_D(t)$ the values of the slope (M) and intercept (Int) can be obtained. The formation permeability is computed from Formula (6):

$$k = \frac{q_{ref} B \mu}{4\pi h M} \qquad (18)$$

Skin factor is calculated from the equation:

$$\frac{Int}{M} = \ln\frac{4k}{\phi\mu c_t r_w^2} - 0.57722 + 2s \qquad (19)$$

5. FIELD EXAMPLE

Production rate during a 48-hour drawdown test (Earlougher, 1977; Example 4.1) declined from 1,580 STB/D (10.47 m^3/hr) to 983 STB/D (6.51 m^3/hr). Reservoir data and well data are presented in Table 1 (to calculate the value of the skin factor we assumed the value of c_t and ϕ). For the first 7.5 hours rate decline can be approximated by a linear equation:

$$q(t) = q_{ref}(a_o + a_1 t)$$

From linear regression analysis it was found that:

q_{ref} = 1613.5 STB/D (10.69 m^3/hr)

a_o = 1.00; a_1 = -0.0174 1/hr

The values of X and Y were calculated (Table 2, Figure 1)

$$X = \ln t - \frac{a_1 t}{a_, + a_1 t}$$

$$Y = \frac{p_i - p_{wf}}{a_, + a_1 t}$$

A linear regression computer program was used to determine the slope and intercept in the equation

$$Y = M X + Int$$

Table 1. Input parameters

h = 12.192 m	μ = 0.0006, Pa·s
ϕ = 0.20	p_i = 20.036, MPa
$c_t = 1.305\cdot10^{-8}$ 1/Pa	r_w = 0.1067, m
B = 1.27	

Table 2. Results of field data processing

t hrs	p_i-p_{wf} MPa	X	Y MPa
1.00	6.0881	8.2064	6.1959
1.50	6.4673	8.6210	6.6406
1.89	6.6534	8.8593	6.8797
3.00	6.9913	9.3424	7.3763
3.45	7.0602	9.4909	7.5111
3.98	7.1223	9.6444	7.6522
4.50	7.1637	9.7777	7.7722
5.50	7.2602	9.9993	8.0285
6.05	7.3291	10.1064	8.1914
6.55	7.3912	10.1968	8.3419
7.00	7.4188	10.2733	8.4477
7.50	7.4394	10.3537	8.5560

It was found that
$M = 1.0748\cdot10^6$ Pa ; Int = $-2.6556\cdot10^6$
The formation permeability is computed from Equation (18)

$$k = \frac{(2.969\cdot10^{-3})(1.27)(6\cdot10^{-4})}{(4)(3.1415)(12.192)(1.0748\cdot10^{6})}$$

$$= 1.374\cdot10^{-14}\,(m^2) = 13.92(md)$$

This result is in good agreement with the value of **k** = 13.6 md obtained by the multiple-rate analysis technique ((Earlougher, 1977). From Equation (19) we determine the value of the **skin** factor s = -1.51.

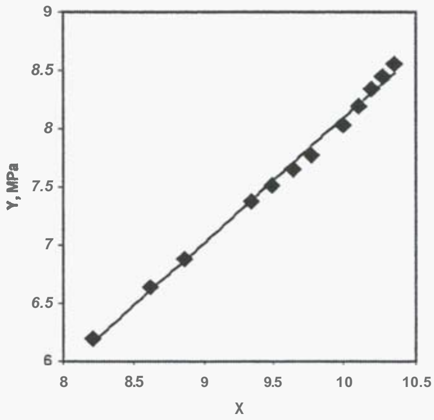

Fig. 1 Plot Y versus X, points - field data

6. CONCLUSIONS

A new method for analysing variable flow rate drawdown tests is suggested. A Cartesian plot or a linear regression computer program can be used to obtain skin factor and formation permeability.

7. ACKNOWLEDGEMENT

We would like to thank Mr. J. (Nino) Zajaczkowski for assistance in preparing this paper.

8. REFERENCES

Carslaw, H.S. and Jaeger, J.C. **(1959).** ***Conduction of Heat in Solids.*** Second Edition, Oxford University Press, London

Earlougher, R.C., Jr. **(1977).*****Advances In Well Test Analysis,*** SPE, New York, Dallas.

Kutasov, I.M. **(1987)** Pressure analysis for a variable flow rate drawdown test, SPE paper **17265.**