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TOWARDS A HYDROTHERMAL ERUPTION FLOW MODEL

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SUMMARY - This paper continues preliminary investigation as begun in Smith & McKibbin (1997). The conservation equations and boundary conditions describing the flow in a hydrothermal eruption are outlined. Methods for solving the system of equations are then investigated for the simpler isothermal case. Results are given for three different geometrical configurations, and, in each case, the flow is quantified.

1. INTRODUCTION

Hydrothermal eruptions are driven by the expansion of hydrothermal fluids to atmospheric pressure. They are violent events which eject a mixture of water, steam and rock particles without warning. Though rare temporally, many have occurred in geothermal fields in New Zealand, as well as around the world. A summary of previous work on the modelling of hydrothermal eruptions and a conceptual model describing the eruption process can be found in Smith & McKibbin (1997).

This paper formulates the problem of modelling hydrothermal eruptions using the principles of conservation of mass, momentum and energy. Some work has already been done on this model in the one-dimensional case [see McKibbin (1989, 1996) and Bercich & McKibbin (1992)]. Here methods for solving such models in two dimensions are examined for the simpler isothermal case.

2. MATHEMATICAL MODEL

2.1 Assumptions

In a hydrothermal eruption, the ground fluid before decompression is at saturated (boiling) conditions. Near-surface fluid is suddenly exposed to atmospheric conditions due to some initiation event forming a zone of reduced pressure. This pressure reduction causes flashing to occur, the fluid expands, and the two-phase mixture is ejected quickly. Because of the rapidity of motion, the flowing fluid is modelled as a homogeneous mixture of liquid and gas, rather than as two separable phases.

2.2 Boundary Conditions

The pressure and temperature conditions at the ground surface and at large distances in the ground are assumed to be ambient (motionless, with hydrostatic pressure distribution, and temperature increasing with depth).

2.3 Conservation Laws

As stated previously, the fluid will be treated as a homogeneous mixture of two phases. Therefore there is a single equation for conservation of momentum. The general equations for transient flow are then as follows, e.g. see O'Sullivan & McKibbin (1989), Nield & Bejan (1992):

<u>Conservation of Fluid Mass</u>

$$\frac{\partial A_m}{\partial t} + \nabla \cdot \mathbf{Q}_m = 0 \qquad (1)$$

where A_m is the fluid mass per unit volume of formation and Q_m is the mass flux per unit area.

<u>Conservation of Momentum (Darcy's Law)</u>

$$\mathbf{v}_f = \frac{k}{\mu_f}(-\nabla p + \rho_f \mathbf{g}) - \frac{c_F}{\nu_f k^{1/2}} |\mathbf{v}_f| \mathbf{v}_f \qquad (2)$$

Here the subscript f is used to represent the fluid mixture, $\mathbf{v}_f$ is the volume flux per unit area, k is the permeability, μ_f is the dynamic viscosity of the fluid, p is the pressure, ρ_f is the fluid density, $\mathbf{g}$ is the acceleration due to gravity, c_F is a dimensionless form-drag constant, and ν_f is the fluid kinematic viscosity which is given by ρ_f / μ_f.

Conservation of Energy

$$\frac{\partial A_e}{\partial t} + \nabla \cdot \mathbf{Q}_e = 0 \tag{3}$$

where A_e is the energy per unit volume of formation and Q_e is the energy flux per unit area.

The mass and energy densities referred to in the conservation equations are given by

$$A_m = \phi \rho_f \tag{4}$$

$$A_e = (1-\phi)\rho_r u_r + \phi \rho_f u_f \tag{5}$$

The subscript r is used here to represent the rock matrix, ϕ is the porosity, and u_r and u_f are the internal energies of the rock and fluid respectively.

The mass and energy flux terms found in the conservation equations are given by

$$\mathbf{Q}_m = \rho_f \mathbf{v}_f \tag{6}$$

$$\mathbf{Q}_e = h_f \mathbf{Q}_m - K\nabla T \tag{7}$$

where h_f is the specific enthalpy of the fluid, K is the thermal conductivity, and T is the temperature.

The density, dynamic viscosity and specific enthalpy of the fluid (a two-phase water mixture) are given by

$$\rho_f = S_l \rho_l + (1-S_l)\rho_g \tag{8}$$

$$\mu_f = \mu_l^{S_l} \mu_g^{(1-S_l)} \tag{9}$$

$$h_f = \frac{S_l \rho_l h_l + (1-S_l)\rho_g h_g}{S_l \rho_l + (1-S_l)\rho_g} \tag{10}$$

where S_l is the liquid saturation (volume fraction that is liquid) and the subscripts l and g generally refer to the liquid and gas phases.

Equation of State

As stated previously, the fluid is assumed to be at saturated (boiling) conditions; the equation of state is given by

$$p = p_{sat}(T) \tag{11}$$

Standard correlations for thermodynamic properties may be used.

Possible Further Simplifications

In earlier investigations, see e.g. Bercich & McKibbin (1992), the following additional assumptions were made:

(a) The eruption is already in progress and is modelled as steady over short periods of time.

(b) The fluid is moving so rapidly that the conductive energy is assumed to be negligible. That is, the heat transfer between the rock and fluid is ignored, and the process is modelled as adiabatic.

With these assumptions, the equations for conservation of mass and energy given in Equations (1) and (3) become, respectively,

$$\nabla \cdot \mathbf{Q}_m = 0 \tag{12}$$

$$\nabla \cdot (h_f \mathbf{Q}_m) = 0 \tag{13}$$

By combining **(12)** and (13) we find

$$\mathbf{Q}_m \cdot \nabla h_f = 0 \tag{14}$$

which implies that enthalpy is constant along the streamlines (in the direction of flow).

3. STEADY ISOTHERMAL FLOWS

In order to devise a robust mathematical procedure for solving the transient non-isothermal model, the simpler case of steady isothermal single-phase flow was solved.

Such a procedure relates to the following: consider the problem of digging a hole for the purposes of extracting water. If water is continually pumped out of the hole, where does the water come from? As with the non-isothermal case, a pressure reduction will cause the flow to commence. In this section we investigate how to quantify the flow around and into the hole in the isothermal case, and determine where the water originates from.

The assumptions in this case are the same as for the non-isothermal case but now we also have k, $\rho = \rho_f$, $\mu = \mu_f$ all constant (the fluid is liquid water). Treating the problem as seepage flow, we also neglect the non-linear term in the conservation of momentum equation **(2).**

Then from Equations (6) and (2), the mass flux vector is

$$\mathbf{Q}_m = \frac{k}{\nu}(-\nabla p + \rho \mathbf{g}) \tag{15}$$

which may be written in the form:

$$\mathbf{Q}_m = \nabla \Phi \tag{16}$$

where the potential Φ for the specific mass flux $\mathbf{Q}_m$ is given by

$$\Phi = -\frac{k}{v}\left(p - p_{atm} + \rho g z\right) \qquad (17)$$

and z is the vertical coordinate.

Substitution into the conservation of mass equation (12) gives

$$\nabla^2 \Phi = 0 \qquad (18)$$

which is Laplace's equation for Φ.

The solution to this isothermal problem is found below in three different two-dimensional geometrical configurations.

3.1 Semi-Circular Horizontal Trench

First, consider the case of digging a semi-circular horizontal trench for the extraction of water. The axis of the trench lies along the surface and its radius is a. We assume that conditions are ambient at distance b from the centre. The solution to the flow in this problem can be found by solving Laplace's equation (18) in polar coordinates perpendicular to the trench.

In polar coordinates, with the usual notation, Laplace's equation for $\Phi(r,\theta)$ is given by

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2} = 0 \qquad (19)$$

where $a \le r \le b$, $\pi \le \theta \le 2\pi$.

The boundary conditions at the surface are taken to be very idealised. The pressure is atmospheric and remains so. At large r, the pressure is hydrostatic. In this case the boundary conditions are

$$\Phi(r,0) = \Phi(r,\pi) = \Phi(b,\theta) = 0 \qquad (20)$$

$$\Phi(a,\theta) = \frac{-k\rho g z}{v} = \frac{-k\rho g a \sin 8}{v}$$

The analytic solution for equation (19) with boundary conditions (20) is as follows

$$\Phi = C_p\left(\frac{r}{b} - \frac{b}{r}\right)\sin\theta \qquad (21)$$

where the coefficient C_p for this polar coordinate solution is given by

$$C_p = \frac{k\rho g a^2 b}{v(b^2 - a^2)} \qquad (22)$$

By equation (16) the mass flux then becomes

$$\mathbf{Q}_m = C_p\left(\frac{r^2 + b^2}{br^2}\right)\sin\theta\mathbf{e}_r + C_p\left(\frac{r^2 - b^2}{br^2}\right)\cos\theta\mathbf{e}_\theta \qquad (23)$$

where $\mathbf{e}_r$, $\mathbf{e}_\theta$ are unit vectors in the r, 8 directions respectively.

If the total mass flux per unit length of the cavity is denoted by Q, then by integrating the normal component of the mass flux vector the following equation for Q is obtained:

$$Q = \frac{2k\rho g a}{v}\frac{a^2 + b^2}{b^2 - a^2} \qquad (24)$$

A **stream** function, which is related to the mass flux by

$$\mathbf{Q}_m = \frac{1}{r}\frac{\partial \Psi}{\partial \theta}\mathbf{e}_r - \frac{\partial \Psi}{\partial r}\mathbf{e}_\theta \qquad (25)$$

and which thereby enables Equation (12) to be satisfied exactly, may be found for this problem in the form

$$\Psi = -C_p\left(\frac{r}{b} + \frac{b}{r}\right)\cos\theta \qquad (26)$$

As the outer radius b of the solution region becomes large, for finite r

$$\Phi \to -\frac{k\rho g a^2}{v}\frac{1}{r}\sin\theta \qquad (27)$$

$$\mathbf{Q}_m \to \frac{k\rho g a^2}{v}\left[\frac{1}{r^2}\sin\theta\mathbf{e}_r - \frac{1}{r^2}\cos\theta\mathbf{e}_\theta\right] \qquad (28)$$

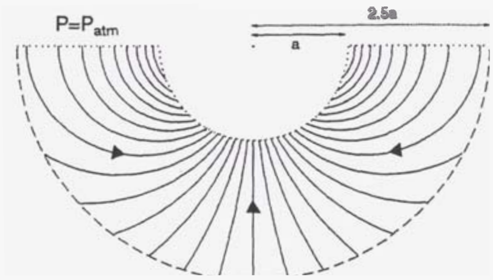
$$Q + \frac{2k\rho g a}{v} \qquad (29)$$

$$\Psi \to -\frac{k\rho g a^2}{v}\frac{1}{r}\cos 8 \text{ and} \qquad (30)$$

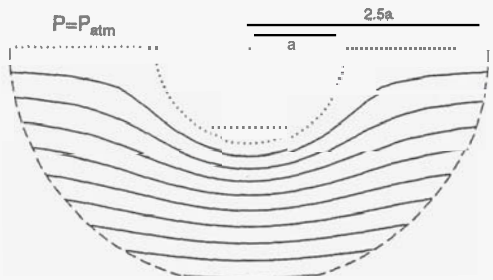
$$p \to p_{atm} + \rho g\left(\frac{a^2}{r} - r\right)\sin\theta \qquad (31)$$

Note that the total mass flux per unit length, Q, is proportional to the depth of the trench, a.

The value of b for which the equations describing the flow near the hole converge to those found in Equations (27) – (31) is of interest. While in this case an analytic solution can be found, such a value for b may provide a guide to domain size where only a numerical solution exists. Experimentation reveals that for $b = 10a$, the



(a) Cross-section of stream surfaces



(b) Cross-section of isobaric surfaces

Figure 1. Isolines for stream-function and pressure for semi-circular horizontal trench. (See text for details.)

numerical and analytic solutions agree to within 2%.

The streamlines (cross-section of the stream surfaces) and isobars (cross-section of isobaric surfaces) are shown in Figures 1a and 1b. These were calculated on the simulation field $a \leq r \leq 10a$, and the region out to $r = 2.5a$ is depicted. Equipotential lines have also been calculated and can be shown to be everywhere perpendicular to the streamlines. However, for clarity the equipotential lines are not shown.

Calculation shows that 80% of the flow originates from the surface while 20% is derived from ground-water.

This problem has also been solved numerically for pressure and the potential function using finite difference methods. Streamlines were then found by plotting lines which were everywhere perpendicular to the potential lines. The results found match the analytic solution.

[Note: The method used for plotting the streamlines in this case may also prove useful in solving the non-isothermal case. Once the conservation equations have been solved for either h_f or $\mathbf{Q}_m$, the orthogonality of ∇h_f and $\mathbf{Q}_m$, see Equation (14), may be used to complete the solution.]

3.2 Hemi-Spherical Hole

Next, consider the case of digging a hemi-spherical hole for the extraction of water. The radius of the hole is a and at some distance b from the centre of the hole conditions are ambient. The solution to the flow in this problem can be found by solving Laplace's equation in spherical coordinates with symmetry about the vertical axis.

Laplace's equation in spherical coordinates with axial symmetry is given for $\Phi(r,\theta)$ by

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial \Phi}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right)=0 \qquad (32)$$

where r is the radial distance from the origin and θ is measured from the vertical axis of symmetry, $a \leq r \leq b$, $\pi/2 \leq \theta \leq \pi$.

The boundary conditions in this case become

$$\Phi\left(r,\frac{\pi}{2}\right)=\Phi(b,\theta)=0 \qquad (33)$$

$$\Phi(a,\theta)=\frac{-k\rho g z}{v}=\frac{-k\rho g a\cos\theta}{v}$$

The analytic solution to equation (32) with boundary conditions (33) is

$$\Phi = C_s\left(\frac{r^3-b^3}{r^2}\right)\cos\theta \qquad (34)$$

where the coefficient C_s for this spherical coordinate solution is given by

$$C_s=\frac{k\rho g a^3}{v(b^3-a^3)} \qquad (35)$$

By equation (16) the mass flux then becomes

$$\mathbf{Q}_m = C_s\left(\frac{r^3+2b^3}{r^3}\right)\cos\theta\mathbf{e}_r + C_s\left(\frac{b^3-r^3}{r^3}\right)\sin\theta\mathbf{e}_\theta \qquad (36)$$

The total mass flux into the hole, Q, is then found by integrating the normal component of the mass flux vector, to give

$$Q=\frac{k\rho g\pi a^2}{v}\frac{a^3+2b^3}{b^3-a^3} \qquad (37)$$

The stream function in this case is of the form

$$\Psi = C_s\left(\frac{r^3 + 2b^3}{2r}\right)\sin^2\theta \tag{38}$$

As the outer radius b of the solution region becomes large, for finite r

$$\Phi \to -\frac{k\rho g a^3}{\nu}\frac{1}{r^2}\cos\theta \tag{39}$$

$$\mathbf{Q}_m \to \frac{k\rho g a^3}{\nu}\left[\frac{2}{r^3}\cos\theta\mathbf{e}_r + \frac{1}{r^3}\sin\theta\mathbf{e}_\theta\right] \tag{40}$$

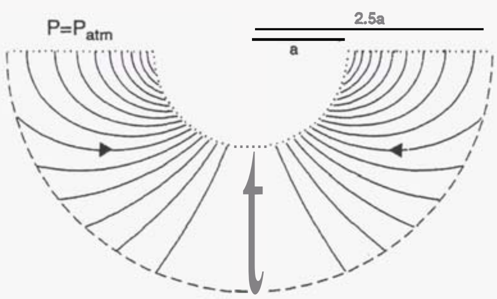
$$Q \to \frac{2k\rho g\pi a^2}{\nu} \tag{41}$$

$$\Psi \to \frac{k\rho g a^3}{\nu}\frac{1}{r}\sin^2\theta \text{ and} \tag{42}$$

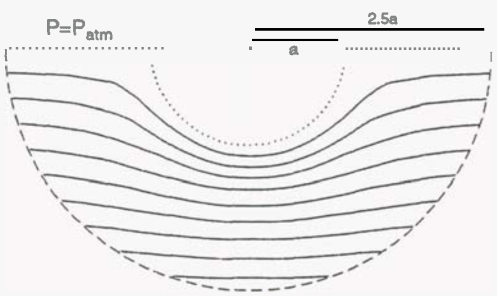
$$p \to p_{atm} + \rho g\left(\frac{a^3}{r^2} - r\right)\cos\theta \tag{43}$$

Note that in this case Q is proportional to the square of the depth of the hole, a^2.

Investigation into the convergence of the flow near the hole shows that an outer radius of $b = 10a$ is sufficient for numerical and analytic solutions to agree to within 1%.



(a) Cross-section of stream surfaces



(b) Cross-section of isobaric surfaces

Figure 2. Isolines for stream-function and pressure for hemi-spherical hole.

The cross-section of the stream surfaces and cross-section of isobaric surfaces are given in Figures 2a and 2b. These were calculated on the simulation field $a \le r \le 10a$ and the region out to $r = 2.5a$ is depicted.

Characteristics similar to those found in Figures 1a and 1b can be seen in Figures 2a and 2b. In this case *85%* of the flow originates from the surface while 15% is derived from ground-water.

This problem has also been solved numerically using finite difference methods and results again match those found analytically.

3.3 Cylindrical Hole

Finally, consider the case of sinking a cylindrical hole for the purpose of gathering ground-water. The hole has a radius a and a depth d. We assume that conditions are ambient at a radius b from the centre of the hole and at a depth D from the surface. The solution to the flow in this problem can be found by solving Laplace's equation (18) in cylindrical coordinates with symmetry about the vertical axis.

Laplace's equation in cylindrical coordinates with axial symmetry is given for $\Phi(r,z)$ by

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right) + \frac{\partial^2\Phi}{\partial z^2} = 0 \tag{44}$$

where r is the distance from the vertical z-axis.

The boundary conditions in this case become

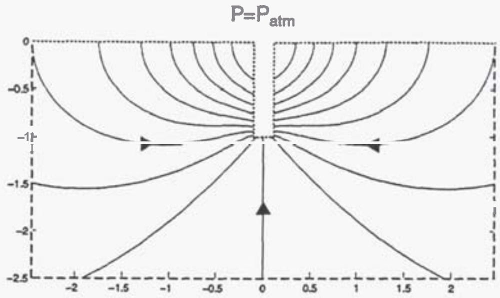
$$\begin{aligned}
&\Phi(b,z) = 0 \text{ for } -D \le z \le 0 \\
&\Phi(r,-D) = 0 \text{ for } 0 \le r \le b \\
&\Phi(r,0) = 0 \text{ for } a \le r \le b \\
&\Phi(a,z) = \rho g z \text{ for } -d \le z \le 0 \\
&\Phi(r,-d) = -\rho g d \text{ for } 0 \le r \le a \text{ and} \\
&\Phi_r(0,z) = 0 \text{ for } -d \le z \le -D
\end{aligned} \tag{45}$$

Streamlines for this case can be found by solving

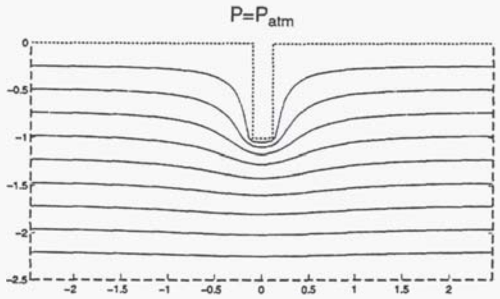
$$\frac{\partial^2\Psi}{\partial r^2} - \frac{1}{r}\frac{\partial\Psi}{\partial r} + \frac{\partial^2\Psi}{\partial z^2} = 0 \tag{46}$$

for $\Psi(r,z)$ with the boundary conditions

$$\begin{aligned}
&\Psi_r(b,z) = 0 \text{ for } -D \le z \le 0 \\
&\Psi_z(r,-D) = 0 \text{ for } 0 \le r \le b \\
&\Psi_z(r,0) = 0 \text{ for } a \le r \le b \\
&\Psi_r(a,z) = \rho g a \text{ for } -d \le z \le 0 \\
&\Psi_z(r,-d) = 0 \text{ for } 0 \le r \le a \text{ and} \\
&\Psi_z(0,z) = 0 \text{ for } -D \le z \le -d
\end{aligned} \tag{47}$$



(b) Cross-section of stream surfaces



(b) Cross-section of isobaric surfaces

Figure 3. Isolines for stream-function and pressure for cylindrical hole.

Equation (44) with boundary conditions (45) and Equation (46) with boundary conditions (47) were solved numerically using finite difference methods. The streamlines (cross-sections of the stream surfaces) and isobars (cross-section of the isobaric surfaces) are given in Figures 3a and 3b. Guided by the considerations for the previous two geometric configurations, a numerical solution was found on a domain of ten times the depth of the hole, d. The region out to $r = 2.5d$ and down to $z = -2.5d$ is depicted.

For this cylindrical case, 90% of the flow originates from the surface while 10% is derived from ground-water. Calculations also show that 86% of the flow emerging from the hole comes from the sides, while only 14% comes from the bottom.

4. SUMMARY

The conservation equations have been set out for the description of flow in hydrothermal eruptions in order to investigate suitable methods for solution in the general case. Simple two-dimensional isothermal flows have been studied. Analytic solutions, where possible, have been compared with numerical solutions for steady flows.

The results for each of the three different geometries show the main pressure reduction to be local (see Figures 1b, 2b, 3b). It is this pressure lowering which will have a significant effect on flashing in a hydrothermal eruption. We would therefore expect hydrothermal eruption fluid to originate locally.

In the case of steady isothermal flow, the results also show the majority of flow originating fiom the surface (see Figures 1a, 2a, 3a). However, in a hydrothermal eruption full steady-state flow will not be established. In this case, as stated above, we would expect the sources of the flow to be more local in character and the far field flow of the isothermal case to be inapplicable. Nevertheless, the present analysis may show how the ground flow will recover. The replenishment of the depleted zone is likely to be from ground-water and not from depth initially. This will have a cooling effect on the region and may explain long recovery times before subsequent eruptions.

Current work in this investigation centres on numerical solution of the full non-isothermal two-phase flows. It is also hoped to perform some laboratory experiments of rapid transient flashing in a porous medium and to find semi-analytic and numerical solutions for comparison. A difficult part of the modelling process remains in connecting the flow in the porous matrix with erosion of the surface.

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