# FLUID FLOW IN A FLASHING CYCLONE SEPARATOR

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**SUMMARY** – In geothermal steam-gathering systems for electric power generation, separators are used to provide a turbine working fluid which is *dry* or slightly superheated steam (water vapour). **This** paper describes the steam flow patterns and pressure distribution within a typical cyclone separator which is operating at saturation (boiling or flashing) conditions. The **mess** and volume fractions of the gas (steam) and liquid phases which must be separated are quantified for a typical two-phase flow example. The steam fraction occupies most of the separator vessel volume; analysis of the motion of **this** steam phase allows depiction of the stream surfaces within the flow and **an** estimate of the pressure drop across the separator. Solutions of a set of simplified conservation equations are found by analytical and numerical methods.

#### 1. INTRODUCTION

Cyclone separators are used widely in geothermal technology to separate the liquid and gas phases in two-phase streams from well-bores. While the main component in the flows is water, there may also be dissolved components such as chlorides and carbonates as well as non-condensible gases such as carbon dioxide and hydrogen sulphide.

Turbine designs are usually based on a *dry* or slightly superheated fluid at inlet; the liquid must be removed from flows when the thermodynamic conditions at the wellhead are such that the flowing enthalpy is less than that of the gas phase.

In this paper, some of the considerations which underlie the thermodynamic design of a separation system are outlined. These are illustrated by a typical example. The fluid dynamical equations which result from physical conservation laws for the flow within a separator are set out; some simple analytic and numerical solutions are found.

### 2. A TYPICAL EXAMPLE

A pipeline carries a two-phase water flow with a total mass flux of M = 100 kg/s. At the well-head, the flowing mixture specific enthalpy is measured to be  $h_f$  = 1500 kJ/kg. The flow passes into a separator which has an inlet pressure of 10 bar gauge, or 11 bar abs. The corresponding separator inlet temperature is  $T_{sat}(11)$  = 184 "C.

Data available from steam tables give the following thermodynamic properties for the liquid and gas (vapour, steam) phases of water at the specified inlet conditions:

**Table 1:** Thermodynamic properties of water at p = 11 bar abs,  $T = T_{sat}(p) = 184$  "C.

property	SI units	$\begin{array}{c} \text{liquid} \\ (\ell) \end{array}$	steam (g)
sp. volume, $\nu$ density, $p$ sp. enthalpy, $h$ dyn. viscosity, $\mu$ kin. viscosity, $\nu$	m <sup>3</sup> /kg	0.001138	0.1772
	kg/m <sup>3</sup>	878	5.64
	kJ/kg	781	2781
	kg/m s	146.e–6	15.1e-6
	m <sup>2</sup> /s	0.17e-6	2.68e-6

#### 2.1 Mass, energy and volume fluxes

The flowing quality, or dryness, X, is the steam (gas phase) **mass** flow fraction of the total mass flow M of the two-phase fluid mixture.

The separate phase <u>mess</u> fluxes are then given by

$$\mathbf{M}_{\ell} = (1 - \mathbf{X})\mathbf{M}$$

$$Mg = XM$$

The respective <u>energy fluxes</u> are measured by the enthalpy flows:

$$H_{\ell} = M_{\ell} h_{\ell} = (1 - X) h_{\ell} M$$

$$Hg = Mghg = Xh_g M$$

with the total energy flux

$$H = H_{\ell} + H_g = [(1 - X) h_{\ell} + X h_g] M$$
 (1)

The specific flowing enthalpy of the two-phase mixture  $h_f = H/M$  is expressed in terms of the respective liquid- and gas-phase values and X, as

$$h_f = (1 - X) h_{\ell} + Xhg$$
 (2)

After rearrangement and substitution of the parameter values given in Table 1, the flowing quality of the example mixture at the separator inlet is found to be:

$$X = \frac{h_f - h_\ell}{h_g - h_\ell} = \frac{1500 - 781}{2000} \approx 0.36$$
 (3)

This means that the gas phase of the two-phase flow carries 36 % of the mass, while 64 % of the mass is carried in the liquid phase.

For a total mass flow of M = 100 kg/s, the steam mass flowrate is  $M_g = 36 \text{ kg/s}$ . The steam energy flux (relative to a datum of 0 °C) is then given by

$$M_g h_g = (36 \text{ kg/s})(2781 \text{ kJ/kg}) = 100 \text{ MW}_{th}$$

With a thermal-to-electric energy conversion factor of 15 %, the equivalent electrical energy flux of the steam flow is therefore 15 MW<sub>e</sub> in this case. The liquid energy flux (relative to datum 0 "C) is

$$M_{\ell}h_{\ell} = (64 \text{ kg/s})(781 \text{ kJ/kg}) = 50 \text{ MW}_{th}$$

The <u>volume fluxes</u> corresponding to each of the phase **streams** are

$$V_{\ell} = M_{\ell} v_{\ell} = (1 - X) v_{\ell} M$$

$$Vg = M_g v_g = X v_g M$$

with total volume flux

$$V = V_{\ell} + V_g = [(1 - X) v_{\ell} + X v_g] M$$
 (4)

The average specific volume flux of the two-phase flow,  $v_f = V/M$ , is then

$$v_f = (1 - X) v_\ell + x v g$$
 (5)

The total volume flux for **our** example is, **from** Equation **(4)** and Table 1,

$$V = [(1-0.36)(0.001138) + (0.36)(0.1772)] M$$
  
= (0.0645 m<sup>3</sup>/kg) (100 kg/s)  
= 6.45 m<sup>3</sup>/s

The volumetric quality  $\beta$  (the fraction of the total volume flux due to the gas phase) is

$$\beta = \frac{Vg}{V} = \frac{1}{1 + \frac{1 - X}{X} \frac{v_{\ell}}{v_{\sigma}}}$$
 (6)

For our example, using X = 0.36 and values from Table 1,  $\beta = 0.989$ .

In **summary**, while the liquid phase carries **64**% of the mass, it carries **33**% of the thermal energy (relative to **datum** 0 °C) and only about 1% of the volume. The volume flux of the steam, 99% of the total, dominates the flow in the separator.

### 2.2 Void fraction in the pipeline

Although not necessary for the separator calculations which follow, it is interesting to consider the two-phase flow geometry in the inlet pipeline. For our example, we use a pipeline of radius D/2 = 25 cm = 0.25 m, area  $A_p = 0.20$  m<sup>2</sup>.

Commonly, the liquid and gas phases move in a pipeline in **an** annular flow regime, with the gas phase flowing in the central region surrounded by the slower-flowing liquid phase **near** the pipe wall. The ratio  $\mathbf{k} = \mathbf{v_g}/\mathbf{v_\ell}$  of the gas speed to that of the liquid is **termed** the slip ratio. [If the fluid moves in a mist flow, the slip ratio  $\mathbf{k} = 1$ .]

In our example, the superficial liquid and gas velocities are, respectively,  $v_{s\ell} = V_{\ell}/A_p = 0.35$  m/s, and  $v_{sg} = V_g/A_p = 32$  m/s. From a map for horizontal flow (Manhane et al., 1974) the regime is annular.

The void fraction a is the fraction of the pipe cross-sectional area  $A_p$  which is occupied by the gas phase. The separate phase mass fluxes are then given by

$$M_{\ell} = (1 - X) M = \rho_{\ell} v_{\ell} (1 - \alpha) A_{D}$$

$$M_g = XM = \rho_g v_g \alpha A_p$$

Elimination of M (and  $A_p$ ) from these equations gives an expression for the void fraction a in terms of the flowing quality X, the phase densities and the slip ratio  $k_r$  as follows:

$$\alpha = \frac{1}{1 + \frac{1 - X}{X} \frac{\rho_g}{\rho_\ell} k}$$
 (7)

Using a correlation due to **Harrison** (1975) for the slip ratio in annular flow, the expression in Equation (4) may be written in the form:

$$\alpha = \frac{1}{1 + \left(\frac{1 - X}{X}\right)^{0.8} \left(\frac{\rho_g}{\rho_\ell}\right)^{0.515}}$$
(8)

For our example, with X = 0.36 and parameters as in Table 1, the slip ratio is k = 10.3 and the void fraction is  $a \approx 0.89$  or about 89 %.

**So**, while the liquid phase carries 64 % of the mass and 1 % of the volume flux, it occupies 11 % of the cross-sectional flow area, flowing in an annulus which occupies the outside 5 % of the radius of the pipeline.

### 2.3 Flow in the pipeline

The Reynolds number for the inlet flow is

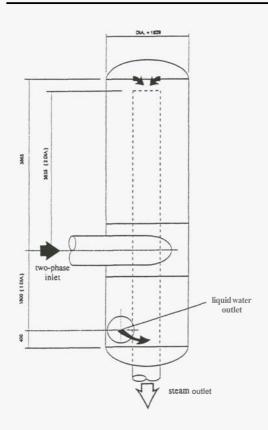
Re = 
$$\frac{vD}{v}$$
 =  $\frac{(32)(0.50)}{2.68 \times 10^{-6}} \approx 10^{7}$ 

**This** means that inertial effects dominate over viscous effects, but that the flow is turbulent. The viscous terms in the Navier-Stokes equations may be neglected, but the turbulent Reynolds stresses may be important.

### 2.4 Flow in the separator

Within the separator itself, different length and velocity scales pertain. Typical dimensions for a cyclone separator as used at Wairakei Geothermal Field are shown in Figure 1. The outer case has a radius of about  $R_0 = 1\,\mathrm{m}$ , while that of the steam outlet pipe is about  $R_i = 0.3\,\mathrm{m}$ . The height  $L_g$  of the steam flow region is in the range  $4-6\,\mathrm{m}$ .

The fluid enters tangentially from the side; the liquid water is flung to the outer surface by centrifugal force. The gas phase (steam) spirals inwards and upwards to the top of the outlet tube; meanwhile, the liquid falls under gravity to the bottom of the vessel, from where it is piped for disposal or for further flashing and separation at a lower pressure.



**Fig. 1:** Typical layout and dimensions of a cyclone separator. [Dimensions: mm]

#### 2.5 Fluid residence times

With the assumption that there is little phase change within the separator, the mass fluxes of the two phases remain nearly constant during their passage through the vessel. Assuming also that the density of the gas phase does not change too much, the volume flux of the steam is

$$V_g = X v_g M = (0.36)(0.1772)(100)$$
  
 $\approx 6.4 \text{ m}^3/\text{s}$ 

The volume  $S_g$  of the steam-filled region in the separator is, typically

$$S_g = \pi (R_0^2 - R_i^2) L_g = \pi [(1)^2 - (0.3)^2](4)$$
  
 $\approx 12 \text{ m}^3$ 

**So** the mean residence time of the steam in the separator vessel is approximately 2 seconds. Similarly, the volume flux of the liquid water can be calculated to be about 0.073 m<sup>3</sup>/s, and it occupies a volume of about 1.5 m<sup>3</sup> in the vessel for a typical residence time of 20 seconds.

There is therefore a contrast in the residence times of the two phases as they flow through the separator.

### 3. EQUATIONS OF MOTION

Attention here is focussed on the steam flow after the liquid phase is removed by centrifugal action,

In order to take into account turbulent variations in the velocity, it is written  $\mathbf{v} = \overline{\mathbf{v}} + \mathbf{v}'$  where  $\mathbf{v}'$  is the fluctuation to the mean flow  $\overline{\mathbf{v}}$ . Similarly the pressure is written  $\mathbf{p} = \overline{\mathbf{p}} + \mathbf{p}'$ . The average values, indicated by an overbar, are taken over a long period of time:

$$\overline{\alpha} = \frac{1}{22} \int_{0}^{\tau} \alpha \, dt$$
 and  $\alpha' = \alpha - \overline{\alpha}$ 

where the time scale  $\tau$  is large. Note that  $\overline{\alpha}' = 0$ .

In the model of the steam flow presented here, it is assumed that:

- the mean flow is steady  $(\partial \overline{\mathbf{v}}/\partial \mathbf{t} \equiv \mathbf{0})$ ;
- there is cylindrical symmetry in cylindrical coordinates (r, 8, z) with z vertically upwards, the mean fluid velocity vector  $\overline{\mathbf{v}} = (\overline{\mathbf{v}}_r, \overline{\mathbf{v}}_\theta, \overline{\mathbf{v}}_z)$  is of the functional form  $\overline{\mathbf{v}}(r, z)$ , while the mean pressure may be expressed as  $\overline{\mathbf{p}} = \overline{\mathbf{p}}(\mathbf{r}, z)$ ;
- the inlet and outlet flows are uniformly distributed around  $0 \le \theta < 2\pi$ ;
- the system is isothermal and the fluid is barotropic i. e. ρ = ρ(p̄).

The Navier-Stokes equations, which express conservation of **mess** and momentum for the flow in cylindrical coordinates, with the above notation and assumptions are (see, for example, Hughes & Gaylord, 1964):

Conservation of mass:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho\overline{v}_{r}) + \frac{\partial}{\partial z}(\rho\overline{v}_{z}) = 0$$
 (9)

Conservation of momentum:

$$\overline{v}_{r} \frac{\partial \overline{v}_{r}}{\partial r} + \overline{v}_{z} \frac{\partial \overline{v}_{r}}{\partial z} - \frac{\overline{v}_{\theta}^{2}}{r} = -\frac{1}{\rho} \frac{\partial \overline{p}}{\partial r} + F_{r} \quad (10)$$

$$\overline{v}_r \frac{\partial \overline{v}_{\theta}}{\partial r} + \overline{v}_z \frac{\partial \overline{v}_{\theta}}{\partial z} + \frac{\overline{v}_r \overline{v}_{\theta}}{r} = F_{\theta}$$
 (11)

$$\frac{\overline{v}_{r}}{v_{r}}\frac{\partial \overline{v}_{z}}{\partial r} + \overline{v}_{z}\frac{\partial \overline{v}_{z}}{\partial z} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial z} - g + F_{z}$$
 (12)

where g is the gravitational acceleration and the terms  $F_i$  are derived from the components of the Reynolds stress tensor and are given in terms of the components of  $\mathbf{v}'$  as follows:

$$F_{r} = -\frac{\partial}{\partial r} (\overline{v_{r'}^{2}}) - \frac{1}{r} \frac{\partial}{\partial \theta} (\overline{v_{r'}v_{\theta'}}) - \frac{\partial}{\partial z} (\overline{v_{r'}v_{z'}}) - \frac{\overline{v_{r'}^{2}}}{r} + \frac{\overline{v_{\theta'}^{2}}}{r}$$
(13)

$$\mathbf{F}_{\boldsymbol{\theta}} = -\frac{\partial}{\partial \mathbf{r}} (\overline{\mathbf{v}_{\mathbf{r}}' \mathbf{v}_{\boldsymbol{\theta}'}}) - \frac{1}{r} \frac{\mathbf{a}}{\partial \boldsymbol{\theta}} (\overline{\mathbf{v}_{\boldsymbol{\theta}'}^{2}}) - \sqrt[6]{\mathbf{v}_{\boldsymbol{\theta}'} \mathbf{v}_{\mathbf{z}'}}) - \frac{2}{r} \sqrt[6]{\mathbf{v}_{\boldsymbol{\theta}'} \mathbf{v}_{\mathbf{z}'}}$$

$$- \frac{2}{r} \sqrt[6]{\mathbf{v}_{\boldsymbol{\theta}'} \mathbf{v}_{\boldsymbol{\theta}'}} \qquad (14)$$

$$F_{Z} = -\frac{\partial}{\partial r} (\overline{v_{r}'v_{z}'}) - \frac{1}{r} \frac{\partial}{\partial \theta} (\overline{v_{\theta}'v_{z}'}) - \frac{\partial}{\partial z} (\overline{v_{z}'^{2}}) - \frac{\overline{v_{r}'v_{z}'}}{r}$$

$$- \frac{\overline{v_{r}'v_{z}'}}{r}$$
 (15)

The turbulent stress terms may be evaluated in different ways. The analysis of turbulent fluid flows is not an easy matter. Various correlations between the turbulent stress components and the mean velocity field have been proposed, but the closure of the problem with appropriate boundary conditions remains difficult.

#### 4. FLOW SOLUTIONS

Simplifying assumptions may be made in order to find approximate solutions to the equations. To find whether density variation with pressure is significant, it is first assumed that the flow is in the  $(r, \theta)$  directions only and the density is uniform; the corresponding pressure distribution is then calculated.

#### 4.1 Radial flow approximation

<u>Approximation</u>:  $\rho = \rho_0$ ;  $v_z = 0$ ; mass flow  $M_g$  from outer radius  $R_o$  to a line sink at inner radius  $R_i$ , distributed over height L; turbulent stresses ignored.

With negligible viscous dissipation, conservative body forces (gravity) and uniform density, assumption of a uniform velocity distribution at inlet implies that  $V \times V = 0$  for flow within the separator. Then  $V = \nabla \phi$  where the potential  $\phi$  is a harmonic function ( $V \cdot V = 0 \Rightarrow \nabla^2 \phi = 0$ ) of the form  $\phi = A\theta - B \ln r$ , with parameters A and B determined by the boundary conditions. The radial and tangential velocity components are then

$$v_\theta \,=\, \frac{A}{r} \text{ and } v_r \,=\, -\, \frac{B}{r} \,, \tag{16}$$

where A = 
$$v_{\theta \circ} R_{\circ}$$
 and B =  $\frac{M_g}{2\pi \rho_0 L}$ 

and where the tangential inlet speed  $v_{\theta 0} = \frac{M_g}{\rho_0 A_p}$  at  $r = R_0$  is determined by the cross-sectional area  $A_D$  of the inlet pipe.

The expressions (16) for the velocity components satisfy Equations (9) and (11) identically. The components of the pressure gradient can be deduced from Equations (10) and (12) to be

$$\frac{\partial p}{\partial r} = \rho_0 \frac{A^2 + B^2}{r^3}$$

$$\frac{\partial p}{\partial z} = -\rho_0 g$$

which give a Bernoulli equation:

$$P = C - P_0 \frac{A^2 + B^2}{2r^2} - \rho_0 gz$$

The pressure difference Ap between the outer radius  $r = R_0$  and the inner radius  $r = R_i$  at any elevation z is

$$\Delta p = \rho_0 \frac{A^2 + B^2}{2} \left( \frac{1}{R_i^2} - \frac{1}{R_0^2} \right)$$

For the typical flow parameters given for the example above,  $A = 32 \text{ m}^2/\text{s}$  and  $B = 0.25 \text{ m}^2/\text{s}$ . With  $\rho_0 \approx 5.6 \text{ kg/m}^3$ ,  $R_c = 1 \text{ m}$  and  $R_i = 0.3 \text{ m}$ , the pressure difference Ap between the outer and inner radii of the circular vessel is about 0.3 bar.

The corresponding steam density variation through such a flow where density variation due to pressure variation is taken account of, is then approximately 0.15 kg/m³, a 3 % change in value. This indicates that reasonable results should be predicted by a model with uniform steam density.

Note also that the pressure will drop radially inwards through the vessel due to the cyclonic flow and the centripetal acceleration induced by the rotation. This will tend to "dry" the gas by effective superheating of the steam as its pressure drops below the saturation value for the temperature.

# 4.2 A Simplified 2-D Model

Characteristics of the model: Steady flow of a fluid of uniform density  $\rho_0$ , with rotational symmetry  $(\partial \mathbf{v}/\partial \theta = \mathbf{0})$  and with negligible viscous effects; neglect turbulent stresses; both the inlet and outlet flows are **uniformly** distributed throughout  $0 < \theta$  12n and for a < z c b and d < z < L respectively; neglect the liquid water flow; assume uniform flow in inlet piping (irrotational flow). The fluid flows between the radii  $r = R_i$  and  $r = R_0$ , and the upper and lower surfaces z = 0 and z = L.

The boundary conditions, translated **from** the flow conditions at inlet and outlet and that of zero normal flow at solid walls, are, assuming that speeds are uniform with z at the inflow and outflow sections:

on  $r = R_0$ :

$$v_r = \left\{ \begin{array}{ll} 0 \;, & \text{Oczla} \\ - \; \frac{M_g}{2\pi\rho_0 R_0 (b-a)}, & \text{a$$

on  $r = R_i$ :

$$v_r = \begin{cases} 0, & 0 < z \le d \\ -\frac{M_g}{2\pi\rho_0 R_i (L-d)}, & d < z < L \end{cases}$$

on  $r = R_0$ :  $v_\theta = v_{\theta 0}$ 

on z = 0, **L**:  $v_z = 0$ .

The steam **flow**, by Kelvin's theorem, remains irrotational within the separator, i.e.  $\nabla \mathbf{x} \mathbf{v} = \mathbf{0}$  and the flow velocity can then be expressed **as** the gradient of a scalar potential function  $\mathbf{v} = \nabla \phi$ .

The equation for conservation of mass for the steady flow of a fluid of uniform density reduces to  $\nabla \cdot v = 0$ . Substitution of  $v = \nabla \varphi$  then gives  $\nabla^2 \varphi = 0$ , and  $\varphi$  is **a** harmonic function.

In cylindrical coordinates, rotational symmetry implies that  $v_r = \frac{\partial \varphi}{\partial r}$ ,  $v_\theta = \frac{1}{r} \frac{\partial \varphi}{\partial \theta}$  and  $v_z = \frac{\partial \varphi}{\partial z}$  are all functionally dependent on (r, z) only, while  $\varphi$  satisfies the equation

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}\right) \phi = 0$$
 (17)

There are at least two approaches to solving the mathematical problem. One is to solve Equation (17) analytically, using the boundary conditions as set out above. In that case, the general solution which satisfies the above constraints is of the analytical form:

$$\begin{split} \varphi &= A_0 \, \theta - B_0 \, \ell \, n \, r + C_0 \, + \\ &+ \sum_{n=1}^{\infty} \left[ A_n I_0 \left( \frac{n \pi r}{L} \right) + B_n K_0 \left( \frac{n \pi r}{L} \right) \right] \cos \frac{n \pi z}{L} \end{split}$$

while the velocity components are given by

$$\begin{aligned} v_r &= - \mathop{B}_{r} o + \\ &+ \sum_{n=1}^{\infty} \frac{n\pi}{L} \left[ A_n I_1 \left( \frac{n\pi r}{L} \right) - B_n K_1 \left( \frac{n\pi r}{L} \right) \right] \cos \frac{n\pi z}{L} \\ v_\theta &= \frac{A_0}{r} \quad \text{and} \end{aligned}$$

$$\begin{array}{l} v_Z = \\ -\sum_{n=1}^{\infty} \frac{n\pi}{L} \left[ A_n I_0 \left( \frac{n\pi r}{L} \right) + B_n K_0 \left( \frac{n\pi r}{L} \right) \right] \sin \frac{n\pi z}{L} \end{array}$$

where  $I_0$ ,  $I_1$  and  $K_0$ ,  $K_1$  are the Bessel functions of the First and Second Kinds, of order zero and one respectively. The constant coefficients  $A_n$  and  $B_n$  ( $n = 0, 1, ..., \infty$ ) and  $C_0$  are evaluated by using the boundary conditions.

A stream-function for the flow may also be defined, **as** follows. Rearrangement of Equation (9) gives

$$\frac{\partial}{\partial r} (rv_r) + \frac{\partial}{\partial z} (rv_z) = 0$$
 (18)

After writing

$$\operatorname{rv}_{r} - \frac{\partial \psi}{\partial z}$$
 and  $\operatorname{rv}_{z} = -\frac{\partial \psi}{\partial r}$ , (19)

where  $\psi = \psi(r, z)$ , to identically satisfy Equation (18), it takes only a small amount of calculation to show that

$$\begin{split} \psi &= -B_0 z + \\ &+ \sum_{n=1}^{\infty} r \left[ A_n I_1 \left( \frac{n \pi r}{L} \right) - B_n K_1 \left( \frac{n \pi r}{L} \right) \right] \sin \frac{n \pi z}{L} \end{split} \tag{20}$$

The constant coefficients are the same as occur in the velocity potential  $\phi$ . A reference value  $\psi = 0$  is taken on z = 0.

It is easily shown that  $\mathbf{v} \cdot \nabla \psi$  (=  $\nabla \phi \cdot \nabla \psi$ ) = 0. This means that the flow velocity vector is tangential to the surface  $\psi(r,z)$  = constant, and that  $\psi$  therefore acts as a stream-function for the flow. Fluid particles move inwards from the inlet to the outlet on spirals which lie on a stream surface  $\psi$  = constant.

As a variable which may be used to describe the whole flow pattern, the stream-function  $\psi$  satisfies the equation

$$\left(\frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\right) \psi = 0$$
 (21)

with the following boundary conditions:

on 
$$r = R_0$$
:

$$\psi = \begin{cases} 0, & \text{Oczla} \\ -\frac{M_g^-}{2\pi\rho_0} {z-a \choose b-a}, & \text{aczcb} \\ -\frac{M_g}{2\pi\rho_0}, & \text{b} \le z < L \end{cases}$$

on  $r = R_i$ :

$$\psi \ = \ \left\{ \begin{array}{ll} 0 \ , & 0 < z \leq d \\ \\ -\frac{M_g}{2\pi\rho_0} \left(\frac{z-d}{L-d}\right), & d < z < L \end{array} \right. \label{eq:psi_def}$$

on **z** = 0: 
$$y = o$$

on z = L: 
$$\psi = -\frac{M_g}{2\pi\rho_0}$$

Note that values of the stream-function  $\psi$  are proportional to  $M_g/\rho_0 = v_{\theta o}A_p$ , the volume flux of the vapour.

### 4.3 A numerical solution

Figure 2 shows a cross-sectional view of a set of stream and isobaric surfaces found from the numerical solution of Equation (21) using the boundary conditions set out above. The solution was confinned by the analytic form given in §4.2. The velocity components were inferred from (19) and the pressure distribution from a Bernoulli equation.

The shape of the **stream** surfaces depends only on vessel aspect ratio and the inlet and outlet configurations; here, the dimensions of the steam flow region are L = 4 m,  $R_0 = 1 \text{ m}$ ,  $R_i = 0.3 \text{ m}$ , while the lower and upper boundaries of the inlet and outlet are given by a = 0.9 m, b = 1.10 m and d = 3.80 m; the vertical heights of the inlet and outlet are equal, in this case, with b - a = L - d =0.20 m. The calculated pressure within the vessel ranges from the inlet value of **11.0** bar abs to the outlet value of 10.71 bar abs, a total drop of 0.29 bar. This is close to the estimate found from the simple approximation calculated in the first part of **§4.1** above, and also near to the design pressure drop used commercially (Kevin Koorey, personal communication). The isobaric surfaces (surfaces of equal pressure) are nearly vertical cylinders, which indicates the strong control of the pressure by the cylindrical motion.

### 5. SUMMARY

A simplified model for flow of the steam phase in a flashing cyclone separator has been solved to give the flow and pressure distribution patterns. The calculated overall pressure drop across the vessel agrees well with design values. Further work to investigate the flow and effects of the separated water is planned.

#### **ACKNOWLEDGEMENT**

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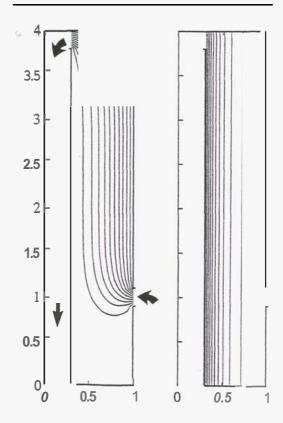


Fig. 2: Cross-sectional view of stream (left) and isobaric surfaces (right) for flow of a gas phase in a cyclone separator vessel. [Dimensions: m]