ANALYSIS OF THE HALF WAY RULE

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SUMMARY - The Half Way Rule is an analytic "tool" used in geothemal practice for indicating the optimum separation pressure of single flash geothermal power process. This paper presents an investigation of the rule, which has been performed through an analysis of the process. This investigation has revealed limitations on the use of the rule in geothermal engineering practice. A quantitative comparison between optimising using the rule against optimising using a numerical simulation, which has been done separately and is not covered here, found vast discrepancies between the results. It is believed that the limitations of the Half Way Rule presented in this paper are the main reason for these discrepancies.

1. INTRODUCTION

Geothermal fluid can be delivered from the ground in one of three possible forms; steam, water or two phase mixture. Two phase fluid is the most common form in geothermal practice. To get out steam the two phase mixture is first separated to its components, steam and brine. Separated steam is then delivered to the turbo-generator where it expands to condenser pressure generating mechanical power converted further into electrical energy.

The pressure at which two phase fluid is separated (separation pressure) is one of the most important parameters of the process. The Half Way Rule (HWR) is an analytic "tool" in use for optimising it. The rule states that:

> "The optimum separation pressure has the corresponding saturation temperature just at the half way between the condenser temperature and the resource enthalpy saturation temperature ".

The HWR is simple and easy to use. The author of this paper believes that the HWR is being used for giving rough and provisional indications. However, it has been noticed recently that an optimising method using modelling and numerical simulation is giving results that contradict the HWR. These discrepancies have inspired an investigation of the HWR.

A study on the HWR has been undertaken to answer the following three questions:

(a) How the HWR is originated?
(b) Is the HWR universally applicable or not?
(c) If not, why not?

To find answers to (a), (b), (c) single flash geothermal process has been analysed. The analysis was steered toward developing an algebraic expression for the HWR. To make that development possible certain assumptions were accepted. Some of them are considered as problematic and, according to the author of the paper, are causing major limitations on the HWR field of application. Those assumptions have been explained and discussed later on in the paper.

2. EXAMPLE FOR USING THE HWR

The following hypothetical parameters are used to illustrate the HWR:

1200 [kJ/kg]	resource enthalpy, i.e. enthalpy of the fluid delivered at the wellhead;
0.1 [bara]	pressure in the condenser.

The boiling water has the enthalpy of 1200 kJ/kg when it is under the pressure of 57.6 bara. The corresponding saturation temperature for 57.6 bara is 272.8 ^{0}C.

The Corresponding saturation temperature for 0.1 bara is 45.8 ^{0}C.

According to the HWR the optimum separation temperature would be:

$$t_{optimum} = \frac{272.8 + 45.8}{2} = 159.3 \quad [^\circ C]$$

The optimum separation pressure for this example would be the corresponding saturation pressure for the temperature of 159.3 °C, which is 6.07 bara.

3. ANALYSIS OF SINGLE FLASH GEOTHERMAL POWER PROCESS

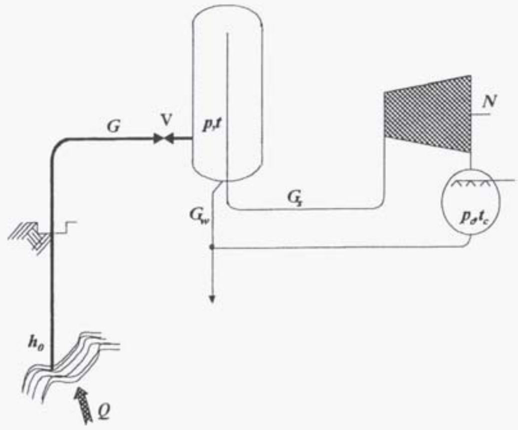
3.1 Analysis

The analysis presented here can be outlined as:

A. to develop function $N(p)$, i.e. a mathematical expression for turbine output N as a function of separation pressure p;

B. to determine which separation pressure gives the maximum for function $N(p)$.

The analysis remains at the surface of both theoretical thermodynamic and analytical mathematics. All the consequent simplifications applied here are well studied and could be explained in some other context.



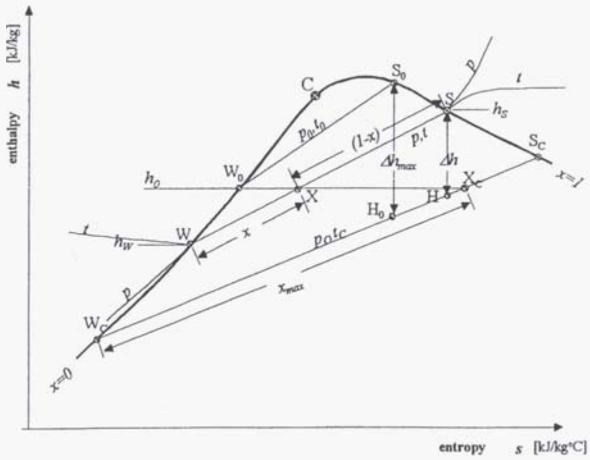
Q [kJ] geothermal heat;
h_o [kJ/kg] enthalpy reached by underground heat transfer (resource enthalpy);
p [bara], separation pressure and the
t [°C] corresponding saturation temperature;
G [kg/s] geothermal fluid delivery;
G_s [kg/s] steam phase - separated;
G_w [kg/s] brine phase - separated;
p_c [bara], condenser pressure and the
t_c [°C] corresponding saturation temperature;
N [kW] turbine output.
V valve

Figure 1. A simplified diagram of single flash geothermal power process with symbols explained

The following notation is used:

- physical properties are presented by italic bold letters ($p, t, \Delta h, g, \ldots$);
- geometrical points are presented by capital bold letters (**S, H, X, W₀**,...);
- lines are presented by their end points (**SH, X_C W_C**,...);
- triangles are presented by their corner points (**W_0 X W, W_0 X_C W_C**,...);

The diagram in Figure 1 presents simplified single flash geothermal power process. The geothermal reservoir has been tapped by a well, with geothermal fluid flow G being delivered in a two phase form. The fluid is further transported to the separator, where separation to steam and water (brine) takes place at pressure p. Steam flow G_s is delivered to the turbo-generator, where a part of its heat energy is transformed into mechanical work by expansion to pressure p_c. *An* electric generator transforms mechanical work further into electrical energy. Brine flow G_w is reinjected back into the ground, together with remaining condensate.



C the critical point;
h_w [kJ/kg] enthalpy of brine separated at pressure p;
h_s [kJ/kg] enthalpy of steam separated at pressure p;
p_o [bara] the highest theoretical separation pressure
t_o [°C] and the corresponding saturation temperature;
x steam share separated out at pressure p;
$(1-x)$ brine share separated out at pressure p;
x_{max} [kg/s] the maximum theoretical steam share that could be separated out from geothermal fluid of enthalpy h_o;
Ah [kJ/kg] available heat potential to be transferred by steam turbine into mechanical work if the separation pressure is p;
Δh_{max} [kJ/kg] the maximum transferable heat potential if geothermal fluid has enthalpy h_o.

Figure 2. Simplified single flash geothermal power process presented in a qualitative form of the Mollier Chart

The following basic assumptions have been accepted:

i. the process is adiabatic;
ii. there are no transport pressure drops;
iii. there is no limitation on separation pressure p;
iv. expansion in the steam turbine is isentropic.

The process from Figure 1 is presented in the Mollier Chart in Figure **2.** The Mollier Chart is a thermodynamic diagram showing the geometric proportion between water and steam physical properties that enables analysis of the thermodynamic process to be done through elementary geometry.

All the isobars and the corresponding saturation isotherms within the saturation zone are collinear. Isobar/isotherm p, t from Figure 2 presents any theoretically possible separation pressure and the corresponding saturation temperature. The lowest theoretically possible separation pressure for this process is the pressure in the condenser. Line **W_C S_C** presents the isobar/isotherm for that case (isobar/isotherm p_c, t_c). The highest theoretically possible separation pressure is the pressure at which resource enthalpy h, is the enthalpy of the boiling water, presented by line W_0 S_0 (isobar/isotherm p_o, t_o).

Point X in Figure 2 indicates geothermal fluid properties at the moment just before separation to steam and brine takes place. The point is located at the intersection of the horizontal straight line $W_0 X_C$, (h, constant enthalpy) and the actual separation pressure isobar/isotherm p, t. Point X splits line W S into two segments, X W and X S. This geometrical split is just analogous to the physical separation ratio for fluid of enthalpy h_o separated at pressure p. The separation ratio is the exact rate of steam (of enthalpy h_s) and water (brine) (of enthalpy h_s) separated out fiom one kilogram of two phase mixture. The following relationships are valid:

$$\frac{WX}{WS} = \frac{G_s}{G} = x$$
$$\frac{XS}{WS} = \frac{G_w}{G} = (1-x)$$
$$WS = XW + XS$$
$$G = G_w + G_s$$
$$G_s = G \cdot x$$
$$G_{s\,max} = G \cdot x_{max}$$
$$G_w = G \cdot (1-x) \quad (1)$$

If we separate out the same flow rate of the same geothennal fluid at a lower and lower pressure, steam share increases while brine share decreases, following the shapes of the Mollier Chart saturation zone. The largest share of steam x_{max}, presented by line X_C W_C, is separated out at the lowest theoretical pressure, i.e. the pressure in the condenser. No steam would be separated out by an attempt to separate the same geothermal fluid at pressurep,. In that case x is equal 0.

The basic assumption iv enables expansion within the steam turbine to be presented as a vertical line. If separation is done at pressure p the available heat potential to be transferred into electrical energy per kilogram of steam is Ah, presented by line S H. If any steam were separated out at pressurep, it would have the maximum heat potential, Δh_{max}. However as no steam could be separated out no electricity can be generated by a separation at p_o.

The maximum steam flow separated out, $G_{s\,max}$, also would not generate any electrical energy, because there is no heat potential available, i.e. Ah at p_c is equal 0.

Turbine output can be expressed by the following equation:

$$N = G_S \cdot \Delta h = G \cdot x \cdot \Delta h \quad \text{[kW]} \quad (2)$$

It has already been explained that both parameters x and Ah are dependent on separation pressure p. Hence x and Ah are some functions of separation pressurep:

$$N = N(p) = G \cdot x(p) \cdot \Delta h(p) \quad \text{[kW]} \quad (3)$$

Steam share x goes from 0 to x_{max}, following separation pressure p. If the pressure is p_o then $x = 0$ and if the pressure is p_c then $x = x_{max}$. If we assume that W_C W_0 from Figure 2 is a straight line, the following geometry is valid.

Line X W is a side of triangle W_0 X W. As separation pressure drops fi-om p_o to p_c line X W expands from zero to X_C W_C. On the way down line X W, together with lines W W_0 and W_0 X, form a series of triangles. If we assume that lines X W are always parallel to each other then all the triangles formed that way become the members of a single similar triangles family. A characteristic of similar triangles is that they always maintain a proportion between the sides.

Available heat potential also varies along separation pressure. The largest potential Δh_{max} is available when geothermal fluid is being "separated" at pressure p_o, while there is a zero potential at separation pressure p_c.

If we assume that S_0 S_C is a straight line, a geometrical analogy, similar to the one fi-om the previous paragraph, is valid here. Line H S is a side of triangle S S_C H. As the separation pressure goes fi-om p_o to p_c line H S gets from H_0 S_0 to zero. If the lines are always parallel than H S on its way right/down, together with lines S S_C and S_C H, form a series of similar triangles.

The geometrical analogies fiom the previous paragraphs still cannot be used for further developing of (3). That will be possible after expressions (4) to (8) and the following few paragraphs.

Pressure is not even close to being proportional either to enthalpy or to entropy. To illustrate this we could say simply that the isobars within the Mollier Chart saturation zone are "more spread" in the area of lower enthalpies while being "more compact" in the region where enthalpies are higher.

Unlike pressure, temperature could be considered as proportional to enthalpy, as long as we accept the following:

$$h = c \cdot t \quad \text{[kJ/kg]} \quad (4)$$

Where:

h [kJ/kg]	enthalpy;
c [kJ/kg°C]	specific heat capacity;
t [°C]	temperature.

This proportionality exists as long as c is a constant. Let us consider that this assumption is acceptable at this level of approximation.

The fact that all isobars and the corresponding saturation isotherms are collinear (within the saturation zone) can be expressed by the following equations:

$$p = C_n \cdot t \quad \text{[bara]} \quad (5)$$

And accordingly:

$$t = \frac{1}{C_n} \cdot p \quad \text{[°C]} \quad (6)$$

Where:

C_{i1}	constant;
p **[bara]**	saturation pressure;
t **[°C]**	saturation temperature correspondent to p.

Expressions *(5)* and (6) are valid along every single isobar/isotherms within the saturation zone.

By substituting *(5)* into (3) we get:

$$N(C_{i1}\cdot t) = G\cdot x(C_{i1}\cdot t)\cdot \Delta h(C_{i1}\cdot t) \text{ [kW]} \quad (7)$$
$$N(t) = C_{i2}\cdot G\cdot x(t)\cdot \Delta h(t) \text{ [kW]} \quad (8)$$

Where:

C_{i2}	constant

Equation **(8)** now expresses the output as a mathematical function of saturation temperature instead of pressure. It is not a concern as by using *(5)* or (6) we can transform it either way.

To find out the form of output/temperature function **(8)** we can now use the geometric analogies (similar triangles) that have been explained before.

Going back to Figure 2. If t goes down fkom t_o to t_c, then x expands proportionally to the change of t. This proportionality is a consequence of expression (4) and the already explained similar triangles analogy developed around triangle **W_0 X W.** In mathematical terms it means that $x(t)$ is a linear function **of** t. Therefore it can be presented in the following form:

$$x(t) = A\cdot t + B \quad [\text{-}] \quad (9)$$

This function has two characteristic states that are used to calculate out coefficients A and B:

if $t = t_o$ **then** $x(t) = 0$ (10)
if $t = t_c$ **then** $x(t) = x_{max}$ (11)

Substituting (10) and (1 1) into (9) results in a simple system of two equations with two unknowns:

$$0 = A\cdot t_0 + B \quad (12)$$
$$x_{\max} = A\cdot t_C + B \quad (13)$$

After solving system (12), (13) we get:

$$A = \frac{x_{\max}}{t_C - t_0} \quad (14)$$
$$B = -\frac{x_{\max}}{t_C - t_0}\cdot t_0 \quad (15)$$

By (14) and (15) into (9) followed by a simple transformation (9) takes the following form:

$$x(t) = \frac{x_{\max}}{t_C - t_O}(t - t_0) \quad (16)$$

A similar procedure can be performed for heat potential Δh. It can also be considered as a linear function of t and presented in the following form:

$$\Delta h(t) = A\cdot t + B \quad \text{[kJ/kg]} \quad (17)$$

By a **similar** development as applied for $x(t)$ equation (17) becomes:

$$\Delta h(t) = \frac{\Delta h_{\max}}{t_0 - t_C}(t - t_C) \quad \text{[kJ/kg]} \quad (18)$$

Substituting (16) and (18) into **(8)** turbine output becomes the following function of separation saturation temperature:

$$N(t) = C_{i2}\cdot \frac{G\cdot x_{\max}\cdot \Delta h_{\max}}{(t_0 - t_C)\cdot(t_C - t_0)}\left[(t - t_C)\cdot(t - t_0)\right] \text{ [kW]} \quad (19)$$

Equation **(19)** expresses the output as a second order function of t. The analysis continues by deriving function (19) over t and making the derivation equal 0. The null point found that way will be t that gives the maximum $N(t)$:

$$\frac{dN(t)}{dt} = C_{i2}\cdot \frac{G\cdot x_{\max}\cdot \Delta h_{\max}}{(t_0 - t_C)\cdot(t_C - t_0)}(t - t_C + t - t_0) \quad)$$
$$\frac{dN(t)}{dt} = 0 \quad (21)$$
$$C_{i2}\cdot \frac{G\cdot x_{\max}\cdot \Delta h_{\max}}{(t_0 - t_C)\cdot(t_C - t_0)}(t_{optimum} - t_C + t_{optimum} - t_0) = 0 \quad (22)$$
$$t_{optimum} - t_C + t_{optimum} - t_0 = 0 \quad (23)$$
$$t_{optimum} = \frac{t_C + t_0}{2} \quad (24)$$

Substituting (6) into (24) gives the final expression:

$$t_{optimumt} = \frac{1}{C_{i1}}\cdot p_{optimum} = \left(\frac{t_C + t_0}{2}\right) \quad (25)$$
$$p_{optimum} = C_{i1}\cdot\left(\frac{t_C + t_0}{2}\right) \quad (26)$$

Equation (26) is a mathematical expression for the optimum separation pressure, i.e. the pressure that gives the maximum process output.

One could now easily recognise that this simple expression is actually **an** algebraic form of the HWR.

3.2 Discussion

The result of any analysis is as good and accurate as the assumptions involved are sustainable.

The assumptions accepted during the analysis from section 3.1 can be divided into two groups.

- The basic assumption i to iv, quoted at the beginning of the analyses (bellow Figure 2);
- Other assumptions that have implicitly been accepted.

The basic assumptions i to iv have been carefully studied. None of them could be the reason for a possible malfunction of the HWR.

However, those assumptions are the source of some minor HWR limitations. For example, acceptance of assumption ii explains why the HWR does not consider pressure drops along pipelines.

Finally we look at the problematic assumptions that have been accepted implicitly during the development of (26). Three critical assumptions have been accepted that way. They are:

1) the assumption that certain parts of the saturation zone boundaries in the Mollier Chart are straight lines;

2) the assumption that isobars/isotherms within the saturation zone in the Mollier Chart are parallel;

3) the assumption that the geothermal fluid flow rate is constant.

The assumption that the specific heat capacity for water and steam is a constant, which was accepted by expression (4), is not listed here. Indeed, c might be considered as a constant only in the context of very rough analyses. *An* analysis of this assumption has been omitted because the physics of the fact that the specific heat capacity is a variable but not a constant has already been incorporated into 1) and 2) here. This might be explained further in a separate context.

Each of the three listed critical assumptions is explained and analysed below.

3.2.1 Assumption that Certain Parts of the Saturation Zone Boundaries in Mollier Chart are Straight Lines

During the development of the expression (26) it was assumed that the following parts of the saturation zone boundaries presented in the Mollier Chart are straight lines. Referring to Figure 2 these lines are:

- line $W_0\,W_C$, which is a part of the left side boundary (x=0);
- line $S_0\,S_C$, which is a part of the right side boundary (x=1).

According to Figure 2 this assumption is well acceptable. However Figure 2 presents a qualitative form of the Mollier Chart only. The exact geometric proportions of the Mollier Chart saturation zone are. presented in Figure **3**.

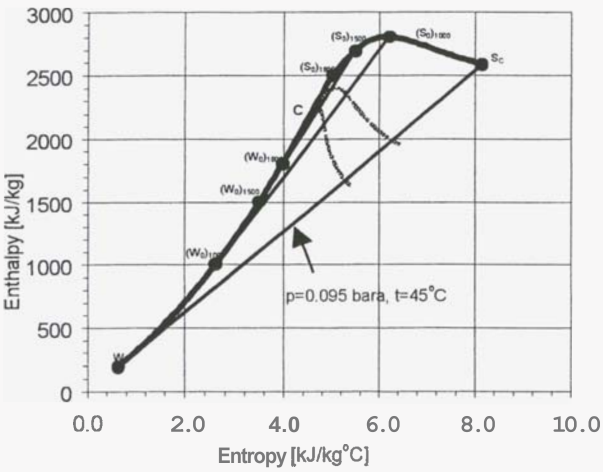


Figure 3. Exact geometric proportions of the Mollier Chart.

Isobar 0.095 bara, as drawn in Figure **3**, presents isobar/isotherm p_c, t_c from Figure 2. There are also three hypothetical resource enthalpies isobars/isotherms drawn within the saturation zone. The resource enthalpies selected for illustration are 1000 kJ/kg, 1500 kJ/kg and 1800 kJ/kg with their main characteristics presented in table 1.

Table 1. Characteristic parameters for isobar/isotherms that have the boiling water enthalpies, 1000,1500 and 1800 kJ/kg

resource enthalpy [kJ/kg]	P_o [bara]	t_o [°C]	the saturation zone end points
1000	29.1	232.1	$(W_0)_{1000}$ $(S_0)_{1000}$
1500	122.0	325.9	$(W_0)_{1500}$ $(S_0)_{1500}$
1800	194.3	363.5	$(W_0)_{1800}$ $(S_0)_{1800}$

Points W_0 and S_0 (the saturation zone end points of isobar/isotherm p_o, t_o from Figure **2**) have been presented in Figure **3** for each of the resource enthalpies. Numbers 1000, 1500 and 1800 adequately tabulate the points.

Using simple qualitative visual analysis we could say that lines $(W_0)_{1000}\,W_C$ and $(S_0)_{1000}\,S_C$ look close enough to a straight line. That indicates that the analysed assumption might be valid for a case of resource enthalpy 1000 kJ/kg.

To accept the assumption that either $(W_0)_{1500}\,W_C$ or $(S_0)_{1500}\,S_C$ is a straight line would not be easy to sustain. For the case of 1800 kJ/kg it would be even more difficult.

The error that this assumption introduces to the HWR has not been quantified in this context. For the higher enthalpy cases (1500 and 1800 kJ/kg) the lines concerned are far fiom being straight and the error might be significant.

3.2.2 The Assumption that Isobars/Isotherms within the Saturation Zone in the Mollier Chart are Parallel

While developing (26) we assumed that the family of triangles (Figure **2**) made by shifting line **X W** down is geometrically similar. To make that possible lines **X W** would always need to be parallel to each other. Line **X W** is a part of line **SW**, which presents isobar/isotherm p, t.

Figure 3 is again used for visual presentation. The characteristic isobars/isotherms for resource enthalpies of 1000, 1500 and 1800 kJ/kg have been presented there. It is obvious that none of the isobars/isotherms is parallel to isobar 0.095 bara.

A quantification of the error introduced by accepting this assumption is again not considered in this context. It was sufficient to identify that the isobar/isotherms are not geometrically parallel which further contributes towards the increase of the HWR total error.

3.2.3 Assumption that the Geothermal Fluid Flow Rate is Constant

There is a sentence in section 3.1, after expressions (1), that says:

"If we separate out the same flow rate of the same geothermal fluid at a lower and lower pressure, steam share increases while brine share decreases, following the shapes of the Mollier Chart saturation zone..."

This sentence spells out the assumption of a constant geothermal flow rate over analysed pressure range. That assumption enabled introducing G into equation (3) as a constant and treating it the same way in the further stream of the analysis. If G were not assumed as a constant but a function of pressure p the analysis would have not resulted in developing the HWR in its algebraic form. In that case the expression for the optimal separating pressure would have been more complex and affected by the relationship between G andp.

However, in the real geothermal world G is practically never a constant but regularly a function ofp. In other words geothermal fluid delivery almost always varies with wellhead pressure. A constant delivery over a range of wellhead pressures (a flat type characteristic) occurs only occasionally. That type of characteristicis considered as an exception likely being a result of the critical flow already reached before the wellhead itself.

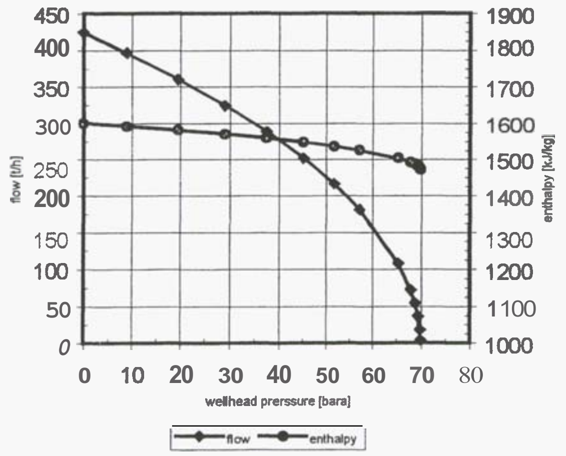


Figure 4. Approximated wellhead characteristics for an exact geothermal well in New Zealand.

The variation of geothermal fluid delivery is indicated by wellhead characteristics. The flow-pressure wellhead characteristic, along with enthalpy-pressure, is the most important geothermal resource indicator. The characteristics for a specific well are presented in Figure 4.

Figure 4 illustrates an additional complication involved here. Resource enthalpy might not be a constant. That is not uncommon in geothermal practice. To incorporate this feature into any analytical expression would be a difficult task.

Returning to discussing the assumption of a constant G. To illustrate how this assumption is not sustainable the pressure-enthalpy characteristic from Figure 4 will be ignored and the resource enthalpy approximated by a constant of 1500kJ/kg. At that resource enthalpy and the pressure in the condenser of 0.095 bara, the HWR range for optimising separation pressure would be set fiom 0.095 bara to 122 bara. In that range geothermal fluid delivery goes from approximately 430 t/h (at 0 bara) to 0 t/h (at approximately 70 bara). At the hypothetical separation pressures above 70 bara the fluid flow would be zero.

The assumption of a constant geothermal fluid delivery over the analysis range, which is fundamental to the HWR, is clearly not sustainable. This assumption brings the largest error into the HWR seriously limiting its range of application.

4. CONCLUSION

The analysis of convention geothermal power process presented in the paper resulted in developing an algebraic form of the HWR.

It has been found that three critical assumptions accepted during the analysis introduce errors into the HWR.

The first two assumptions, about the saturation zone boundaries being straight lines and the isobars being parallel, introduce a moderate error. Such errors are usually acceptable for analytic expressions.

The third one, assuming a constant geothermal fluid delivery, introduces a major error that imposes serious limitations on using the HWR in geothermal practice.

A qualitative analysis, which has been done separately, indicates that the HWR might give accurate results in the case of a relatively low resource enthalpy (approx. 1000kJ/kg) and a flat wellhead characteristic.

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