GEOTHERMAL TWO-PHASE PRESSURE LOSS IN A BEND

A. MUNDAKIR[1, 2] & K.C. LEE[1]

[1]Geothermal Institute, University of Auckland
[2]Pertamina, Jakarta, Indonesia

SUMMARY - When a two-phase fluid flows through a bend, its flow pattern is disturbed. The Chisholm's **(1980)** pressure drop correlation for bends was tested using geothermal two-phase flow data. Chisholm's **(1980)** correlation under-predicts the measured pressure drops by 50%. Other correlations: Chisholm C-type **(1967)**, Sookprasong **(1980)**, Kuhn **&** Morris **(1997)** and Paliwoda **(1992)** all use a two-phase multiplier method. **A** new correlation **is** proposed using frictional and momentum components for the total pressure drop through a long radius (LR) elbow. The new correlation predicts the measured data to within **5%.**

1. INTRODUCTION

The flow characteristics and pressure drops of a single-phase fluid through pipe fittings are well understood, but not so for a two-phase fluid. Two-phase flow studies in pipe fittings are relatively few in number. Many researchers have investigated and observed that the flow behaviour in a bend is very complicated due to the presence of secondary flows. When the fluid flows through a bend, the curvature of the pipe bend causes a centrifugal force directed from the centre of curvature to the outer wall and together with fluid adhesion to the wall produce the secondary flow (ideally organised in two identical eddies) as described by Azzi et al (2000). The two-phase flow pattern is also disturbed when it flows through a bend as reviewed by Lee **(1979).**

Because of the flow complexity it is very difficult to model the two-phase flow through a bend, to derive a correlation analytically, and to provide a systematic calculation method for pressure drop across a bend. *So* a simplified physical concept in deriving a simplified energy and momentum balance is needed. **As** a result, many proposed correlations for predicting pressure drops in bends are empirical. Chisholm's **(1980)** correlation adopted by Engineering Science Data Unit (ESDU) **(1989)** is the most popular correlation.

In using two-phase flow correlations, it is important to consider the two-phase flow parameters that affect the pressure drops. ESDU **FMI 2 (1989)** reviewed qualitatively the variables affecting two-phase flow in a straight pipe. For two-phase flow in a bend, the parameters affecting the pressure drop was reviewed by Azzi et al (2000). The pressure drop is considered to depend on the flow rates of both phases, macroscopic phase state and transport properties (density and viscosity) of each phase, local geometric parameters of the bend and the gravity. All of them are called primary parameters. Since the flow pattern is the result of interaction **of** the primary parameters, so it is not considered as an independent parameter. Wall roughness (E) has a minor influence compared to those of single-phase flow. Surface tension ($ is also ignored because its magnitude is generally small compared to the inertial and pressure forces in a system above the atmospheric pressure. The physical relationship between the primary parameters is as follows:

a) The **mass** flow rate of each phase **is** found from the total mass flow and the dryness.
b) The densities and viscosity of both phases are included in a density and viscosity ratio. The gas phase viscosity effect is small compared to that **of** liquid phase viscosity, so it acts as a means for non-dimensionalization.
c) The bend radius curvature is usually normalised by the pipe diameter.

The qualitative interrelationships between the parameters and the pressure loss are:

1. The pressure drop will increase with larger total mass flow rate since the velocity of each phase is higher.
2. On reducing the density ratio by increasing the system pressure, the pressure loss will decrease.
3. A higher or lower liquid phase viscosity induces corresponding changes to the wall shear stress, while the momentum exchange between the phases would change in opposite direction.
4. An increase in bend curvature radius would decrease the intensity of the induced secondary flow hence lower the pressure loss.

2. PRESSURE DROP CORRELATION FOR TWO-PHASE FLOW IN BEND

Two-phase pressure drop in bends is due mainly to friction and momentum effect. Since most of the variables of the two-phase flow are correlated to those of single-phase flow and by convention the respective bend pressure loss is related to that in single-phase flow hence the pressure drop correlations. By taking this approach, Chisholm (1980) approximated ratios of two-phase momentum flux (MF) to those of single-phase when the density ratios are almost unity and expressed as:

$$\frac{MF}{MF_{LO}} = 1+\left(\frac{\rho_L}{\rho_G}-1\right)\left(\frac{1}{K}x(1-x)+x^2\right) \quad (1)$$

For a uniform cross-section pipe, momentum flux change due to velocity ratio change (K) can be defined:

$$\frac{\Delta(MF)}{MF_{LO}} = \left(\frac{\rho_L}{\rho_G}-1\right)x(1-x)\Delta\left(\frac{1}{K}\right) \quad (2)$$

With the following assumptions:

1. bend separates the two phases hence the velocity ratio within the bend,
2. further loss exists downstream as the momentum flux increases to the equilibrium value,
3. downstream effects are small in the single-phase flow,
4. single-phase loss occurs within the bend,
5. sufficient accuracy of the homogeneous model two-phase multiplier,

the two-phase pressure drop in a horizontal bend can be defined as:

$$\Delta p_b = \Delta p_{bLO}\phi_{LO}^{\ 2} + \Delta(MF) \quad (3)$$

where:

$$\Delta p_{bLQ} = k_{LO}\frac{G^2}{2\rho_L} \quad (4)$$

$$\phi^2{}_{LO} = 1+\left(\frac{\rho_L}{\rho_G}-1\right)x \quad (5)$$

so:

$$\frac{\Delta p_b}{\Delta p_{bLO}} = 1+\left(\frac{\rho_L}{\rho_G}-1\right)\left(Bx(1-x)+x^2\right) \quad (6)$$

The right-hand side of this equation is known as two-phase multiplier ($\Phi^2{}_{LO}$), where:

$$B = 1 + \frac{2}{k_{LO}}\Delta\left(\frac{1}{K}\right) \quad (7)$$

and $\Delta(1/K)$ parameter corresponding to the change in velocity ratio of the phases. For a 90° bend, it was suggested empirically that:

$$\Delta\left(\frac{1}{K}\right) = \frac{1.1}{2+(R/D)} \quad (8)$$

therefore

$$B = 1 + \frac{2.2}{k_{LO}(2+(R/D))} \quad (9)$$

In summary, the correlation for two-phase flow pressure drop in a 90° bend can be defined from equation (6) as follows:

$$\Delta p_b = \Delta p_{bLO}\cdot\Phi^2_{LO} \quad (10)$$

where Δp_{LO} is defined in equation (4) and:

$$\Phi^2_{LO} = 1+\left(\frac{\rho_L}{\rho_G}-\right)\left(Bx(1-x)+x^2\right) \quad (11)$$

Table 1. Two-phase multiplier correlations for 90° bend (Azzi et al, 2000).

ESDU (Chisholm, 1980): $\Phi^2_{LO} = 1+\left(\frac{\rho_L}{\rho_G}-1\right)\left(Bx(1-x)+x^2\right)$; $B = 1+\frac{2.2}{k_{LO}(2+(R/D))}$
Chisholm B-type (2000): $\Phi^2_{LO} = \frac{1}{(1-x)^2}\{1+\left(\frac{\rho_L}{\rho_G}-1\right)\left(Bx(1-x)+x^2\right)\}$
Chisholm C-type (1967): $\Phi^2_{LO} = 1+\frac{C}{X}+\frac{1}{X^2}$; $C = C^*\left(\left(\frac{\rho_L}{\rho_G}\right)^{0.5}+\left(\frac{\rho_G}{\rho_L}\right)^{0.5}\right)$, $C^* = 1+25\frac{D}{L}$; $X = \left(\frac{1-x}{x}\right)^{0.9}\left(\frac{\mu_L}{\mu_G}\right)^{0.1}\left(\frac{\rho_G}{\rho_L}\right)^{0.5}$
H.G.Kuhn and S.D. Morris (1997): $\Phi^2_{LO} = \frac{1}{(1-x)^2}\left(\frac{\rho_L}{\rho_{Hom}}\left(1+x\left(\frac{\rho_{Hom}}{\rho_G}-1\right)\left(B_{90}-1\right)\right)\right)$; $\frac{1}{\rho_{Hom}} = \frac{x}{\rho_G}+\frac{1-x}{\rho_L}$
P. Sookprasong (1980): $\Phi^2_{LO} = \frac{(\rho_L V_L+\rho_G V_G)(V_G+V_L)}{\rho_L V_L^2}$
A. Paliwoda (1992): $\Phi^2_{GO} = [\varphi+A(1-\varphi)x][1-x]^{0.333}+x^{2.276}$; $\varphi = \frac{\rho_G}{\rho_L}\left(\frac{\mu_L}{\mu_G}\right)^{0.25}$, A=2.7

Up to now most researchers have developed many correlations for bends based on the Chisholm's approach. They just modified some parameters of the Chishom's two-phase multiplier. Azzi et al (2000) summarised the two-phase multiplier correlations for 90° bend as tabulated in Table 1.

3. ANALYSIS

In analysing the pressure drop correlation in a bend, it is important to consider the basic assumptions, the theoretical consequences and the affecting parameters.

Basic assumptions:
All of the above correlations are based on the Lockhart and Martinelli (1949) concept of two-phase flow multiplier. Therefore the correlations are all correlated to the single-phase liquid or gas flow which is calculated based on the single-phase loss coefficient of bend (K), which is determined empirically from a particular type of bend. In deriving the correlation, Chisholm (1980) also assumed single-phase pressure drop occurred within the bend. This assumption is physically incorrect regarding the existence of the two phases in the flow. In addition for flows with high density ratios, this assumption could lead to dissipation. Another questionable assumption is the derivation of the $\Delta(1/K)$ parameter that corresponding to the velocity ratio change to the empirical equation as defined in (8).

Theoretical consequences:
There are two basic theoretical consequences:

1. Boundary conditions of the two-phase flow: when the value of $x=0$ the pressure drop should be equal to that of single-phase liquid flow. On the other hand when the value of $x=1$ the pressure drop should be equal to that of single-phase gas flow.
2. Boundary conditions of the bend curvature, when the bend angle equal zero theoretically the pressure drop should be equal to that of the straight pipe with an equivalent length of straight pipe. This consideration is not explicitly integrated in all of the above correlations.

Parameters:
It is also important to examine the relationship between the primary parameters with the available measured pressure drop data. This relationship would help to determine the significance of the parameters in the correlations.

4. NEW CORRELATION FOR LR ELBOW

The following assumptions are made for the proposed new correlation:

1. Pressure drop within bend can be treated as the sum of three separate components: i.e. friction, momentum change and gravity hence can be expressed as:

$$\Delta p_b = \Delta p_f + \Delta p_m + \Delta p_g \tag{12}$$

2. Only two-phase pressure drop occurs when a two-phase fluid flows through a bend.

The following discussion focuses on geothermal two-phase flow and for horizontal LR elbow. For the prediction of frictional pressure drop in a horizontal straight pipe, a correlation was developed by Harrison (1975) as follows:

$$dp_f = \tfrac{1}{2} \cdot \lambda \cdot \frac{L}{D} \cdot \frac{\rho_L \cdot \overline{V}^2}{(1 - AC)} \tag{13}$$

$$\overline{V} = \frac{W(1-x)}{\rho_L (1-\alpha) A} \tag{14}$$

$$\alpha = \frac{1}{1 + \left(\frac{1-x}{x}\right)^{0.8} \left(\frac{\rho_G}{\rho_L}\right)^{0.515}} \tag{15}$$

$$AC = \frac{W^2 x^2 v_G}{\alpha \cdot A^2 \cdot P} \tag{16}$$

This correlation was tested using geothermal steam and water data of Freeston et al (1983) in Mundakir (1997). It showed a good agreement with the measured data. By proposing a new correlation for void fraction (α) and an equivalent liquid phase flow velocity ($\overline{V}$) as defined below, Zhao (1998) improved the prediction method. The equivalent liquid phase flow velocity is defined as:

$$\overline{V} = \frac{(1-\alpha)}{(1-\alpha^{0.5})^{8/7}(1+\tfrac{8}{7}\alpha^{0.5})} \cdot \overline{V_f} \tag{17}$$

where:

$$\overline{V_f} = 1.1(1-x)\frac{W(1-x)}{\rho_L(1-\alpha)A} \tag{18}$$

and α is calculated from the following equation:

$$\frac{1-\alpha}{\alpha^{7/8}} = \left[\left(\frac{1}{x}-1\right)\left(\frac{\rho_G}{\rho_L}\right)\left(\frac{\mu_L}{\mu_G}\right)\right]^{7/8} \tag{19}$$

It can be concluded that both methods are reliable for two-phase geothermal flow in a horizontal straight pipe. The equivalent liquid phase velocity parameter defined in equation **(17)** shows a very good correlation with the measured data of pressure drops in a LR elbow as shown in Figure 1. This indicates that two-phase frictional component is well correlated to the bend pressure drop. None of the primary parameters such as density, viscosity and mass quality show a good correlation with the measured data.

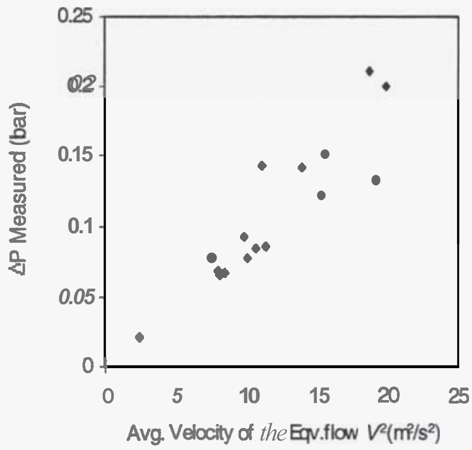

Figure 1. Average velocity of eqv. single-phase flow ($\overline{V}^2$) vs. Measured pressure drop in LR elbow.

By using a fairly simple physical argument, Moms **(1992)** developed a model for two-phase momentum flux from which the corresponding pressure gradient may be calculated. The momentum flux is defined as:

$$J_m = G^2 \cdot v_e \qquad (20)$$

where v_e is the effective specific volume and is defined as:

$$v_e = \left[x v_G + K(1-x) v_L\right]\left[x + \frac{(1-x)}{K}\left\{1 + (K-1)^2 \cdot \phi\right\}\right] \qquad (21)$$

$$K = \left(\frac{v_H}{v_L}\right)^{1/2} \qquad (22)$$

$$v_H = x v_G + (1-x) v_L \qquad (23)$$

$$\phi = \frac{1}{\left[\left(v_G / v_L\right)^{1/2} - 1\right]} \qquad (24)$$

The model is independent of any data set and showed best overall prediction of the available data, Morris **(1992).**

A new correlation is proposed by employing the two correlations discussed above with respect to the first assumption. The frictional component of the pressure drop in the bend can be calculated by using equation **(13)** and **(17)-(19)** then multiply it by the equivalent straight pipe length of the bend. For **90°** bend, the equivalent straight pipe length is:

$$L_{eq} = \tfrac{1}{2} \cdot \pi \cdot R \qquad (27)$$

therefore:

$$\Delta p_f = \tfrac{1}{4} \cdot \lambda \cdot \pi \cdot R \frac{\rho_L \cdot \overline{V}^2}{D(1-AC)} \qquad (28)$$

The momentum change of pressure drop component can be calculated from Morris equation **(20)** divided by the non-dimensional bend curvature (R/D) and multiplied by 'flashing factor' defined as:

$$F = \frac{(1-x^2)}{(1-x)} \qquad (29)$$

hence:

$$\Delta p_m = \frac{(1-x^2)}{(1-x)} \cdot \frac{G^2 \cdot v_e}{(R/D)} \qquad (30)$$

Finally, the pressure drop in the bend is:

$$\Delta p_b = \Delta p_f + \Delta p_m \qquad (31)$$

5. DISCUSSION

All of the correlations in Table 1, including the new correlation are tested by using the geothermal data in (Lee et al, **1979).** The flow parameters were set to produce an annular flow regime and the main flow parameters were:

1. Separator pressure: **6.5 – 11** (barg)
2. Steam flow rate : **0.47 – 1.74** **(kg/s)**
3. Water flow rate : **0.54 – 11.3** (kg/s)
4. Quality : **0.05 – 0.33**

The predictions of all correlations against the measured pressured drops data are plotted in Figures **2-8.** The ESDU **(1980)** correlation under-predicts the measured data by about 50% as shown in Figure **2.**

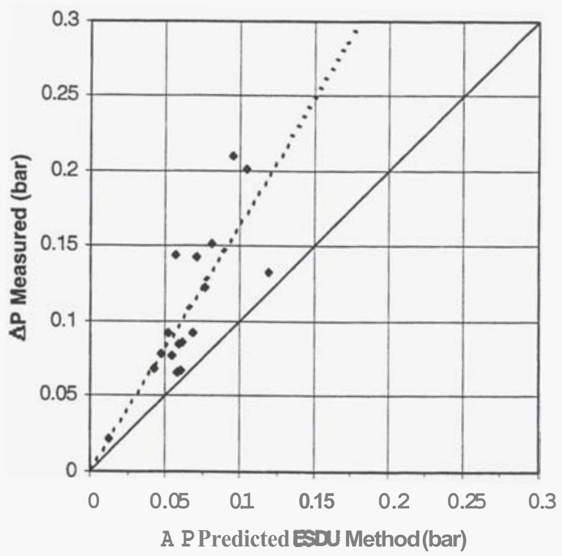

Figure 2. AP Predicted vs. AP Measured for Horizontal LR elbow: ESDU correlation (1980).

The Chisholm C-type (1967) correlation under-predicts the measured data by about 70% as do Paliwoda (1992) and Sookprasong (1982) correlations (Figures 3-5).

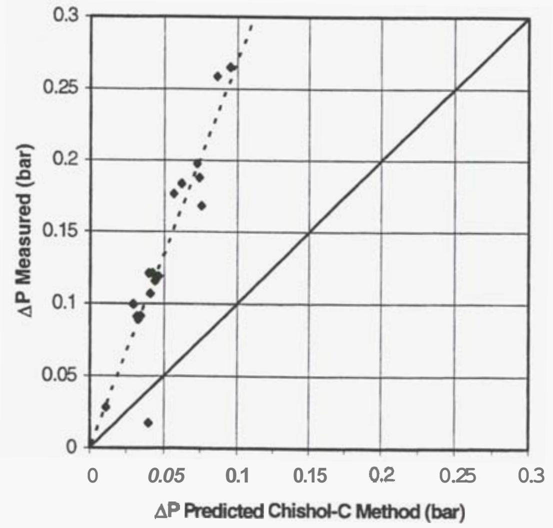

Figure 3. AP Predicted vs. AP Measured for LR elbow: Chisholm C-type correlation (1967).

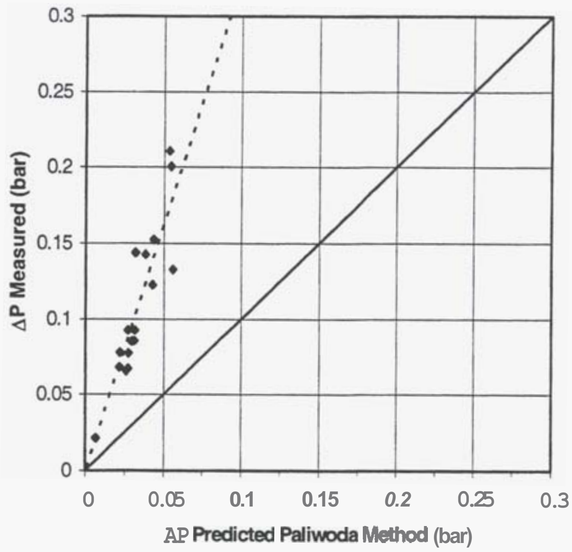

Figure 4. AP Predicted vs. AP Measured for LR elbow: Paliwoda correlation (1992).

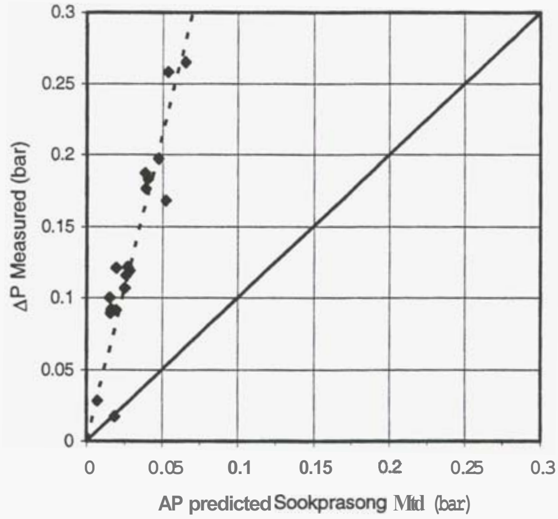

Figure 5. AP Predicted vs. AP Measured for LR elbow: Sookprasong correlation (1980).

The Chisholm B-type (2000) correlation is similar to the ESDU correlation except by a factor of $(1/(1-x)^2)$. This correlation gives better prediction, over-predicting the measured data by only 10%. Surprisingly, the Kuhn and Morris correlation gives very similar prediction as the Chisholm B-type correlation (Figures 6 and 7). However the new method gives the best prediction as shown in Figure 8.

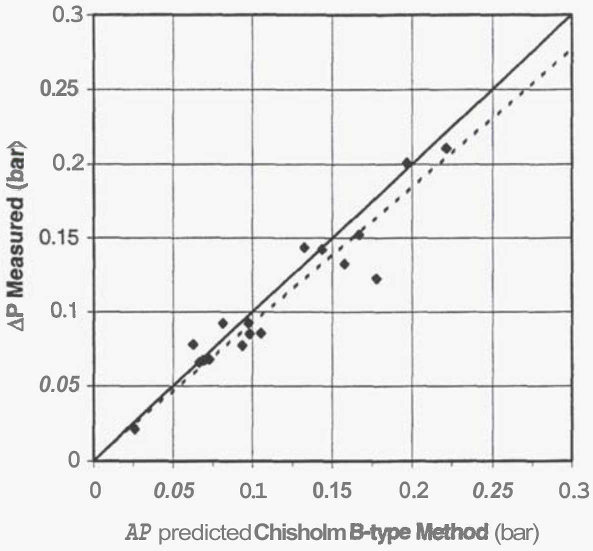

Figure 6. AP Predicted vs. AP Measured for LR elbow: Chisholm B-type correlation (2000).

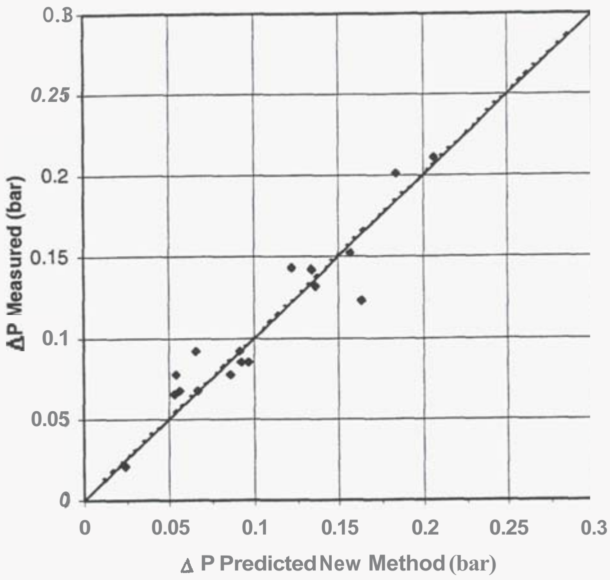

Figure 6. AP Predicted vs. ΔP Measured for LR elbow: New proposed correlation (1992).

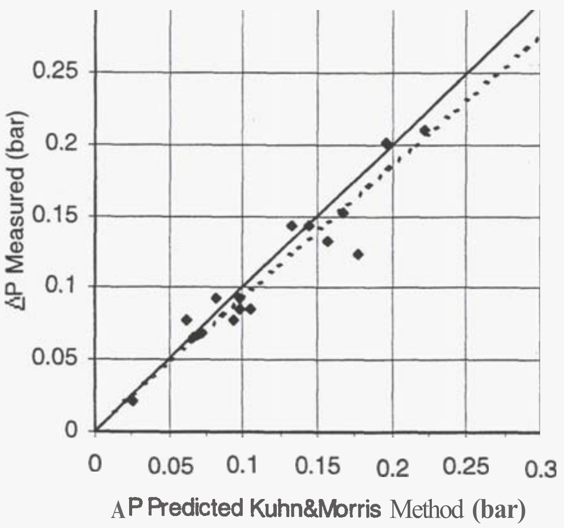

Figure 7. AP Predicted vs. ΔP Measured for LR elbow: Kuhn and Moms correlation (1997).

The theoretical limits of the correlations are also investigated. Only the ESDU and Paliwoda correlations that satisfy the boundary conditions, give a value of pressure drop when $x=0$ and $x=1$. On the other hand, only Chisholm C-type and Sookprasong correlations do not satisfy the two boundary conditions.

Kuhn & Morris, Chisholm B-type and the new correlation satisfy only one boundary condition when $x=0$. However the prediction accuracy of the all correlations at the two boundary conditions with respect to the single-phase pressure drop has yet to be proven.

With the exception of the new correlation, all the correlations use the bend curvature parameter only to determine the single-phase bend pressure loss coefficient (k_{LO}). As stated before, none of the correlations including the new correlation explicitly includes the angle of the bend to satisfy the second theoretical limit, when the bend angle equals zero the pressure drop should be equal to that of the straight pipe.

However the new correlation provides a logical consequence of the first assumption, when the angle of the bend equals zero the momentum change component of the pressure drop become negligible. Therefore the pressure drop is only due to the frictional component defined in equation (28) which is the same as the pressure drop in straight pipe with the equivalent straight pipe of the bend.

6. CONCLUSIONS

A new correlation for predicting two-phase pressure drop in horizontal LR elbow has been proposed based on the frictional and momentum change pressure drop components. The new correlation is independent of any single-phase pressure drop coefficient for bend (k_{LO}). It gives a very good agreement when tested against geothermal two-phase flow data.

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7. NOMENCLATURE

A	Pipe Area	(m^2)
AC	Acceleration term defined in equation (16)	-
B	Bend coefficient defined inequation (9)	-
D	Pipe diameter	(m)
G	Mass flux	($kg/s.m^2$)
J_m	Momentum Flux defined inequation (20)	(Pa)
K	Slip ratio	-
k_{LO}	Single-phase bend coefficient	-
L	Pipe length	(m)
MF	Momentum Flux	
P	Pressure	(Pa)
R	Bend radius	(m)
$\overline{V}$	Average velocity of equivalent single-phase flow	(m/s)
$\overline{V}_f$	Liquid phase velocity	(m/s)
v	Specific volume	(m^3/kg)
v_e	Effective specific volume defined in equation (21)	(m^3/kg)
W	Mass flow rate	(kg/s)
x	Mass flow quality	-
a	Void fraction	-
A	Difference or change	-
Δp	Pressure drop	(Pa)
Φ^2_{LO}	Two-phase multiplier	-
ϕ	Parameter in eqn. (24)	
λ	Friction factor	-
μ	Viscosity	(kg/m.s)
ρ	Density	(kg/m^3)

Subscripts

b	bend
LO	Liquid only
f	Frictional component
m	Momentum component
G	Gas phase
L	Liquid phase
H	Homogeneous
eq	Equivalent

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