Proceedings 18th NZ Geothermal Workshop 1996

ON A CAPILLARY PRESSURE IN TWO-PHASE RESERVOIRS

R.I. PASHKEVICH

Kamchatsky Complex Department of NIPIgeotherm Institute
Petropavlovsk-Kamchatsky, Russia

SUMMARY - An equilibrium state of geothermal fluid in fractured and/or porous reservoirs is discussed, when classic hydrostatic equation, as corollary of Darcy's law, gives wrong results. It is shown, that generally hydrostatic equation must include addition term, which eliminates considered paradox of incomplete physical model. The use of modified hydrostatic equations can increase adequacy of calculation of fluid pressure both in one-phase saturated and two-phase (unsaturated two-component) geothermal reservoirs.

The two-phase modification of hydrostatic equation eliminates known negative absolute fluid pressure paradox for unsaturated permeable medium (two-phase geothermal reservoir), followed from standard hydrostatic equation and gives simplified correlations between capillary pressure and fluid saturation, which satisfactorily compare with laboratory experimental data for porous media.

1. INTRODUCTION

In different general and applied problems of hydrogeothermics, hydrogeology and technology of development of geothermal fields a question about distribution of geothermal fluid pressure inside reservoir arise. In the most of reservoirs, the fluid as a rule, is constantly in free-forced convective motion. Nevertheless, frequently in practical calculations for the some types of reservoirs (f.e., systems of separated permeable zones of horizontal stretch), and also as an initial condition for other cases, a condition of immobility or equilibrium of fluid is applied.

The equilibrium state is often consider also for two-phase geothermal reservoirs in reinjection problems (Pruess, 1991). Then the capillary forces may play an important role. However any proved correlations between capillary pressure and saturation for geothermal reservoirs are presently absent. The known correlations for porous media are used, as well as hydrostatic equation.

The aim of the paper is to show some paradoxes and shortcomings of standard hydrostatic approach and equations for one- and two-phase geothermal reservoirs and to suggest modified equations.

The equilibrium distribution of pressure in reservoir is usually considered equal to hydrostatic, the same for a case of pure fluid, without the presence of rocks. It is just considered, that pressure grows with depth, but is constant along horizontal stretch. As a confirmation, generally a corollary of filtration Darcy's law for case of motionless fluid saturated porous media (Gurevich et. *al.*, 1972) is derived.

Actually, if in Darcy's law a velocity of fluid is zero, therefore,

$$\nabla p = \rho \mathbf{g}, \quad (1)$$

where p is fluid pressure, averaged in a same spatial scale, in which a filtration velocity is determined, ρ is fluid density, g is the vector of gravitational acceleration.

It is obvious that expression (1) correctly describes a distribution of fluid pressure in porous, fractured and porous-fractured media, if fluid in whole medium volume is continuous phase, and the horizontal components of porosity gradient are equal to zero. In that case gradient of local fluid pressure p' will be equal to hydrostatic, then a gradient of averaged pressure will be also equal to hydrostatic, in condition of spatially constant gravity.

Before, in connection with elucidation of conditions of equilibrium of stratified underground waters, S.I. Smirnow (cited by Gurevich et al., 1972, where the criticisms of his results are presented) raised a question of availability of deflections of pressure distribution from hydrostatic, which can be caused by fluid density gradients. Later Gray and Hassanizadeh (1991a,b) had pointed on the shortcomings of (1) (as a condition of equilibrium for cases of unsaturated porous media) connected with capillary effects. These authors suggest modified Darcy's law for unsaturated flows, containing, as compared with classic law, additional term, that is intended to overcome the well-known hydrostatics paradoxes in unsaturated media. Nevertheless, as it is stated by the authors, Gray-Hassanizadeh's equations in case of fully saturated flow lead to standard generalized Darcy's law, and then in hydrostatics for saturated medium will give the expression (1) and hydrostatic distribution of fluid pressure.

However, it is easy to find simple examples, when actual distribution of steady fluid pressure in permeable medium is different from hydrostatic, which is waited by corollary (1) from Darcy's law, and besides for case of full saturation when fluid density is constant.

2. ONE-PHASE RESERVOIRS: SOME HYDROSTATICS PARADOXES

Consider the conditional geothermal reservoir, comprising systems of parallel equidistant wedge-shaped fractures, or porous layers, in which lower plane of fracture's or layer's boundary is parallel to horizontal plane, and upper plane has angle of incidence to the right. For simplicity consider the absolutely impermeable rocks. Then (1) gives that averaged fluid pressure p grows with depth, but does not change in horizontal plane, including along fracture or layer. Nevertheless it is not correct and can be simply verified. Really, average pressure grows also along fracture to the right. Thanks to symmetry, it is sufficient to consider single fracture or layer. In this case, average fluid pressure, included in expression (1), will be equal to fracture (layer) cross section averaged pressure. Then, if a means a fracture angle, and x means a coordinate along fracture, one can obtain for horizontal component of pressure gradient:

$$\partial p/\partial x = \rho g \tan \alpha . \qquad (2)$$

Presence of this component is caused of fracture width gradient, or in macro scale, of porosity gradient along direction x. In the case of absence of horizontal component of porosity gradient, (f.e. for conditional geothermal reservoir consisted of rectangular or cylindrical channels with constant cross section), expression (1) will be absolutely correct.

Then classical Darcy's filtration law for variable porosity of media in hydrostatics case may give incorrect results. This paradox with help of Goldshtik's classification (Goldshtik *et. al.*, 1989) can be attributed to the paradoxes of incompleteness of physical model.

It is to be expected that expression (1) and then the Darcy's law in right hand side must have additional terms.

During the derivation of Darcy's law by means of spatial averaging methods (Slattery, 1972, Nigmatulin, 1987) intermediate expression for filtration velocity v can be presented as follows:

$$\mathbf{v} = -(\mathbf{K}'/\mu)\left[\nabla \varepsilon p - \varepsilon \rho \mathbf{g} + \frac{1}{V}\int_A p' \,\mathbf{n}\, dA\right], \qquad (3)$$

where ε is porosity, K' is intermediate tensor of permeability, which is not equal to tensor of permeability K in Darcy's law, μ is dynamic viscosity of fluid, A is pore or fracture surface in a averaging volume V, $\mathbf{n}$ is unit normal to the surface pointing out of fluid, p' is microscopic fluid pressure. Generally surface integral in (3) and terms containing porosity gradients are implicitly included in tensor of permeability and then the classical form of Darcy's law is obtained:

$$\mathbf{v} = -(\mathbf{K}/\mu)[\nabla p - \rho \mathbf{g}] . \qquad (4)$$

It is necessary to mark, that Darcy's law in form of (4) for fluid filtration in permeable media, as well as in geothermal reservoirs, with use of experimentally obtained or calculated on base of expression (4) the permeability tensor **K**, can giving correct velocity and pressure distributions in case of any porosity gradients. Nevertheless a corollary (1) from this law for motionless fluid in permeable medium in general case, as demonstrated above, may lead to mistakes.

Correct expression for determination of static fluid pressure will be a corollary from equation (3) with zero filtration velocity, as follows:

$$\nabla \varepsilon p = \varepsilon \rho \mathbf{g} - \frac{1}{V}\int_A p' \,\mathbf{n}\, dA . \qquad (5)$$

It is easy to show that equation (5) for discussed above case of conditional geothermal reservoir will give correct expression for horizontal component of fluid pressure gradient (2).

Then, despite the local hydrostatics pressure p' in every point of fluid saturated permeable medium satisfying the main hydrostatics equation in form of (1), average hydrostatic fluid pressure p in general case submits to equation (5), which becomes equation (1) in case of constant porosity.

Equation (5) may be obtained also by direct representative volume averaging of main hydrostatics equation for local pressure in form (1), using Slattery-Whitaker theorem (Slattery, 1972, Nigmatulin, 1987) for connection between average of gradient of local variable and gradient of average variable.

3. TWO-PHASE RESERVOIRS: HYDROSTATICS EQUATION AND RELATIONS FOR CAPILLARY PRESSURE

Now consider a case of hydrostatics in unsaturated porous or fractured medium, corresponding to two-phase geothermal reservoirs, contained both fluid and steam phase, and also to *gassy* single-phase reservoirs with a fluid domination. Use procedure described in above paragraph. Then one can obtain for hydrostatics fluid phase pressure the equation as follows:

$$\nabla \varepsilon s p = \varepsilon s \rho \mathbf{g} - \frac{1}{V} \int_{A_T + A_S} p' \mathbf{n}\, dA\,, \qquad (6)$$

where s is fluid saturation, A_r and A_s are respectively surfaces of fluid-rock and fluid-steam or fluid-gas interfaces. Equation *(6)* **is** different from hydrostatics equation for unsaturated permeable medium in a form of (1), followed for static case from generalized Darcy's law, used in standard theory of unsaturated flow.

In the Gray and Hassanizadeh's unsaturated flow theory (1991a) from their modified **Darcy's** law in static case, an equation, similar to equation (6), is produced. Nevertheless that equation does not explicitly contain porosity gradients, but includes (introduced **by** the authors) wettability potentials, which demand experimental definition. Later point does Gray and Hassanizadeh's hydrostatic equation to be complicated for direct analysis.

For simplicity consider a case with constant porosity. Then from (6), using **Darcy's** law, a corollary from Stokes' theorem and standard definition of a capillary pressure p_c, and also, suggesting steam *(gas)* pressure ***p,*** is constant, we get:

$$\left[p_s - p\right]\nabla s = -\frac{1}{\varepsilon V} \int_{A_s} 2K_M \sigma \mathbf{n}\, dA\,, \qquad (7)$$

$$p_c \nabla s = -\frac{\sigma}{\varepsilon V} \oint_C d\mathbf{l} \times \mathbf{n}\,, \qquad (8)$$

where K_M is mean curvature of surface of fluid-steam (fluid-gas) interface, σ is surface tension of fluid, C is a bounding line for surface A_s in averaging volume.

Therefore hydrostatics equation in form of *(6)* easily leads to relation **(8)** between capillary pressure and a saturation gradient. Problem of definition of such relation had been started with Leverett 's work (Luikov, 1978, Ferrand ***et.*** *al.*, 1990).

4. CAPILLARY PRESSURE - SATURATION RELATIONS AND COMPARISON WITH EXPERIMENTS

For an estimation of adequacy of formula (8), consider a case of rhombohedral packing layer of uniform spheres which diameter is D. Assume contact angle is zero; radius of most narrow channels between spheres is 0.155 of sphere's radius (Luikov, 1978) and is equal to interface surface curvature radius. Then from **(8)** it may obtain for vertical saturation gradient:

$$ds/dz = 0.155\pi\sigma \Big/ \left(p_c \varepsilon D^2\right). \qquad (9)$$

In general the fluid-steam (gas) interface surface curvature radius depends on coordinate along layer height, and definition of such dependence must be an objective of the experiments. Here it **is** sufficient to consider two most simple cases.

1. Let curvature radius be constant. Then carrying out integration of equation (9) over layer height, taking into account that capillary pressure gradient is equal to hydrostatic in case ***of*** constant porosity, it may obtain:

$$p_c / p_{cmax} = \exp\left[\left(S_{min} - S\right)\rho g \varepsilon D^2 / 0.155\pi\sigma\right]. \qquad (10)$$

2. Let curvature radius be linearly growing with an elevation above zero capillary pressure level. Then, carrying out integration of equation (8), it may obtain:

$$p_c / p_{cmax} = \left[\left(1 - S\right)/\left(1 - S_{min}\right)\right]^{\rho g \varepsilon D^2 / 0.155\pi\sigma}, \qquad (11)$$

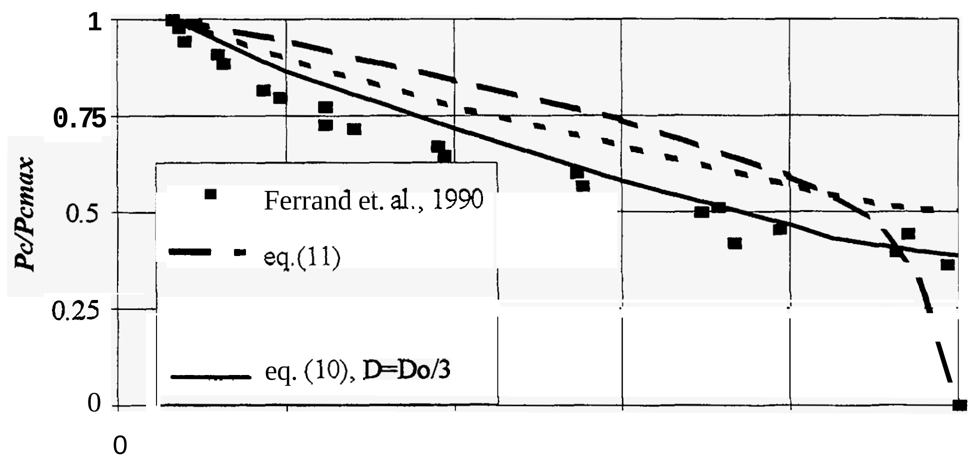

Figure 1. **A** comparison of the calculations by formulae (10) and (11) with experimental **data** by Ferrand ***et.*** *al.* (1990).

where p_{cmax} is a maximum value of the capillary pressure in the layer, S_{min} is a minimum value of the saturation in a layer.

The results of calculation by formulae (10), (11) (see Fig.1) compare satisfactorily with the experimental data of Ferrand *et. al.* (1990) of drainage in water-air saturated sand. D_0 is diameter of sand grains in original experiments.

Therefore, derived here equation of hydrostatics in unsaturated porous of fractured medium (6) allows relations between saturation gradient and capillary pressure (7) and (8) to be obtained, corollaries fkom which (eq. (10) and (11)) even for most strong simplifications are in accordance with experiments.

Equation (6) contains, as compared with classical equation of hydrostatics in form of (1), an additional term which taking into account a force of fluid pressure on fluid-ruck and fluid-steam *(gas)* interface surface, and also explicitly takes into account the porosity and saturation gradients. One can show, following Gray and Hassanizadeh (1991a), that an appearance of additional term in (6) allows to overcome of well-known paradox of negative absolute pressure of fluid phase predicted by equation (1).

5. CONCLUSIONS

One must stress the following. The above discussion of the cases when standard hydrostatics equation (1) may give wrong results and leads to paradoxes both for one-phase saturated and two-phase (or unsaturated two-component) geothermal reservoirs, has objective to pay attention to the need for care in using of equation (1) in general case of variable porosity and fluid saturation. The nonisotermal effects connected with real heat flow in geothermal reservoirs, which are not consider here, do not change this main point. Similarly conventional type of reservoir, considered in the beginning of the paper, was chosen only for simplicity and convenience of analysis.

Nevertheless the conclusion that for consideration of hydrostatics in permeable porous or fractured media, including in geothermal reservoirs, in general it is necessary to use equation (5) for one-phase saturation and (6) for two-phase or two-component saturation, but not the standard equation (1), is correct irrespective of simplifications adopted here in particular examples.

Usage of modified hydrostatic equations (5) and (6) may allow to increase the adequacy of calculations of fluid hydrostatic pressure in both one- and two-phase geothermal reservoirs and to avoid of possible errors connected with incompleteness of standard equation (1).

The above presented equations (6), (8) can be used as a initial base for further theoretical, experimental and field works on correlations between capillary pressure and fluid saturation for geothermal reservoirs.

The remaining theoretical problem is the search for expressions of surface integrals in equations (5), (6), (8) in terms of macroscale parameters -- fluid and steam *(gas)* pressure, geometric propertiers of permeable media, fluid saturation and others.

6. ACKNOWLEDGMENTS

Author thanks Professors W.G. Gray, Notre-Dame University, USA and S.M. Hassanizadeh, Delft University of Technology, The Netherlands, for a discussion of their cited articles, that has lead to a start of above presented work, and Mr. V.A. Yampolsky, Geologist-in-Chief of 'Kamchatskburgeothermia' State Enterprise, Russia for his critical comments.

7. REFERENCES

Ferrand, L.A., Milly, P.C.D., Pinder, G.E. and Turrin, R.P. (1990), A comparision of capillary pressure-saturation relations for drainage in two and three-fluid porous media, *Adv. Water Resour.*, Vol. 13, #2, 54-63.

Goldshtik, M.A., Shtern, V.N. and Yavorsky N.I. (1989). *Viscous Flows with Paradox Features.* Nauka, Novosibirsk, (in Russian).

Gray, W.G. and Hassanizadeh, S.M. (1991a), Paradoxes and realities in unsaturated flow theory, *Water Resour. Res.*, Vol. 27, # 8, 1847-1854.

Gray, W.G. and Hassanizadeh, S.M. (1991b), Unsaturated flow theory including interfacial phenomena, *Water Resour. Res.*, Vol. 27, # 8, 1855-1863.

Gurevich A.A., Kapchenko L.N., Kruglickow N.M. (1972). *Theoretical Principles of Oil Hydrogeology.* Nedra, Leningrad, (in Russian).

Luikov, A.V. (1978). *Heat-Mass Exchange: Reference Book.* Energia, Moscow, (in Russian).

Nigmatulin, R.I. (1987). *Dynamics of Multiphase Media.* Vol. 1, Nauka, Moscow, (in Russian).

Pruess, K. (1991). Grid orientation and capillary pressure effects in the simulation of water injection into depleted vapor zones, *Geothermics,* Vol. 20, No. 5/6, 257-277.

Slattery, J.C. (1972). *Momentum, Energy and Mass Transfer in Continua,* McGraw-Hill, New York.