SIMILARITY SOLUTION FOR DOUBLE NATURAL CONVECTION WITH DISPERSION FROM SOURCES IN AN INFINITE POROUS MEDIUM

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SUMMARY - Scale analysis of coupled heat and mass transfer from a point or a horizontal line source in an infinite saturated porous medium is reported is this paper. Conservation equations are shown to have solutions of similarity form for generalized variables in the case of flow when dispersion is predominant over molecular diffusion. Closed-form solutions are presented for Darcy and non-Darcy natural convection for both point and line sources. The estimations of distances from sources where the solutions obtained will be valid are given.

1. INTRODUCTION

Natural convection phenomena in porous media can be associated with simultaneous heat and mass transfer. Buoyancy forces that drive such flows arise not only from density differences due to variation in temperature but also from those due to variations in solute (chemical species) concentration. Examples are found in many natural and technological applications such as geothermics, geophysics, grain storage installations, the dispersion of chemical contaminants through water-saturated soil, and underground disposal of nuclear wastes. This problem also finds applications in chemical industry.

Transfer processes around concentrated sources have a significant place in research of the mechanism of coupled double diffusive natural convection. Relative to the research activity on the flow around concentrated sources induced by thermal buoyancy alone, the problem of the near-source convection driven by two buoyancy effects has received quite limited attention. Even in the review by Trevisan and Bejan **(1990)** devoted solely to the combined transfer processes by natural convection, and in the book by Nield and Bejan **(1999)** containing an exhaustive bibliography on convection in porous media, only a few papers are cited on this problem. The transient and steady state flow near a point source of heat and mass in the low Rayleigh number regime was the subject of investigation by Poulikakos **(1985).**

The corresponding problem for the vicinity of a horizontal line source was analyzed by Larson and Poulikakos **(1986).** The solutions were obtained by means of perturbation analysis in the thermal Rayleigh number. The effect of species diffusion on the buoyancy induced by temperature and flow fields near the concentrated heat and mass sources in porous medium were discussed. The high Rayleigh number regime of coupled double diffusive natural convection from a line source in porous medium for Darcy flow has been considered by Lai **(1990).** It was shown that the boundary layer equations can be written in terms of the similarity variables for power-law variation of centre-line temperature and concentration and have closed-form solutions for the special case of Lewis number $Le = 1$.

In the comparatively recent paper by Telles and Trevisan (1993) the problem of dispersion in natural convection heat and mass transfer for the case of vertical surfaces embedded in a porous medium was analyzed. The authors focused on the boundary layer regime for Darcy flow through a porous medium and studied the effect of hydrodynamic dispersion in porous media on both heat and mass transfer in natural convection flows. It was shown that a few different classes of problem exist depending on the dispersion coefficients. They presented several numerical solutions of the systems of similarity equations, including the case when the thermal and the mass dispersions supersede the molecular diffusion. However the problems of dispersion heat and mass transfer near the source in porous media have escaped scrutiny.

Before proceeding to an analysis of the combined effect of the molecular and the dispersion mechanisms we have to have some asymptotic solutions for each mechanism individually that permit reasonably simple solutions and proper physical interpretation, and thus serve the purposes of verification and qualitative analysis.

2. MATHEMATICAL FORMULATION

Natural convection heat and mass transfer is considered in this paper in the steady-state regime from a point or a line sources embedded in a porous medium of permeability K saturated with a liquid of viscosity μ, density ρ and the heat capacity c_P. The sources generate heat at a rate q and, at the same time, a substance at a rate m.

For the density variations, the assumption of a Boussinesq fluid has been made, which means that the density is assumed constant everywhere except in the body force term in the momentum equations via the thermal (β)and the concentration (β_c) volumetric expansion coefficients.

Under the boundary layer approximation, the governing equations that describe the flow in the plume above a point and a line source are given as follows

$$\frac{\mu}{K}u + b\rho u^2 = \frac{g}{b}\left[\beta(T - T\infty) + \beta_C(C - C_\infty)\right] \quad (1)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = y^{-n}\frac{\partial}{\partial y}\left(a_\Sigma y^n \frac{\partial T}{\partial y}\right) \quad (2)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = y^{-n}\frac{\partial}{\partial y}\left(D_\Sigma y^n \frac{\partial C}{\partial y}\right) \quad (3)$$

$$\frac{\partial u y^n}{\partial x} + \frac{\partial v y^n}{\partial y} = 0 \quad (4)$$

with the boundary conditions given by

$$\frac{\partial u}{\partial y} = 0, \frac{\partial T}{\partial y} = 0, \frac{\partial C}{\partial y} = 0, \; v = 0 \quad y = o \quad (5)$$

$$T = T_\infty, \; C = C_\infty \; u = 0 \quad y \to \infty \quad (6)$$

The total energy and mass diffusion conservation conditions across any horizontal plane in the plume are

$$\int_0^\infty u\,(T - T_\infty)y^n dy = \frac{q_n}{2\pi^n \rho_f c_p} \quad (7)$$

$$\int_0^\infty u\,(C - C_\infty)y^n dy = \frac{m_n}{2\pi^n \rho_f} \quad (8)$$

Here n=0 and 1 for a line and a point source, respectively; inertial coefficient $b=c_F K^{-1/2}$, where form-drag constant c_F is of the order of magnitude 0.1÷0.55 (Ward, 1964; Nield & Bejan, 1999).

We will focus on two limiting flow regimes. The first one is Darcy flow

$$u = \frac{gK}{\nu}\left[\beta(T - T\infty) + \beta_C(C - C_\infty)\right] \quad (9)$$

In the other case the role of the linear resistance is negligible, so that one can use the next version of the momentum equation (an inertial flow)

$$u^2 = \frac{g}{b}\left[\beta(T - T\infty) + \beta_C(C - C_\infty)\right] \quad (10)$$

The coefficients a_Σ and D_Σ represent overall the thermal and the mass diffusivities, respectively. They embody both molecular diffusion and dispersion.

One can say, at the present time the commonly accepted typical representation of overall diffusivity for boundary-layer-type problems (e.g. the problem of interest) is (Aerov & Umnik, 1951; Ranz, 1952; Wakao & Kaguei, 1982)

$$\lambda_\Sigma = \lambda + c_a \rho_f c_p |\vec{v}| l \quad (11)$$

$$a_\Sigma = \lambda_\Sigma / (\rho c_p) \quad (11a)$$

$$D_\Sigma = D + c_D |\vec{v}| l \quad (12)$$

where λ is the stagnant thermal conductivity of a fluid and a saturated porous medium; D is the molecular mass diffusivity; l is the characteristic length (an analog of the mixing length in turbulence); c_a and c_D represent the dispersion coefficients.

In the next section we consider the boundary-layer heat and mass transfer problem with scale analysis. Then the similarity analysis of the problem when the dispersion mechanism supersedes the molecular dispersion will also be presented.

3. SCALE ANALYSIS

The algorithm of the scale analysis of boundary-layer-type transfer processes was detailed in by Ghukhman (1967). For the problem of the same type for porous media this approach was extensively used by A. Bejan (see, for instance, Nield & Bejan (1999) for details and references).

The simplest case of the dominant diffusion mechanism will be our initial concern.

3.1 Molecular diffusion mechanism

Heat transfer driven flows

Consider the case of heat transfer driven flows when the molecular diffusion is predominant ($|\beta_C \Delta C| \ll |\beta \Delta T|$) in the momentum equation. Using the scale analysis one can obtain the next estimations for a line source and Darcy flow on the basis of the momentum, the energy and the mass conservation equations

$$u \propto g\beta K \Delta T / \nu, \; u \propto aH / \delta_T^2, \; q \propto \rho c_p \Delta T u \delta_T$$

or

$$u \propto \left(\frac{a}{H}\right) Ra_{LD}^{2/3}, \; \Delta T \propto \left(\frac{q}{\lambda}\right) Ra_{LD}^{-1/3}, \; \frac{\delta_T}{H} \propto Ra_{LD}^{-1/3} \quad (13)$$

where $Ra_{LD} = g\beta q K H / (\nu a \lambda)$ is the Rayleigh number for a line source and Darcy flow; δ_T and H are the heat plume thickness and height.

The concentration plume thickness δ_C can be estimated following Bejan & Khair (1985). We obtain from the concentration equation (3)

$$\frac{u \Delta C}{H} \min(\delta_T, \delta_C) \propto D \frac{\Delta C}{\delta_C}$$

For the case $\delta_C \ll \delta_T$ (i.e. the Lewis number $Le=a/D \gg 1$) using δ_C as a scale length and accounting for the estimations (13) we have

$$\delta_C \propto H Ra_{LD}^{-1/3} Le^{-1/2}$$

and for the case $\delta_C \gg \delta_T$ ($Le \ll 1$)

$$\delta_C \propto H Ra_{LD}^{-1/3} Le^{-1}$$

The Table 1 summarizes all for this class of flow. It is worthy of note that these results for the *heat* plume are the same obtained for pure heat transfer natural convection.
Here

$$Ra_{pD} = \frac{g\beta Kq}{\nu a\lambda},\ Ra_{pl} = \frac{g\beta Hq}{ba^2\lambda},\ Ra_{LI} = \frac{g\beta H^2 q}{ba^2\lambda}$$

are the modified Rayleigh numbers.

POINT SOURCE		LINE SOURCE
Darcy flow		
$u \propto (a/H)Ra_{pD}$		$u \propto (a/H)Ra_{LD}^{2/3}$
$\Delta T \propto q/(\lambda H)$		$\Delta T \propto (q/\lambda)Ra_{LD}^{-1/3}$
$\delta_T \propto HRa_{pD}^{-1/2}$		$\delta_T \propto HRa_{LD}^{-1/3}$
$\delta_C \propto HRa_{pD}^{-1/2}Le^{-1/2}$	$(Le >>1)$	$\delta_C \propto HRa_{LD}^{-1/3}Le^{-1/2}$
$\delta_C \propto HRa_{pD}^{-1/2}Le^{-1}$	$(Le << 1)$	$\delta_C \propto HRa_{LD}^{-1/3}Le^{-1}$
non -Darcy flow (inertial regime)		
$u \propto (a/H)Ra_{pl}^{1/2}$		$u \propto (a/H)Ra_{LI}^{2/5}$
$\Delta T \propto q/(\lambda H)$		$\Delta T \propto (q/\lambda)Ra_{LI}^{-1/5}$
$\delta_T \propto HRa_{pl}^{-1/4}$		$\delta_T \propto HRa_{LI}^{-1/5}$
$\delta_C \propto HRa_{pl}^{-1/4}Le^{-1/2}$	$(Le >>1)$	$\delta_C \propto HRa_{LI}^{-1/5}Le^{-1/2}$
$\delta_C \propto HRa_{pl}^{-1/4}Le^{-1}$	$(Le << 1)$	$\delta_C \propto HRa_{LI}^{-1/5}Le^{-1}$

Table 1 Transport scales for molecular diffusion **only**

Mass transfer driven flow

For these flows the buoyancy effect due to variations in solute concentration is dominant ($|\beta_C \Delta C| >> |\beta \Delta T|$ in the momentum equation). New scales we obtain from the momentum and the constituent conservation equations, keeping in mind that the scale length of the flow is δ_C.

The plume thickness scales are

- point source

  $\delta_C \propto HRa_{pD}^{-1/2}Le^{-1}|N|^{-1/2}$ - Darcy flow

  $\delta_C \propto HRa_{pl}^{-1/4}Le^{-3/4}|N|^{-1/4}$ -inertial flow

- linesource

  $\delta_C \propto HRa_{LD}^{-1/3}Le^{-2/3}|N|^{-1/3}$ - Darcy flow

  $\delta_C \propto HRa_{LI}^{-1/5}Le^{-3/5}|N|^{-1/5}$ - inertial flow

Here $N = \dfrac{\beta_C m}{\beta(q/c_P)}$ is the 'buoyancy' ratio.

3.2 Hydrodynamic dispersion

In this section we will consider that the hydrodynamic dispersion supersedes the molecular diffusion. Whereas for the case of non-Darcy flow regime it seems quite reasonable for many kinds of liquids, Darcy flow needs additional estimations to find the field of application.

The Darcy flow regime is realized if $Re = buK/\nu << 1$. Taking into account the velocity and the plume thickness scales (13) we have

$$Ra\,Pr^{-1}Da^{1/2} << 1.$$

But from the relations (10) it follows that

$a, = a + c_a lu = a\left(1 + c_a RaDa^{1/2}\right)$, $Da = K/H^2$

The dispersion is predominant if $RaDa^{1/2} >> 1$.
These estimations are compatible if $Pr >> 1$, which can be realised for oils and chemical solutions.

With the procedure given in *Sec.3.1* one can obtain the next scales of flow (for the safe of simplicity we suggest the mixing lengths for the heat and the mass dispersion are the same. But it is not difficult to carry out the full analysis):

POINT SOURCE	LINE SOURCE
Darcyflow	
$u \propto \frac{a}{H}Ra_{pD}^{1/2}\left(\frac{l}{H}\right)^{-1/2}$	$u \propto \frac{a}{H}Ra_{LD}^{1/2}\left(\frac{l}{H}\right)^{-1/4}$
$\Delta T \propto \frac{q}{\lambda H}Ra_{pD}^{-1/2}\left(\frac{l}{H}\right)^{-1/4}$	$\Delta T \propto \frac{q}{\lambda}Ra_{LD}^{-1/2}\left(\frac{l}{H}\right)^{-1/4}$
$\delta \propto (l/H)^{1/2}$	$\delta \propto (l/H)^{1/2}$
non-Darcy flow (inertial regime)	
$u \propto \frac{a}{H}Ra_{pl}^{1/3}\left(\frac{l}{H}\right)^{-1/3}$	$u \propto \frac{a}{H}Ra_{LI}^{2/3}\left(\frac{l}{H}\right)^{-1/3}$
$\Delta T \propto \frac{q}{\lambda H}Ra_{pl}^{-1/3}\left(\frac{l}{H}\right)^{-2/3}$	$\Delta T \propto \frac{q}{\lambda}Ra_{LI}^{-1/3}\left(\frac{l}{H}\right)^{-1/3}$
$\delta \propto (1/H)^{1/2}$	$\delta \propto (l/H)^{1/2}$

Table 2 **Scales for** dispersion **only**

4. SIMILARITY SOLUTION

Scale analysis allows an understanding of the role of the main parameters controlling the phenomenon, and is very useful for experimental data processing and finding possible self-similar transformations in the governing equations.

The similarity solutions for the particular case where dispersion dominates are now presented.
The energy and the constituent conservation equations take the form

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = ly^{-n}\frac{\partial}{\partial y}\left(uy^n\frac{\partial T}{\partial y}\right) \qquad (14)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=ly^{-n}\frac{\partial}{\partial y}\left(uy^n\frac{\partial C}{\partial y}\right) \quad (15)$$

These equations coupled with the momentum equation (9) (or (10)), mass conservation equation (4) and the boundary conditions (5) – (8) form the complete set of equations.

Furthermore let T^* be the generalized 'temperature'

$$T^*=T-T_\infty+(\beta/\beta_C)(C-C_\infty) \quad (16)$$

Then the problem of interest will be formulated as

$$u=\frac{g\beta K}{\nu}T^* \quad (17)$$

$$u^2=\frac{g\beta}{b}T^* \quad (18)$$

$$u\frac{\partial T^*}{\partial x}+v\frac{\partial T^*}{\partial y}=ly^{-n}\frac{\partial}{\partial y}\left(uy^n\frac{\partial T^*}{\partial y}\right) \quad (19)$$

$$\frac{\partial uy^n}{\partial x}+\frac{\partial vy^n}{\partial y}=0 \quad (20)$$

$$\frac{\partial T^*}{\partial y}=0 \quad y=0 \quad T^*=0 \quad y\to\infty$$

$$\int_0^\infty uT^*y^ndy=\frac{1}{2\pi^n\rho_fc_p}\left(q_n+\frac{\beta}{\beta_C}c_pm_n\right)=\frac{Q^*}{2\pi^n\rho_fc_p}.$$

The foregoing boundary layer scales suggest the following similarity transformations

- for Darcy flow

$$\Psi=\left(\frac{Q_n^*g\beta Kl^{n+1}}{2\pi^n\rho_fc_p\nu}\right)^{\frac{1}{2}}x_*^{\frac{n+1}{4}}F(\eta)=\Psi_Dx_*^{\frac{n+1}{4}}F(\eta)$$

$$T^*=\left(\frac{Q_n^*\nu}{2\pi^n\rho_fc_pg\beta Kl^{n+1}}\right)^{\frac{1}{2}}x_*^{-\frac{n+1}{4}}\Theta(\eta)=T_Dx_*^{-\frac{n+1}{4}}\Theta(\eta)$$

- for inertial flow

$$\Psi=\left(\frac{Q_n^*g\beta l^{2(n+1)}}{2\pi^n\rho_fc_pb}\right)^{\frac{1}{3}}x_*^{\frac{n+1}{3}}F(\eta)=\Psi_Ix_*^{\frac{n+1}{3}}F(\eta)$$

$$T^*=\left(\frac{b}{g\beta}\right)^{\frac{1}{3}}\left(\frac{Q_n^*}{2\pi^n\rho_fc_pl^{n+1}}\right)^{\frac{2}{3}}x_*^{-\frac{n+1}{3}}\Theta(\eta)=T_Ix_*^{-\frac{n+1}{3}}\Theta(\eta)$$

with the independent variables

$$x_*=x/l \quad \eta=y(lx)^{-1/2}$$

and stream function Ψ

$$\frac{\partial\Psi}{\partial y}=uy^n, \quad \frac{\partial\Psi}{\partial x}=-vy^n$$

Using these variables and transformations Eqs (14) - (20) reduce to the following sets of the ordinary differential equations:

- Darcyflow

$$(F'\Theta')'+\frac{n+1}{4}(\Theta F'+\Theta'F)=0$$

$$F'=\eta^n\Theta$$

- inertialflow

$$(F'Q')'+\frac{n+1}{3}(\Theta F'+\Theta'F)=0$$

$$F'^2=\eta^{2n}\Theta$$

where primes denote differentiation with respect to η.

These systems are subject to the boundary conditions

$$\Theta'=0 \ \eta=0, \ \Theta'=0 \ \eta\to\infty$$

and the generalized conservation constraint

$$\int_0^\infty F'\Theta d\eta=0$$

These equations have the following analytical solutions

for a line source

- Darcyflow

$$\Theta(\eta)=\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\mathrm{Cos}\frac{\eta}{2} \quad (21)$$

$$F(\eta)=\left(\frac{8}{\pi}\right)^{\frac{1}{2}}\mathrm{Sin}\frac{\eta}{2} \quad (22)$$

- inertialflow

$$\Theta(\eta)=\left(\frac{3}{8}\right)^{\frac{1}{3}}\mathrm{Cos}^2\frac{\eta}{\sqrt{6}} \quad (23)$$

$$F(\eta)=3^{\frac{2}{3}}\mathrm{Sin}\frac{\eta}{\sqrt{6}} \quad (24)$$

for a point source

- Darcy flow

$$\Theta(\eta)=\frac{\alpha_0J_1(\alpha_0)}{\alpha_0J_1(\alpha_0)}J_0\left(\sqrt{2}\right)=0.8011J_0\left(\frac{\eta}{\sqrt{2}}\right) \quad (25)$$

$$F(\eta)=\frac{\sqrt{2}}{\alpha_0J_1(\alpha_0)}\eta J_1\left(\frac{\eta}{\sqrt{2}}\right)=1.1329\eta J_1\left(\frac{\eta}{\sqrt{2}}\right) \quad (26)$$

- **inertial flow**

$$\Theta(\eta)=AJ_0^2\left(\frac{\eta}{\sqrt{3}}\right) \quad (27)$$

$$F(\eta)=A^{\frac{1}{2}}\eta J_1^2\left(\frac{\eta}{\sqrt{3}}\right) \quad (28)$$

where $A=\left[3\int_0^{\alpha_0}J_0^3(t)tdt\right]^{-2/3}=0.7036$; J_0 and J_1 are the Bessel functions of the first kind, respectively, of zeroth and first-order; $\alpha_0=2.4048$ is the first root of the function J_0.

The distinctive feature of the solution of the statement under discussion is that the temperature (concentration) perturbation is localized with space. This is common with non-linear problems of heat conduction (Landau & Lifshitz, **1987).**

From Eqs **(21)-(28)** it follows that the temperature (concentration) difference falls on the plume of the terminal thickness:

- Darcyflow

$$Y_{lD} = \pi (lx)^{1/2} = 3.142 (lx)^{1/2} \quad \text{- line source} \quad \textbf{(29)}$$

$$Y_{pD} = \sqrt{2}\alpha_0 (lx)^{1/2} = 3.401 (lx)^{1/2} \text{- point source} \quad \textbf{(30)}$$

- inertial flow

$$Y_{lI} = \sqrt{\frac{3}{2}}\pi (lx)^{1/2} = 3.848 (lx)^{1/2} \quad \text{- line source} \quad \textbf{(31)}$$

$$Y_{pI} = \sqrt{3}\alpha_0 (lx)^{1/2} = 4.165 (lx)^{1/2} \text{ - point source} \quad \textbf{(32)}$$

The plume has a parabolic form.

When equations **(29)** and **(30)** are compared with equations **(31)** and **(32)** it is apparent that the quadratic resistance causes an increase in the temperature (concentration) boundary layer.

The temperature on the centre-line has the form:

Darcy flow

- point source

$$T_a^* = \left(\frac{Q_0^* \nu}{\pi \rho_f c_p g \beta K l} \right)^{1/2} x_*^{-\frac{1}{4}}$$

- line source

$$T_a^* = \left(\frac{Q_1^* \nu}{2\pi \rho_f c_p g \beta K l^2} \right)^{1/2} \frac{1}{\alpha_0 J_1(\alpha_0)} x_*^{-\frac{1}{2}}$$

Inertialjlow

- point source

$$T_a^* = \left(\frac{3b}{8g\beta} \right)^{\frac{1}{3}} \left(\frac{Q_0^*}{2\rho_f c_p l} \right)^{\frac{2}{3}} x_*^{-\frac{1}{3}}$$

- linesource

$$T_a^* = A \left(\frac{b}{g\beta} \right)^{\frac{1}{3}} \left(\frac{Q_1^*}{2\pi \rho_f c_p l^2} \right)^{\frac{2}{3}} x_*^{-\frac{2}{3}}$$

The temperature and the concentration profiles can be obtained by reference to the definition of the generalized 'temperature' (16)

$$T - T_\infty = \frac{\beta q_n / c_P}{\beta q_n / c_P + \beta_C m_n} T^*$$

$$C - C_\infty = \frac{\beta_C m_n}{\beta q_n / c_P + \beta_C m_n} T^*$$

The solutions obtained would hold if $\lambda << \rho c_p u l$. This condition gives the next estimations for the distances **from** sources when one can use analytical solutions

$$\frac{Q_n^* l^{1-n} g \beta K}{\lambda_f \nu a_f} x_*^{-\frac{3}{2}(n+1)} \rangle\rangle 1$$

$$\frac{Q_n^* l^{2-n} g \beta}{b \lambda_f a_f^2} x_*^{-2(n+1)} \rangle\rangle 1$$

It is well to bear in mind that these solutions can break down in the neighbourhood of the plume borders for the conductivity component can be of considerable importance in transfer processes because of small convective velocities.

5. CONCLUSIONS

In this paper only the analysis of limiting situations of the dominant effects of either molecular or dispersion diffusion mechanism of coupled heat and mass transfer from point or line sources embedded in a porous medium is presented. The results of scale analysis have demonstrated a key part of these mechanisms on the heat and mass transfer laws and have enabled us to obtain new simple analytical solutions for the case of predominant part of dispersion on the assumption of identical mechanism for heat and mass transfer.

The limited space of the paper has not allowed presentation of an analysis with coupled effects both of these mechanisms.

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