MODELLING THE FLIGHT OF HYDROTHERMAL ERUPTION EJECTA

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SUMMARY – Hydrothermal eruptions occur when hot fluid just below the ground surface is exposed to atmospheric conditions. As it moves to the lower-pressure atmosphere, the fluid boils; the consequent expansion as the denser liquid phase flashes to steam produces high vertical speeds and lifts the rock particles which surround the vent. A mixture of liquid water, vapour and solid particles is ejected into the atmosphere, and returns to the ground surface, where the solid particles are distributed over an area which depends on their size and the vigour of the eruption. This paper includes a discussion about the conceptual modelling of such a process, a proposal for quantitative model equations which might be used to describe the phenomenon, and some preliminary calculations using the mathematical model. Some attention is paid to the fluid conditions which may pertain at the base of such eruptions and which factors are important in the flow model.

1. INTRODUCTION

Preliminary work on developing a mechanistic mathematical model of hydrothermal eruptions was undertaken during the beginning of the last decade (McKibbin, 1989, 1990; Bercich & McKibbin, 1992, 1993). More recent work has investigated the effects of non-condensible gases (McKibbin, 1996) and has developed some aspects of the models of the underground flows which supply hydrothermal eruptions and which contribute to pressure recovery afterwards (Smith & McKibbin, 1997, 1998, 1999, 2000).

The early studies foundered when quantitative modelling of the above-ground eruption stream was tackled. There was difficulty with solving the equations which were formulated (from a variety of sources) to describe the conservation of mass, momentum and energy. This paper sets out a revised formulation in an attempt to get correct and tractable model equations. Included is some discussion about the concepts needed to get the more important terms sorted out from those which are less important, and some results are used to illustrate one simplified version of the model.

2. SOME CONCEPTS

When hot fluid just below the ground surface is exposed to atmospheric conditions, it moves upwards to lower-pressure regions; the consequent expansion as the denser liquid phase boils produces high vertical speeds and ejects rock particles which surround the vent. The mixture of liquid water, vapour and solid particles is erupted into the atmosphere, and then falls to the ground, where the rock particles are distributed over an area which depends on their size and the vigour of the eruption.

A model of the problem will clearly be very complicated if one wanted to follow every solid particle and quantum of fluid during such an eruptive process. A better strategy is to try to work out what factors are important and which can be neglected. For example, the fluid has two phases; should they both be modelled as providing the lift forces or can one be regarded as relatively inert? If so, which one? Are the solid particles important in the overall momentum balance, or are they incidental in what is really just a fluid fountain?

Some clues to help answer the first question can be found by analysing the phase changes that take place during depressurisation of hot ground fluid as it rises to meet the atmosphere. While conditions are two-phase, and if the phases are in thermal equilibrium, there is a unique relationship between the pressure p and the temperature T, given by $T = T_{sat}(p)$ (see Appendix for a correlation over the pressure range expected in this application). The specific enthalpy h_f of a fluid two-phase mixture is given in terms of the quality (vapour mass fraction, dryness) X, by

$$h_f = (1 - X)\, h_\ell + X h_g \qquad (1a)$$

where h_ℓ and h_g are the separate liquid and gas (vapour) specific enthalpies. These last two quantities may be expressed in terms of the pressure p (Appendix gives correlations for water) and so the mixture specific enthalpy is a function of p and X:

$$h_f(p, X) = (1 - X)\, h_\ell(p) + X\, h_g(p) \qquad (1b)$$

Note that these calculations are based on depressurisation of the whole fluid, and do not take account of different speeds for the different phases. If the separate phase speeds are different within a two-phase flow, then the flowing quality X_f must be considered instead (see below).

If it is assumed that fluid rises adiabatically to the surface and that the kinetic energy is small compared with the advected energy, then the process may be regarded as isenthalpic, with the constant mixture specific enthalpy $h_f = h_d$ throughout the fluid. The value of h_d corresponds to conditions deep in the ground and may be found from Equation (1b) with $p = p_d$, $X = X_d$. The vapour mass fraction X may then be calculated from the above as a function of pressure p:

$$X(p) = \frac{h_d - h_\ell(p)}{h_g(p) - h_\ell(p)} \qquad (2)$$

It is reasonable to suppose that the undisturbed pressure p_d will increase approximately linearly, in the fashion of "boiling-point-with-depth", over a few tens of metres, with the ground fluid initially likely to be saturated liquid or very close to those conditions. If so, the volume fraction of vapour underground will be small, and the source fluid may be regarded as hot liquid water, at a temperature above 100°C, the boiling temperature at atmospheric conditions.

This volume fraction, called the gas (or vapour) saturation, is usually denoted S_g, while the liquid volume fraction of the two-phase mixture, or the liquid saturation, is denoted $S_\ell = 1 - S_g$.

The specific volume v_f of a two-phase fluid mixture can be written as:

$$v_f(p, X) = (1 - X)\, v_\ell(p) + X\, v_g(p) \qquad (3)$$

The gas saturation S_g may be found in terms of the dryness X by using Equation (3) and the volumetric mixture density relationship:

$$1/v_f = \rho_f = (1 - S_g)\, \rho_\ell + S_g\, \rho_g \qquad (4)$$

to give, in terms of the pressure and dryness.

$$S_g(p, X) = \frac{X\, v_g(p)}{(1 - X)\, v_\ell(p) + X\, v_g(p)} \qquad (5)$$

Conversely, the dryness is given by:

$$X(p, S_g) = \frac{S_g\, v_\ell(p)}{[1 - S_g]\, v_g(p) + S_g\, v_\ell(p)} \qquad (6)$$

For given underground pressure and gas saturation conditions p_d and S_{gd}, the corresponding value of $X_d = X(p_d, S_{gd})$ may be calculated from Equation (6), with $v_d = v_f(p_d, S_{gd})$, the mixture specific volume, found from Equation (3).

Under the isenthalpic assumption, values of the vapour mass fraction $X_s = X(p_s)$, the vapour saturation $S_{gs} = S_g(p_s, X_s)$ and the specific volume ratio $v_{rats} = v_{fs}/v_d = v_f(p_s, X_s)/v_d$ (the values of these parameters at the surface, where the pressure is p_s = 1 bar absolute) are shown in Figure 1 for various combinations of p_d and S_{gd}.

While there is not much change in the vapour mass fraction (dryness), it is noticeable that both the vapour volume fraction S_g and the specific volume ratio v_{rat} increase markedly with both p_d and S_{gd}. From this, one can deduce that it is the gas phase, or steam, which is the main volumetric flowing component in an eruption, and that it is the expansion of the fluid mixture which produces the high escape velocities observed.

3. IMPLICATIONS FOR MODELS

There are several possibilities for initial attempts at modelling the upflowing eruption stream, which consists of steam, liquid water droplets, rock fragments of various sizes and air which is entrained into the flow from the surrounding atmosphere. Among the possible models are the following:

The Steam-Lift (SL) model

From the calculations in the previous section, the main flow component, from a volumetric point of view, is the gas phase (vapour or steam), at least near the vent. Entrained air may join this flow, and the partial pressure of the steam will decrease, with a corresponding drop in temperature, as the stream rises. Some of the steam will condense, to increase the overall mass flow of liquid water.

Such a model can be regarded as a gas flow of steam + air, with liquid water droplets and rock fragments being lifted by the resulting drag forces. If air entrainment is neglected, and the rock fragments are regarded as thermally inert objects lifted by the steam flow, the model may be simplified further. Such a mechanistic model will be called the Steam-Lift (SL) model.

The Two-Phase-Lift (TPL) model

Another approach is to regard the fluid stream to be a mixture of vapour and liquid water flowing in a mist regime (equal phase speeds) with the two fluid phases being in thermal equilibrium while condensation and/or evaporation take place. Rock fragments are lifted by the drag forces of the fluid stream, but are thermally inert (heat dispersion in the rock particles is negligible over the time taken for ejection). Again, air may be considered as a further entrained component. With air entrainment neglected, this mechanism will be called the Two-Phase-Lift (TPL) model.

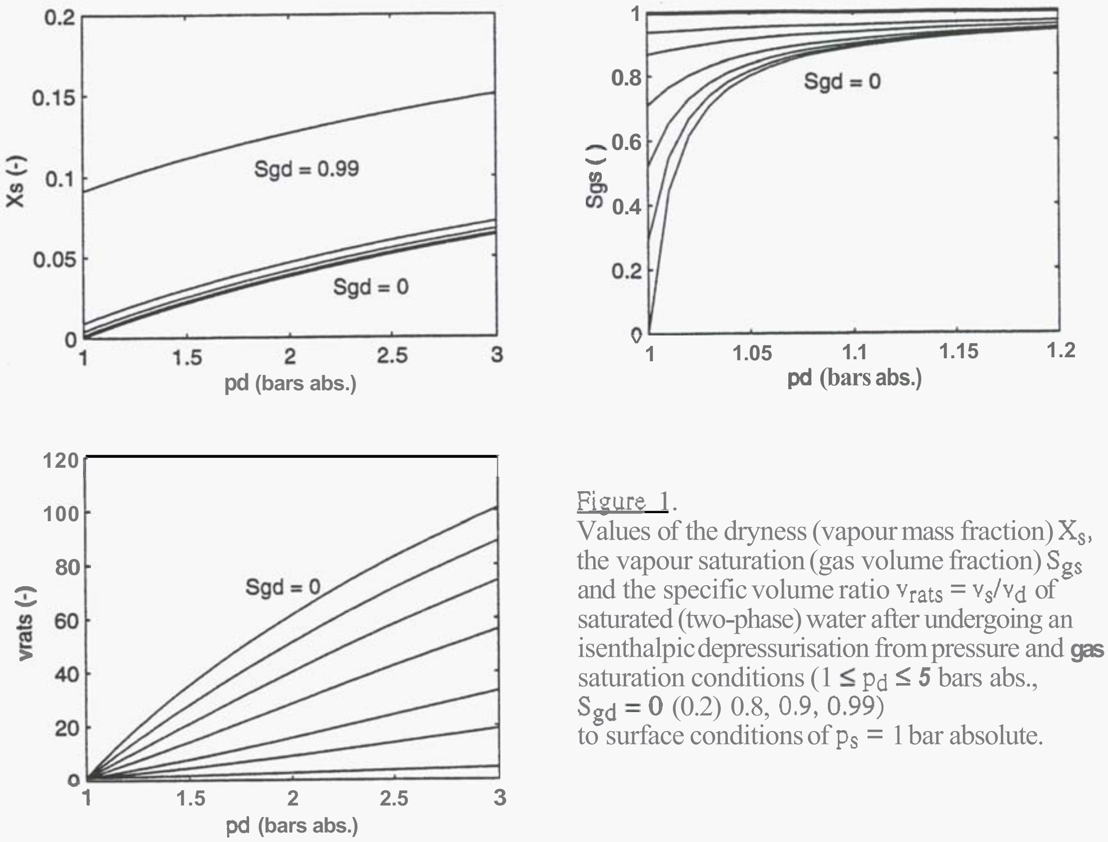

Figure 1.
Values of the dryness (vapour mass fraction) X_s, the vapour saturation (gas volume fraction) S_{gs} and the specific volume ratio $v_{rats} = v_s/v_d$ of saturated (two-phase) water after undergoing an isenthalpic depressurisation from pressure and gas saturation conditions ($1 \leq p_d \leq 5$ bars abs., $S_{gd} = 0$ (0.2) 0.8, 0.9, 0.99) to surface conditions of $p_s = 1$ bar absolute.

In both of the **SL** and TPL cases above, the mass and/or volume fluxes of the steam and liquid water from the vent at ground level may be supposed to be quantified using the X_s and/or S_{gs} values calculated using the methods of **§2** above. Such an assumption relies on all of the exiting fluid being ejected in the eruptive column, rather than, for example, some of the liquid pooling on the ground while the steam flows upwards.

The following quantification of the various fluxes is restricted to a one-dimensional vertical eruptive flow column, with air entrainment neglected, and is based on the Steam-Lift **(SL)** model referred to above.

Mass, volume and energy fluxes

The eruptive column is assumed to be of lateral area A, which may vary with height z above the ground surface, i.e. A = A(z). At ground level, a fraction α_{rs} of the area A(0) will be occupied by solid material, while the vent fractures will occupy an area $(1 - \alpha_{rs})A(0)$.

At any height z in the column, the fractions of the area A that are occupied by the gas (steam), liquid and solid phases will be denoted by α_g, α_ℓ and α_r respectively, where

$$\alpha_g + \alpha_\ell + \alpha_r = 1 \tag{7}$$

The flowing quality of the fluid + solid stream is denoted Y_f. This is the fluid mass flow fraction of the total mass flow M. The flowing quality of the fluid stream itself is denoted by X_f; this is the gas mass flow fraction of the total fluid mass flow, If the mass flows of the fluid and the solid components are denoted M_f and M_r respectively, then the gas and liquid mass fluxes M_g and M_ℓ are

$$M_g = X_f M_f = X_f Y_f M \tag{8}$$

$$M_\ell = (1 - X_f) M_f = (1 - X_f) Y_f M \tag{9}$$

$$M_r = (1 - Y_f) M \tag{10}$$

with

$$M_g + M_\ell + M_r = M_f + M_r = M$$

being the total mass flow. Note that the mass flux of the solid phase M_r may be also given, after using Equation (8), by

$$M_r = \frac{1 - Y_f}{Y_f} M_f \tag{11}$$

The mass fluxes per unit lateral area are denoted by $m_i = M_i/A$, for i = g, ℓ, f and r, and m = M/A.

The volume fluxes corresponding to each of the gas, liquid and solid phase streams are, in terms of M_f and M,

$$V_g = v_g M_g = X_f v_g M_f = X_f Y_f v_g M \tag{12}$$

$$V_\ell = v_\ell M_\ell = (1 - X_f) v_\ell M_f = (1 - X_f) Y_f v_\ell M \tag{13}$$

$$V_r = v_r M_r = \frac{1 - Y_f}{Y_f} v_r M_f = (1 - Y_f) v_r M \tag{14}$$

after using Equations (8) – (11). The quantities v_i, $i = g, \ell$, f and r are the specific volumes of the respective phases. The total volume flux over area A is

$$V = V_g + V_\ell + V_r$$

$$= \left\{ Y_f [X_f v_g + (1 - X_f) v_\ell] + (1 - Y_f) v_r \right\} M$$

The volumetric quality β (the fraction of the total volume flux due to the gas phase) is

$$\beta = \frac{V_g}{V} = \frac{1}{1 + \dfrac{1 - X_f}{X_f}\dfrac{v_\ell}{v_g} + \dfrac{1 - Y_f}{Y_f X_f}\dfrac{v_r}{v_g}}$$

or, in terms of the phase densities, $\rho_i = 1/v_i$,

$$\beta = \frac{1}{1 + \dfrac{1 - X_f}{X_f}\dfrac{\rho_g}{\rho_\ell} + \dfrac{1 - Y_f}{Y_f X_f}\dfrac{\rho_g}{\rho_r}} \tag{15}$$

Since, typically, $\rho_g \approx 1, \rho_\ell \approx 1000, \rho_r \approx 1000–2000$ kg m^{-3}, and X_f and Y_f do not take extreme values, this means that β is very close to unity, and the volumetric flow of steam dominates.

The superficial speed of each phase i is defined as the volume flux per unit lateral area, $\hat{w}_i = V_i/A$, while the actual vertical speed w_i of phase i is the average volume flux over the lateral area that the phase occupies in the stream:

$$w_i = \frac{V_i}{\alpha_i A} \tag{16}$$

The actual speeds of the separate phases are, from Equations (16) and (12) – (14),

$$w_g = \frac{X_f}{\alpha_g} v_g m_f = \frac{X_f Y_f}{\alpha_g} v_g \tag{17}$$

$$w_\ell = \frac{1 - X_f}{\alpha_\ell} v_\ell m_f = \frac{(1 - X_f) Y_f}{\alpha_\ell} v_\ell m \tag{18}$$

$$w_r = \frac{1 - Y_f}{\alpha_r Y_f} v_r m_f = \frac{1 - Y_f}{\alpha_r} v_r m \tag{19}$$

The fluid flowing specific enthalpy is the total enthalpy flux divided by the total mass flux:

$$h_f = \frac{M_g h_g + M_\ell h_\ell}{M_g + M_\ell} = X_f h_g + (1 - X_f) h_\ell$$

with simplification using Equations (8) and (9). Note that this expression is the same as that given by Equation (1a) only if the fluid phase speeds are the same, and then $X = X_f$.

The equations describing conservation of mass, momentum and energy for a steady vertical flow of uniform cross-section are described next.

4. CONSERVATION EQUATIONS

Conservation of mass

For steady one-dimensional vertical flow, in a column which has a constant cross-sectional area A, the mass fluxes per unit area, m_f of the fluid stream (gas + liquid) and m_r of the solid phase, are independent of height z. Use of Equations (17) – (19), and setting the (constant) mass fluxes per unit lateral area equal to their values at the ground surface (z = 0), gives

$$m_f = m_g + m_\ell = \alpha_g \rho_g w_g + \alpha_\ell \rho_\ell w_\ell$$
$$= m_f(0) \tag{20}$$

$$m_r = \alpha_r \rho_r w_r = m_r(0) \tag{21}$$

The rock material density ρ_r is assumed constant. The quantity

$$E(z) = \frac{dm_g}{dz} = -\frac{dm_\ell}{dz}$$

is the mass rate of evaporation of liquid to vapour per unit volume of the eruption flow.

Conservation of momentum

The Reynolds number for the fluid flow is given by

$$Re = \frac{\rho_f w_f d}{\mu_f}$$

where d is an appropriate length scale. For gas flow, Re is large and it is reasonable to assume that internal fluid viscous stresses are small compared with inertial, gravitational and inter-phase drag forces. The model includes buoyancy forces and assumes that the liquid is in droplets; these small droplets and rock particles are propelled by the gas through drag forces. The equations for momentum conservation of the gas, liquid and solid phases are then:

$$[m_g w_g]' = -\rho_g g - p'$$
$$- D_\ell (w_g - w_\ell)|w_g - w_\ell| - D_r(w_g - w_r)|w_g - w_r| \tag{23}$$

$$[m_\ell w_\ell]' = -\alpha_\ell (\rho_\ell - \rho_g) g$$
$$+ D_\ell (w_g - w_\ell)|w_g - w_\ell| \tag{24}$$

$$[m_r w_r]' = -\alpha_r (\rho_r - \rho_g) g$$
$$+ D_r(w_g - w_r)|w_g - w_r| \tag{25}$$

where a prime (') denotes a derivative with respect to z. The drag coefficients D_ℓ and D_r are calculated as the effect for an ensemble of droplets or particles (all of which are assumed to be approximately spherical) which occupy volumes calculated from the separate mass densities of the liquid and solid phases, and are given by

$$D_\ell = \frac{1}{2} C_s \rho_g A_{drop} \frac{\alpha_\ell}{V_{drop}} = \frac{3}{8} C_s \rho_g \frac{\alpha_\ell}{R_{drop}}$$

$$D_r = \frac{1}{2} C_s \rho_g A_{part} \frac{\alpha_r}{V_{part}} = \frac{3}{8} C_s \rho_g \frac{\alpha_r}{R_{part}}$$

The quantities A_i, V_i and R_i (i = drop, part) are respectively the cross-sectional area, volume and radius of the water droplets and rock particles. The drag is assumed proportional to the square of the relative speeds of the gas and particles, with the coefficient for a single particle taken to be (for $Re \geq 1500$) $C_s = 0.38$.

Conservation of total momentum is expressed by adding Equations (23) – (25):

$$[m_g w_g + m_\ell w_\ell + m_r w_r]' = -[\alpha_g \rho_g + \alpha_\ell \rho_\ell + \alpha_r \rho_r] g - p' \qquad (26)$$

Conservation of energy

The flux of the total kinetic + internal energy of the system is changed only through work and heat. Assuming that the solid particles are thermally inert, and that there is no viscous dissipation, energy conservation may be expressed as

$$\begin{aligned}
&[\tfrac{1}{2}(m_g w_g^2 + m_\ell w_\ell^2 + m_r w_r^2) \\
&\qquad + m_g u_g + m_\ell u_\ell]' = -[w_g p]' \\
&\quad - [w_g \rho_g + w_\ell \alpha_\ell (\rho_\ell - \rho_g) + w_r \alpha_r (\rho_r - \rho_g)] g \\
&\quad - D_\ell (w_g - w_\ell)^2 |w_g - w_\ell| - D_r (w_g - w_r)^2 |w_g - w_r|
\end{aligned}$$

$$\begin{aligned}
&= -mg - [w_g p]' \\
&\quad - [\alpha_\ell (w_g - w_\ell) + \alpha_r (w_g - w_r)] \rho_g g \\
&\quad - D_\ell (w_g - w_\ell)^2 |w_g - w_\ell| - D_r (w_g - w_r)^2 |w_g - w_r|
\end{aligned}$$

Subtraction from this equation of multiples w_g, w_ℓ, w_r, of Equations (23), (24), (25), respectively, and using (20), gives

$$\begin{aligned}
[m_g h_g + m_\ell h_\ell]' &= \tfrac{1}{2}(w_g^2 - w_\ell^2) m_g' - p w_g' \\
&\quad + [(m_g v_g + m_\ell v_\ell) p]' \qquad (27)
\end{aligned}$$

where the h_i, $i = g, \ell$ are the specific enthalpies of the gas and liquid phases.

The above equations differ slightly from the formulation used in previous papers (e.g. McKibbin, 1979) but we regard them as forming a better model than that used before.

5. A SIMPLE CALCULATION

Because the general model is very complex, with many parameters, only one simple example is illustrated here.

Using the ideas of the Two-Phase-Lift (TPL) model, it is assumed that the liquid volume fraction of the eruption fluid stream is small, and that the fluid is flowing in a mist regime, with small liquid droplets moving at the same speed as the vapour, i.e., $w_\ell = w_g$ $(= w_f)$.

Mass conservation gives, from (20) and (21),

$$m_f = \alpha \rho_f w_f = m_f(0) \qquad (28)$$

$$m_r = (1 - \alpha) \rho_r w_r = m_r(0) \qquad (29)$$

where α is the fraction of the total flow area which is occupied by the two-phase fluid mixture:

$$\alpha = \alpha_g + \alpha_\ell \quad \text{and} \quad \rho_f = \frac{\alpha_g \rho_g + \alpha_\ell \rho_\ell}{\alpha}$$

Momentum conservation gives

$$\begin{aligned}
m_f w_f' &= -[\alpha \rho_f + (1 - \alpha) \rho_g] g - p' \\
&\quad - D_r (w_f - w_r) |w_f - w_r| \qquad (30)
\end{aligned}$$

$$m_r w_r' = -(1 - \alpha)(\rho_r - \rho_g) g + D_r (w_f - w_r) |w_f - w_r| \qquad (31)$$

with

$$D_r = \frac{3}{8} C_s \rho_f \frac{(1 - \alpha)}{R_{part}}$$

The energy equation (28) becomes

$$m_f h_f' = [\alpha w_f p]' - p w_f' \qquad (32)$$

with the fluid mixture specific enthalpy given by

$$h_f = X h_g + (1 - X) h_\ell$$

The dependent variables are $p(z)$ and $\alpha(z)$, with initial values $p(0)$, $\alpha(0)$ at the surface $z = 0$. The densities and specific enthalpies of the separate fluid phases are functions of pressure; correlations are given in the Appendix.

A set of results is shown in Figure 2, for the case where the fluid originates from the underground conditions $p_d = 4$ bars abs. and $S_{gd} = 0$. This means that the fluid specific enthalpy at the source is $h_d = 605$ **kJ/kg.**

It is supposed that the surface pressure is just above atmospheric, at $p_s = 1.02$ bars abs. At the surface, the dryness **X** of the fluid is about 0.0823, while the vapour saturation is about 0.993, again illustrating the volumetric domination of the steam phase in the flow stream.

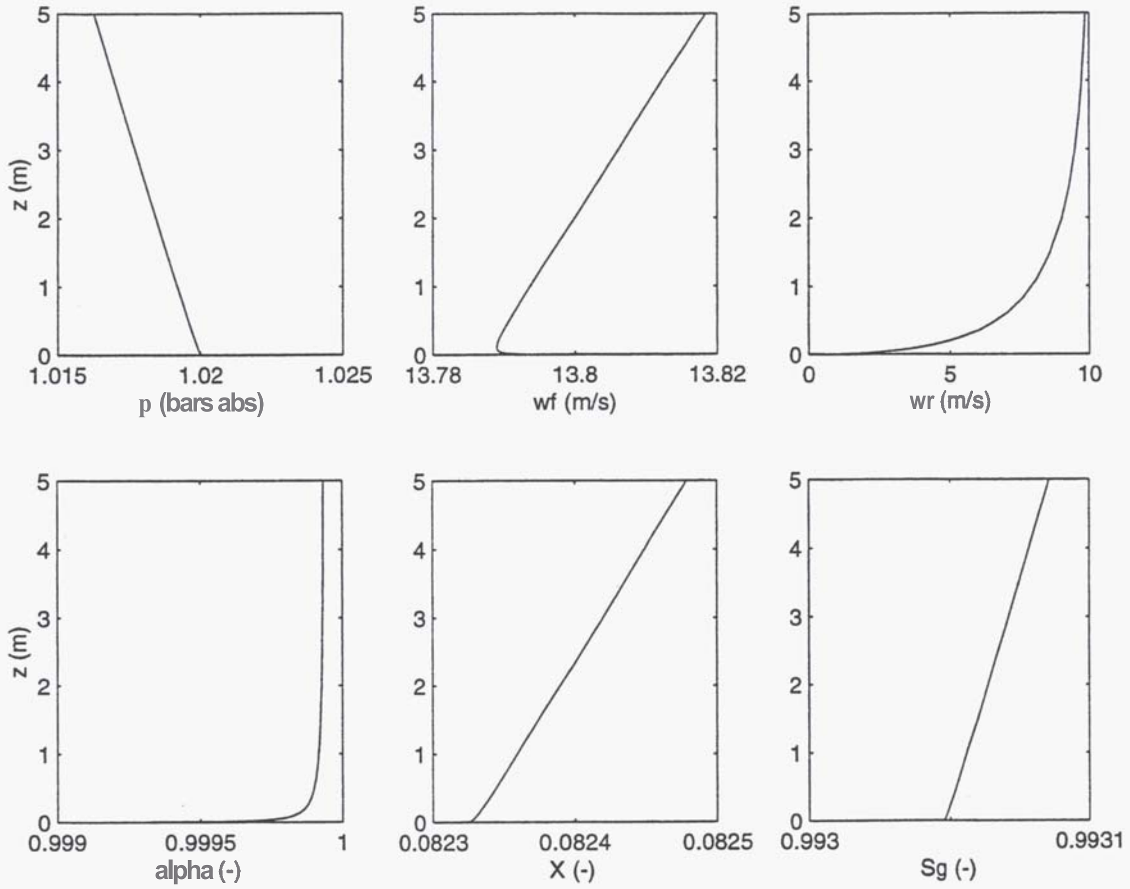

Figure 2. A set of results for the case $p_d = 4$ bars abs., $S_{gd} = 0$, $p_s = 1.02$ bars abs., $m_f = 100$ kg/s/m^2, $m_r = 1$ kg/s/m^2, $\alpha(0) = 0.999$, $\rho_r = 1500$ kg/m^3, $R_{part} = 1$ mm. The values of pressure **p**, fluid speed w_f, rock particle speed w_r, fluid volume fraction α, fluid dryness X and vapour saturation S_g are plotted against elevation z, for $0 \leq z \leq 5$ m.

For (somewhat arbitrary) mass flows $m_f = 100$ kg/s/m^2 and $m_r = 1$ kg/s/m^2, with $\alpha(0) = 0.999$ and rock particle density and size $\rho_r = 1500$ kg/m^3 and $R_{part} = 1$ mm, the values of pressure p, fluid speed w_f, rock particle speed w_r, fluid volume fraction α, and the dryness X and vapour saturation S_g of the two-phase fluid are plotted against height z, for $0 \leq z \leq 5$ m.

The graphs show how the rock particles are accelerated in the stream, while their volumetric contribution, measured by the value of $1 - \alpha$, reduces; the two-phase composition of the fluid does not change much, indicating that there is little evaporation from the liquid to the gas phase as the column rises.

Over the **5** m height of the calculation, the fluid speed is approximately constant at about 13.8 m/s, while the rock particles are accelerated to their terminal speed of about 10 m/s. As the rock particles are accelerated, their spatial density falls, and α, the areal fraction of the column occupied by the fluid portion of the flow, increases to about **99.99** %.

SUMMARY

Equations which may be used to describe mass, momentum and energy conservation in a steady 3-phase flow **of** liquid water droplets, vapour and solid particles in a 1-D vertical flow of uniform cross-section have been detailed.

Insofar as they might be used to model hydrothermal eruptions, it should be noted that air entrainment has been neglected.

An illustrative example for the case where the fluid is flowing in a mist regime **shows** some of the features which belong to solutions of such a model. The mist flow simplification is thought to be reasonable, since the fluid jet issues forth at high speed, and the liquid is likely to be divided into small droplets by the turbulent motion near the surface.

More work is presently under way to refine the models and explore further solutions for comparison with field data.

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APPENDIX

Some useful correlations for the thermophysical properties of water near atmospheric conditions

The following correlations provide simple formulae which, for near-surface water (down to 40 m, say), may be used to calculate the saturation temperature $T_{sat}(p)$, and, for saturated conditions, the liquid and gas (vapour) specific enthalpies $h_\ell(p)$ and $h_g(p)$, the liquid and gas specific volumes $v_\ell(p)$ and $v_g(p)$, and the liquid and gas dynamic viscosities $\mu_\ell(p)$ and $\mu_g(p)$.

Using data from Rogers & Mayhew (1980), log-log plots of the property values against the pressure p show closely linear relationships in nearly all cases, and produce the power-law formulae given below. There is close agreement between the steam table data and the following correlations.

For $1 \leq p \leq 5$ bars absolute:

Saturation temperature

$$T_{sat}(p) = 99.6\, p^{0.2655} \ {}^\circ\mathrm{C}$$

Specific enthalpies

$$h_\ell(p) = 417\, p^{0.2689} \ \mathrm{kJ\ kg^{-1}}$$

$$h_g(p) = 2675\, p^{0.01683} \ \mathrm{kJ\ kg^{-1}}$$

Specific volumes

$$v_\ell(p) = 0.001044\, p^{(0.005942 \log p + 0.01895)} \ \mathrm{m^3\ kg^{-1}}$$

$$v_g(p) = 1.694\, p^{-0.9365} \ \mathrm{m^3\ kg^{-1}}$$

Dynamic viscosities

$$\mu_\ell(p) = 280 \times 10^{-6}\, p^{-0.2817} \ \mathrm{kg\ m^{-1}\ s^{-1}}$$

$$\mu_g(p) = 12.0 \times 10^{-6}\, p^{0.09262} \ \mathrm{kg\ m^{-1}\ s^{-1}}$$

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