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NON-BALANCED COUNTER FLOW; THE EFFECT OF NET MASS FLUX

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SUMMARY - Equations describing water-steam counterflow in geothermal reservoirs are reduced to a quadratic equation for either pressure gradient or water relative permeability. Based on the analytical solution of this equation , necessary conditions for the existence of two-phase counterflows are obtained and the effects of varying reservoir permeability are investigated. Some numerical exploration of the effects of varying net mass flux is undertaken.

1 INTRODUCTION

Two-phase water-steam geothermal systems form complex natural phenomena which involve high temperatures and pressures. An essential feature of these systems is the presence of two-phase zones in which steam and water coexist in equilibrium. If most of the total mass flow condenses to liquid on the upper boundary of the two-phase region, a balanced water-steam counterflow can occur: steam flows upward driven by the density difference between liquid and vapor phases, and the condensate flows downward driven by gravity. Such heat pipe models were introduced by White *et al.* **(1971)** and further developed by Schubert and Straus **(1979).** An impermeable upper boundary (or a caprock) is thought to be necessary for the formation of a water-steam counterflow. Many reservoirs, however, are unconfined, or communicate with lateral flows of groundwater. In such cases a non-zero vertical transport of mass can occur across a two-phase region.

Mathematical modelling of two-phase geothermal systems is based on the continuum mechanics approach (Bear, **1972;** Nield and Bejan, **1992).** A two-phase fluid is modelled as two co-existing continuous media filling the same pore-space. A mathematical model is written in the form of partial differential equations which admit one-dimensional steady solutions. The study of the simplified steady-state case is helpful for two reasons: firstly, it frequently occurs in nature and, secondly, it provides better understanding of the essential features of complex time-dependent problems as illustrated further.

2 BALANCED COUNTERFLOW

If reservoir boundaries are impermeable, then the net mass flux is zero at any horizontal level z. Conservation of mass and energy in the steady-state with zero capillarity and with a heat flux Q give

$$u_v + u_l = 0, \quad u_v h_v + u_l h_l = Q, \tag{1}$$

where u is mass flux density and h is specific enthalpy, and subscripts v and l denote the vapour and liquid phases respectively. Darcy's equation is assumed to apply to both liquid and vapour phases separately,

$$u_l = -\frac{kk_{rl}}{\nu_l}\left(\frac{\partial P}{\partial z} + \rho_l g\right), \tag{2}$$

$$u_v = -\frac{kk_{rv}}{\nu_v}\left(\frac{\partial P}{\partial z} + \rho_v g\right), \tag{3}$$

where k_{rl} and k_{rv} denote liquid and vapour phase relative permeabilities, respectively.

The usual approach is to eliminate pressure derivatives from equations (1), and obtain a solvability condition relating saturation, pressure and heat flux (e.g,, Weir and Kissling, **1996).** Here, however, terms are non-dimensionalised as detailed below, and it is further assumed that $k_{rl} + k_{rv} = 1$. Then dependence on relative permeability is eliminated from equations (1), giving the following quadratic equation for the dimensionless pressure gradient, $q = (dp/dz)/\rho_l^* g$ (Pestov, **1996):**

$$aq^2 + bq + c = 0, \tag{4}$$

where

$$a = 1 + \frac{\mu\tilde{m}\tilde{\kappa}F}{\tilde{F}\hat{l}m\hat{\rho}_v} - \frac{\tilde{m}\tilde{\kappa}F}{\tilde{F}\hat{l}}, \tag{5}$$

$$b = -1 - m\hat{\rho}_v - \frac{\mu\tilde{m}\tilde{Q}}{\hat{l}m\hat{\rho}_v} + \frac{\tilde{m}\tilde{Q}}{\hat{l}} - \frac{\mu\tilde{m}\tilde{\kappa}F}{\tilde{F}\hat{l}m\hat{\rho}_v} + \frac{\tilde{m}\tilde{\kappa}F}{\tilde{F}\hat{l}}m\hat{\rho}_v, \tag{6}$$

$$c = m\hat{\rho}_v + \frac{\mu\tilde{m}\tilde{Q}}{\hat{l}m\hat{\rho}_v} - \frac{\tilde{m}\tilde{Q}}{\hat{l}}m\hat{\rho}_v, \tag{7}$$

Coefficients (5)–(7) involve the following non-dimensional parameters: $m = \rho_v^*/\rho_l^*$ – the density ratio at the lower boundary, $\tilde{m} = \tilde{\rho}_v/\rho_l^*$ – the density ratio at the upper boundary $z = 0$, $\mu = \mu_v^*/\mu_l^*$ – the dynamic viscosity ratio, $\tilde{Q} = \&Q/\tilde{m}kg\rho_l^{*2}l^{*2}$ – the non-dimensional heat flux, and $\tilde{\kappa} = \mu_l^*\alpha^*\tilde{F}/\tilde{m}k\rho_l^* l^*$ – the non-dimensional conductive heat flux. Here $F = dT/dp$ is the slope of the saturation line, $\tilde{F}$ is the value of F at $z = 0$, T is temperature, p is pressure, l is latent heat, Q is vertical heat flux, k is permeability, g is acceleration due to gravity, ρ is density, μ is dynamic viscosity, ν is kinematic viscosity, 'asterisk' marks the characteristic values and 'hat' stands for non-dimensional quantities.

Using the quadratic formula, we find that the roots of equation (4) are

$$q_{1,2} = \frac{1}{2a}\left(-b \pm \sqrt{d}\right), \qquad (8)$$

where $d = b^2 - 4ac$ is the discriminant of equation (4).

The above quadratic equation is obtained under the assumption of constant ρ ~ρ_v, , μ_l, and cy, which is often the case for geothermal reservoirs. A similar equation can be obtained for liquid relative permeability (see Pestov, 1996).

2.1 Rational solutions

If both $\tilde{Q}$ and $(\tilde{Q} - \tilde{\kappa})$ are not large, then discriminant d becomes a complete square and roots (8) can be rewritten in the form of rational expressions:

$$q_1 = m\hat{\rho}_v + \frac{\mu(\tilde{Q} - \tilde{\kappa}F m\hat{\rho}_v/\tilde{F})}{\hat{l} m\hat{\rho}_v/\tilde{m} + \tilde{\kappa}F(\mu - m\hat{\rho}_v)/\tilde{F}}, \qquad (9)$$

$$q_2 = 1 - \frac{m\hat{\rho}_v(\tilde{Q} - \tilde{\kappa}F/\tilde{F})}{\hat{l} m\hat{\rho}_v/\tilde{m} + \tilde{\kappa}F(\mu - m\hat{\rho}_v)/\tilde{F}}. \qquad (10)$$

Similar rational expressions can be obtained for liquid relative permeability (see Pestov, 1996).

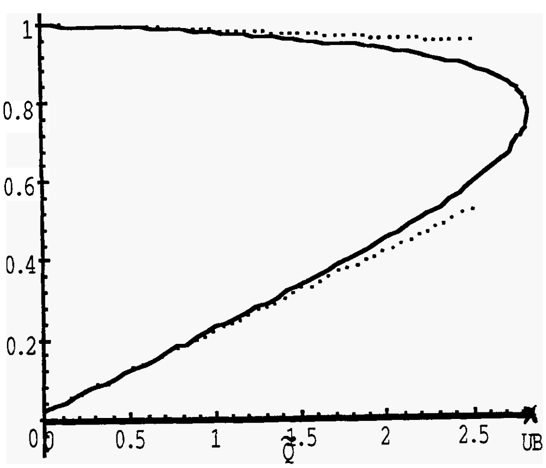

Figure 1. Dimensionless pressure gradient versus $\tilde{Q}$.

Fig. 1 presents dimensionless pressure gradient q at $z = 0$ as a function of $\tilde{Q}$. The exact solution (8) is shown by a solid line. The rational solutions (9) and (10) are shown by points. Parameter $\tilde{m}$ is taken to be 0.02 and parameter $\tilde{\kappa}$ is taken to be 0.016. This corresponds to $T = 513°K$ at $z = 0$ and $k = 10^{-14}\ m^2$. The upper branches with q close to 1 are liquid-dominated solutions. The lower branches with relatively small q are vapour-dominated ones. The solid and dotted curves agree closely with an error less than 1% when $a < 1.5$ (or when $Q < 37.5\ W/m^2$ for these values of T and k). Since typical values of Q are rather small, rational expressions (9)–(10) can be used to approximate the steady-state conditions for many geothermal fields. According to (9)–(10), q varies only slightly with depth for most geothermal reservoirs with the only exception being a low-permeability and high heat flow reservoir ($\tilde{Q} \sim 1$ and $\tilde{Q} - \tilde{\kappa} \sim 1$).

2.2 Necessary condition

The physical meaning of q implies that $q_1 \geq m\hat{\rho}_v$ and $q_2 \leq 1$. The first inequality means that in a vapour-dominated counterflowing zone the pressure gradient is not less than the vapourstatic pressure gradient. The second inequality indicates that in a liquid-dominated counterflowing zone the pressure gradient does not exceed the hydrostatic pressure gradient. The latter together with the fact that q must be real give the following necessary condition for the existence of two-phase counterflow (vapour or liquid-dominated):

$$\tilde{\kappa} \leq \tilde{Q} \leq \tilde{Q}^{cr}. \qquad (11)$$

However, liquid-dominated counterflow is not possible and only vapour-dominated counterflow may exist when

$$\tilde{m}\tilde{\kappa} \leq \tilde{Q} \leq \tilde{\kappa}. \qquad (12)$$

The upper bound is

$$\tilde{Q}^{cr} = \frac{1 - \tilde{m}}{(\sqrt{\mu} + \sqrt{\tilde{m}})^2} + \tilde{\kappa}\frac{\sqrt{\mu} + \tilde{m}\sqrt{\tilde{m}}}{\sqrt{\mu} + \sqrt{\tilde{m}}}. \qquad (13)$$

Formula (13) is in agreement with that obtained by Bau and Torrance (1982) in the absence of conductivity ($R = 0$). There are also two lower bounds R and $\tilde{m}\tilde{\kappa}$ for liquid and vapour-dominated counterflows respectively. The latter bound is relatively small. For $T = 513°K$ at $z = 0$ and cy $= 3.2\ W/m\ °K$, the inequality (12) becomes:

$$0.01\, W/m^2 \leq Q \leq 0.43\, W/m^2.$$

The upper and lower bounds calculated for $\tilde{m} = 0.02$, $\mu = 0.2$, $a = 3.2\ W/m\ °K$ and different k are given in Table 1.

$k\ m^2$	$\tilde{\kappa}$	$\tilde{Q}^{cr}$
—	0	2.8284
10^{-12}	1.6×10^{-4}	2.8285
10^{-13}	1.6×10^{-3}	2.8296
10^{-14}	1.6×10^{-2}	2.8406
10^{-15}	1.6×10^{-1}	2.9507
10^{-16}	1.6	4.0516

Table 1. Upper and lower bounds on heat flux.

2.3 Effects of reservoir permeability

Examining the liquid-dominated solution (10) reveals that conduction may be neglected when $\tilde{Q} \gg \tilde{\kappa}$. On the other hand, according to (11), $\tilde{Q}$ must not exceed the critical value $\tilde{Q}^{cr}$, which, in turn, depends on permeability k through parameter $\tilde{\kappa}$ as given by (13). For low permeabilities parameter k becomes relatively large, so that $\tilde{\kappa} \sim \tilde{Q}^{cr}$. Then $\tilde{Q} \sim \tilde{\kappa}$ and conduction is a substantial part of the total heat transport. For high permeabilities $\tilde{\kappa} \ll \tilde{Q}^{cr}$, the necessary condition (11) allows for much larger $\tilde{Q}$, and the inequality $\tilde{Q} \gg \tilde{\kappa}$ may be satisfied. For instance, if $T = 513°K$ at $z = 0, \alpha = 3.2\, W/m\,°K$ and $k = 10^{-14}\ m^2$, then $\tilde{\kappa} = 0.016$ which is much less than the corresponding value of $\tilde{Q}^{cr} = 2.8$ (see Table 1).

The above comments may be further illuminated by eliminating the pressure gradient term from equations (1), to obtain the solvability condition (in dimensional terms)

$$Q = Q_m \frac{(\lambda_l h_l + \lambda_v h_v)}{\lambda_l + \lambda_v} - \frac{g\lambda_l\lambda_v(\rho_l - \rho_v)l}{\lambda_l + \lambda_v} - \lambda g F \frac{(\lambda_l \rho_l + \lambda_v \rho_v)}{\lambda_l + \lambda_v}. \quad (14)$$

Here a net mass flux Q_m has been included in the mass conservation equation, $\lambda_l \equiv (kk_{rl})/\nu_l$ and $\lambda_v \equiv (kk_{rv})/\nu_v$.

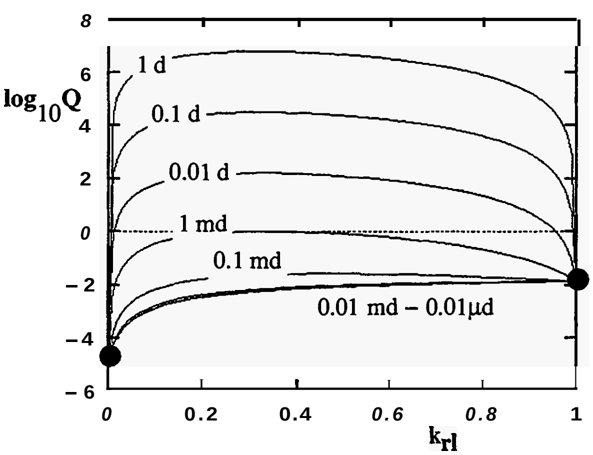

Figure 2. Log of steady state heat flux against liquid relative permeability, for various permeabilities.

Fig. 2 shows how the steady heat flux Q varies with liquid relative permeability, for zero net mass flux Q_m, and for various values of permeability k. This, and the following figures, have been obtained by numerically calculating equation (14) at a temperature of 280°C, with $\lambda = 2$W/(m°C). Full and accurate dependence of thermodynamic properties on temperature and pressure has been included.

Note that the values of heat flow at the ends $k_{rl} = 0$ and $k_{rl} = 1$ remain fixed in place (marked by the black disks in Fig. 2), at conductive values, as permeability varies. As permeability is reduced, conductive effects dominate, and Q becomes single-valued instead of double-valued.

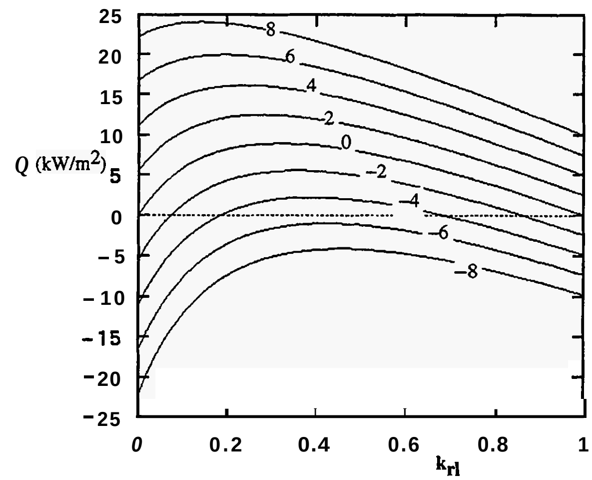

Figure 3. Steady state heat flux against liquid relative permeability, for various net mass fluxes (g/s/m^2), when $k = 1$d.

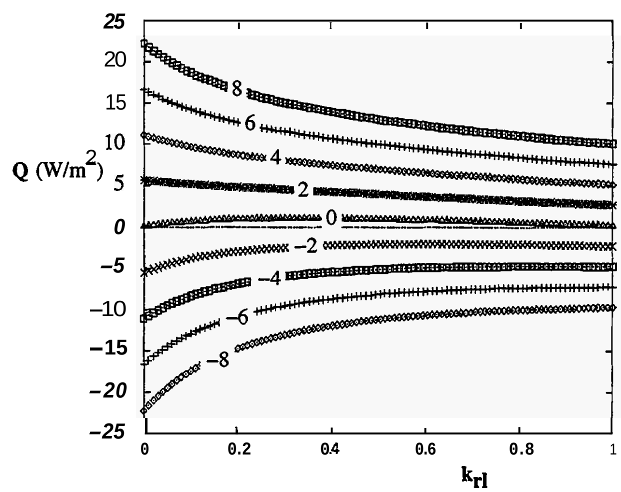

Figure 4. Steady state heat flux against liquid relative permeability, for various net mass fluxes (mg/s/m^2), when $k = 0.1$md.

3 EFFECT OF NET MASS FLUX

The terms making up the steady heat flow in equation (14) can be physically identified. The first is due to net mass flux, with the expression multiplying Q_m being a two-phase flowing enthalpy. The second is the contribution of counterflow, and leads to the possibility of two solutions for a given heat flow. The third is the contribution of conduction, with the dependence of temperature gradients on the Clausius-Clapeyron equation made explicit.

The effect of varying the net mass flux is demonstrated in Figs 3 and 4, for permeabilities 1 d and 0.1 md respectively. As Q_m increases, so does Q, and the effect is more marked near pure steam because the flowing two-phase enthalpy is higher there. For large mass fluxes, counterflow effects are negligible, and Q becomes single-valued instead of double-valued.

4 CONCLUSIONS

The governing conservation equations, which describe water-steam counterflow in geothermal reservoirs, can be reduced to a quadratic equation for either pressure gradient or water relative permeability. This equation, in the case of zero net mass flux (balanced counterflow), admits an analytical solution. For parameters typical of geothermal reservoirs the analytical solution can be written in the form of rational expressions. Given the rational solutions, pressure gradient and relative permeabilities do not vary significantly with depth. Hence, in most practical cases they can be taken equal to their values at the upper boundary of a two-phase region with the only exception being a low-permeability and high heat flow reservoir.

The analytical approach developed for the case of zero net mass flux yields upper and lower bounds for the dimensionless heat flux $\tilde{Q}$. When the inequality

$$\tilde{\kappa} \leq \tilde{Q} \leq Q^{cr}$$

is satisfied, two-phase counterflow (vapour or liquid-dominated) may exist. However, when the inequality

$$\tilde{m}\tilde{\kappa} \leq \tilde{Q} \leq \tilde{\kappa}$$

is satisfied, liquid-dominated counterflow is not possible and only vapour-dominated counterflow may exist.

In a liquid-dominated counterflow conduction is not important when $\tilde{Q} \gg \tilde{\kappa}$ and $\tilde{\kappa} \ll \tilde{Q}^{cr}$ (high heat flow, high permeability). In a vapour-dominated counterflow conduction is non-dominant when $\tilde{Q} >> \tilde{m}\tilde{\kappa}$ which includes most practical cases.

Note that non-dimensional parameters $\tilde{m}$, $\tilde{\kappa}$, $\tilde{Q}$ and $\tilde{Q}^{cr}$ can be easily calculated for a given geothermal reservoir.

When permeability is reduced, with zero net mass flux, conductive effects lead to a single-valued Q rather than double-valued. When net mass flux is large enough, this also leads to single-valued Q dependence on either liquid relative permeability or pressure gradient. This is of interest when addressing questions of which solution is selected, vapour-dominated or liquid-dominated. In particular, as permeability is reduced, the liquid-dominated solution disappears. As heat flow increases in the upwards direction, the vapour-dominated solution disappears. As heat flow increases in the downwards direction, the liquid-dominated solution disappears. These are a matter of interpretation, since in all cases the full range of relative permeabilities is still available, depending on heat flow.

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6 REFERENCES

Bau, H.H. and Torrance K.E. (1982) Boiling in low-permeability porous materials. ***Int. J. Heat Mass Transfel;* 25,** 45-55.

Bear, J. (1972) ***Dynamics of fluids inporous media.*** Dover Publications, New York.

Nield, D.A. and Bejan, A. (1992) ***Convection in porous media.*** Springer-Verlag, New York.

Pestov, I. (1996) ***Mathematical modelling of two-phase geothermal systems.*** PhD thesis, Victoria University of Wellington.

Schubert, G. and Straus, J.M. (1979) Steam-water counterflow in porous media. ***J. Geophys. Res.,*** 84, 1621-1628.

Weir, G.J. and Kissling, W.M. (1996) Enhanced conduction in steady, vertical flows of water, steam and carbon dioxide in a porous medium, in press, ***Transp. in*** Porous ***Media.***

White, D.E., Muffler, L.J.P. and Truesdell, A.H. (1971) Vapor-dominated hydrothermal systems compared with hot-water systems. ***Economic Geology,*** Vol. 66, 75-97.