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ANALYSIS OF MAGNETOTELLURIC DATA USING THE REILLY DECOMPOSITION TECHNIQUE

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ABSTRACT - Results of magnetotelluric measurements made over complex three-dimensional structures are usually expresses as an impedance tensor which has 8 independent components, all containing useful information. The Reilly (1979) decomposition technique represents these 8 parameters as an admittivity, which is the sum of two real tensors: an apparent conductivity (or apparent resistivity) tensor, and an apparent permittivity tensor. The apparent permittivity can be used to illustrate the phase changes associated with complex structures. This decomposition is demonstrated for an idealised model of geothermal system. For this model, the apparent resistivity tensor shows similar properties to the standard MT presentation techniques. However, the apparent permittivity tensor shows variations that appear to delineate the model boundaries with some precision. This decomposition may prove a useful tool in the analysis of MT data in complex structures such as geothermal systems.

INTRODUCTION

Magnetotelluric (MT) methods are becoming an increasingly popular means of investigating deep geothermal resources. In such demanding environments where electrical structures are complex and predominantly three-dimensional, interpretation is difficult and remains one of the major stumbling blocks. As a primary aid to interpretation, visualisation techniques, which allow images of the structure to be produced prior to interpretation, can be extremely useful. In this paper we demonstrate one such technique.

The decomposition technique proposed by Reilly (1979) starts from the outset assuming a complex, three dimensional earth, and thus the method should be well suited to complex structures such as are found in a geothermal environment. However, because at the time of its proposal, three dimensional modelling techniques were in their infancy, the Reilly decomposition technique has never been fully tested. Making use of the modelling techniques now available, we present here a comparison of the Reilly decomposition technique with standard methods, using a model of an idealised geothermal system.

MAGNETOTELLURIC MEASUREMENTS

In the usual application of MT, measurements (consisting of 2 components of electric field and 2 components of magnetic field vector) are analysed to determine the components of an impedance tensor Z. That is

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \begin{pmatrix} H_x \\ H_y \end{pmatrix} \qquad (1)$$

where (E_x, E_y) are the component of the electric field vector at a given frequency, and (H_x, H_y) are the corresponding magnetic field components. In general, the components of the impedance tensor are determined at a series of discrete values of radian frequency ω (= 2π frequency). The components of Z are normally written as complex numbers, where the amplitude and phase relationships between E and H are given by the magnitude and argument of the components of Z.

There are many approaches to defining apparent resistivity from the 8 components of Z. In the seminal work on magnetotellurics, Cagniard (1953) defines a scalar apparent resistivity as:

$$\rho_a = [E_x/H_y]^2/\mu\omega = 0.2\,T\,|Z_{xy}|^2 \qquad (2)$$

In a horizontally layered earth, this apparent resistivity would be independent of the orientation of the measurement array. In the general situation, however, all components of Z are non-zero, and a multitude of definitions exist. Examples of some of these are discussed by Spies and Eggars (1986). Given the tensor nature of Z, in generality, tensor functions are necessary to define apparent resistivity (as is used with DC resistivity, Bibby 1986). Early attempts at defining a more general apparent resistivity as

$$\rho_{ij} = 0.2\,T\,|Z_{ij}|^2 \qquad (3)$$

(Vozoff 1972, Abramovici 1974) have the disadvantage that ρ_{ij} defined in this way is not a tensor at all.

In full, the impedance tensor is described by four complex numbers, or eight parameters. Thus eight parameters are required to fully describe the data contained in Z. Indeed Z may be described as the sum of two tensors consisting of the real, and complex parts.

$$\text{i.e. } Z = \mathrm{Re}(Z) + i\,\mathrm{Im}(Z) \qquad (4)$$

In a two-dimensional earth (and along axes of symmetry) there is a natural coordinate system defined by the strike of the features, in which only two components of Z are non-zero. In order for this to occur, each of the two tensors of equation **(4)** must have the same principal axes.

For three-dimensional structures, there is no requirement that the principal axes of the real and complex portions of the impedance tensor should be identical. Thus, there is no simple decomposition of the impedance tensor. **As** a consequence most modem processing techniques rely on determining a coordinate system that best approximates a two dimensional situation, by determining a set of axes that gives a compromise between the two possible sets of principal axes. **A** comprehensive approach needs to recognise the full complexity of the impedance tensor with 8 (independent) components all of which contains useful information.

REILLY DECOMPOSITION OF THE IMPEDANCE TENSOR

Reilly (1979) recognised the need for more than a single (real) tensor quantity in order to represent the eight parameters of the impedance tensor. Furthermore, he suggested that, in line with normal definitions of apparent resistivity, any such representation should be able to be equated to physical properties of a uniform, anisotropic earth. In recognition of this, Reilly suggested the use of uniform earth model in which the electrical properties were contained in a complex apparent admittivity tensor γ_{ij}. The admittivity tensor (Ward and Hohmann, 1987) is the sum of two (non-complex) tensors: a conductivity tensor σ_{ij} and a permittivity tensor ε_{ij}.

$$\gamma_{ij} = \sigma_{ij} + i\omega\varepsilon_{ij} \tag{5}$$

In the real earth, the contribution of the terms involving permittivity is very small at the frequencies used in MT, and it is usually equated to zero. However the phase variations produced by complex structures cannot be represented by a uniform earth with anisotropic resistivity. Reilly gets over this problem by extending the concept to include an apparent permittivity in the representation of Z. Thus the measured properties (contained in the impedance tensor) are presented **as** an apparent conductivity and an apparent permittivity tensor.

This means of presentation is analogous to the direct current case, where the behaviour of a three-dimensional earth cannot be represented by simple scalar apparent resistivity (Bibby 1986). The apparent permittivity term is able to represent the phase changes that result from the accumulation of charges on complex structures in the earth, without suggesting that the actual permittivity of the ground is necessarily large, or indeed is variable.

We use the term apparent Permittivity, although it is recognised that in doing so, ε_{ij} is merely a device that enables a measured impedance tensor to be represented by (apparent) physical properties of the earth. This may be considered as a convenient way of repackaging the data contained in the impedance tensor into terms that can be represented by properties of a hypothetical uniform earth.

For presentation purposes, rather than using apparent permittivity tensor ε_{ij}, it is easier to use the tensor parameter $\omega\varepsilon_{ij}$ which has the units of conductivity.

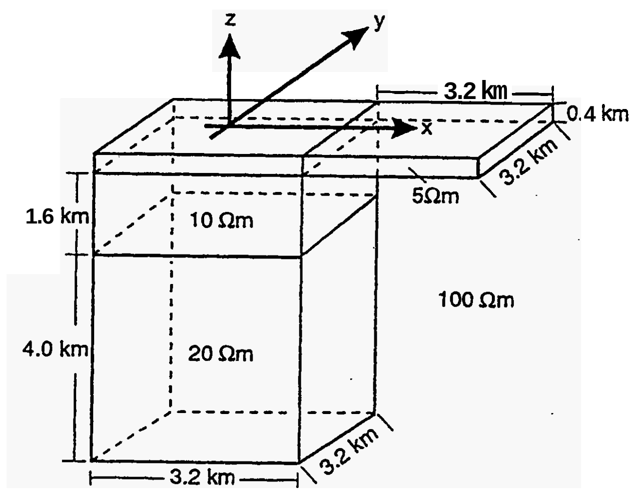
The full theoretical development of the decomposition of an observed impedance tensor Z into the apparent admittivity tensor is found in Reilly (1979) and is only summarised here. The components of the admittivity tensor γ_{ij} can be written **as:**

$$\begin{aligned}
\gamma_{11} &= i\omega\mu\,[Y_{22}Y_{11} - Y^2_{21}] \\
\gamma_{12} &= i\omega\mu\,Y_{22}[Y_{12} - Y_{21}] \\
\gamma_{21} &= i\omega\mu\,Y_{11}[Y_{21} - Y_{12}] \\
\gamma_{22} &= i\omega\mu\,[Y_{22}Y_{11} - Y^2_{12}]
\end{aligned} \tag{6}$$

where Y_{ij} are the components of the inverse of the measured impedance tensor,

i.e. $$\mathbf{Y} = \mathbf{Z}^{-1} \tag{7}$$

(a) ***Perspective view***



(b) Map view

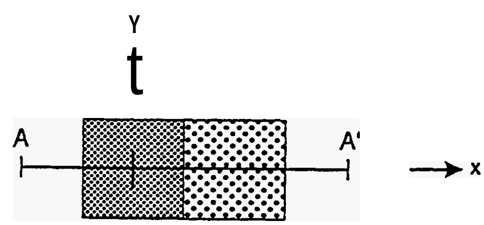


Figure 1: a) Three-dimensional resistivity model of an idealised geothermal system and associated outflow tongue. The geothermal reservoir or upflow zone is represented by the 10 and 20 Ωm bodies. The 5 Ωm body represents the upper part of the reservoir and an outflow in the x direction.
b). The map view, Fig. 1b, shows the profile along which the MT parameters have been illustrated.

Thus the apparent resistivity and the apparent permittivity tensors can be written:

$$\sigma_{ij} = \mathrm{Re}\,(\gamma_{ij})$$

and $$\varepsilon_{ij} = \mathrm{Im}\,(\gamma_{ij})/\,\omega$$

These parameters behave as expected in simple situations such as a horizontally layered earth, or two dimensional structures. In particular, for a layered earth, σ_{ij} is diagonal, and $\omega\varepsilon_{ij}$ is identically zero. We illustrate here the parameters in a more complex geometry.

Decomposition into two real tensors makes it possible to apply standard methods for illustrating these parameters. Thus, the apparent conductivity (or its inverse, apparent resistivity) can be represented geometrically as an ellipse, or by various tensor invariants (e.g. Bibby 1986). It is also worth noting that, with the separation into two tensors, the number of independent invariants for each tensor is three, giving a total number of 6 independent invariants, not seven as suggested by Szarka and Menvielle (1996).

THE THREE-DIMENSIONAL MODEL

The three-dimensional model shown in Fig. 1 is a representation of the resistivity structure of a geothermal reservoir with an associated outflow of thermal fluid. This model was chosen because of the increasing need in New Zealand to be able to distinguish upflow zones from outflow zones. Furthermore, as exploitation moves to deeper levels, improved resistivity images of the deeper parts of the geothermal reservoir and their surroundings will be needed.

The resistivity values used are based upon the resistivity structure of the Ohaaki geothermal system inferred from bipole-dipole surveys (Bibby, 1978). To this has been added an outflow zone. This model is broadly similar to the model used in Pellerin *et al.* (1992) to evaluate the detectability of a reservoir or 'upflow zone' lying beneath **a** more extensive area of low resistivity. The conclusion of the Pellerin ***et*** *al.*(1992) study of several different EM techniques was that LOTEM and MT methods are capable of detecting an underlying reservoir. Three-dimensional effects dominate the response and the signal due to the underlying reservoir is very weak. A similar modelling study conducted by ourselves (Caldwell and Bibby; 1993, 1995) showed that a DC multiple-source bipole-dipole survey would also detect the reservoir.

Computational Method

All calculations were made using the 3D integral equation modelling method described in Xiong (1992) and Xiong and Tripp (1995). The electric and magnetic fields were computed at 25 values of time, logarithmically spaced between 1 ms and 10 s. Field vectors were calculated at 200m intervals along the 9.6 km profile, AA'.

MODELLING RESULTS

For comparison purposes, results are shown for the profile AA' of Fig. 1, because this line crosses the boundary between the upflow and outflow regions of our model geothcrmal system. This profile is along the axis **of** symmetry of model and, as a consequence, the impedance tensor can be resolved into T_E (electric field perpendicular to the profile) and T_M (electric field parallel to the profile) modes. In the terms of the previous discussion, the principal axes of the two parts of the tensor are the same (the tensor behaves like that of a two dimensional model). Relative to a set of axes parallel and perpendicular to the line of symmetry, the tensor contains only four (two complex) components. In field examples such simple symmetry seldom occurs.

Traditional Analysis

The normal presentation of such data is in the form of pseudo-sections (Fig. 2) in which various parameters are plotted against period. As the period of the electromagnetic waves increases the penetration also increases. A pseudo-section of apparent resistivity therefore gives an image that can be compared with the known resistivity - depth structure of the model. The apparent resistivities for the two modes are shown in Fig. 2a, c and the corresponding phase angles are given in Fig. 2b, d. When the electric field is perpendicular to the profile (T, Fig. 2a, b) the near surface structure dominates so that the apparent resistivity pseudo-section does not show any structure below the superficial layers. The deeper structure becomes clearer when the electric field is parallel to the profile; that is, it is perpendicular to the boundary of the geothermal plume beneath the outflow (Fig. 2c, d). The phase pseudo-sections also show indications of the deep structure, with phases greater than 45° (indicative of conductors) occumng in the region of the geothermal plume.

An alternative way of processing MT data uses the determinant of the impedance tensor (Fig. 2e, f) which is an invariant (Ranganayaki 1984). This parameter has been used for analysis of MT data at Ohaaki geothermal field by Ingham (1989). For a two-dimensional situation, the apparent resistivity derived from the determinant is equal to the geometric mean of the apparent resistivity parallel and perpendicular to strike. Both the apparent resistivity and phase (Fig. 2e, f) show indications of the deeper structure, although the signature is not strong. The advantage of the use of the determinant is in the elimination of some of the extreme values,

Reilly decomposition

There are a number of ways of illustrating the results from the Reilly decomposition. Each of the tensors (σ_{ij} and $\omega\varepsilon_{ij}$) may be treated independently to derive principal axes, extreme values, or other tensor invariants (as is done with multiple-source apparent resistivity tensors). Fig. 3 shows three of these. In this analysis, we have derived the apparent resistivity tensor (σ_{ij}^{-1}), so as to be consistent with the traditional analysis. The apparent permittivity is shown as $\omega\varepsilon_{ij}$, which has the dimensions of conductivity. For a uniform earth $\omega\varepsilon_{ij}$ = 0, so that for this model the background apparent permittivity is zero.

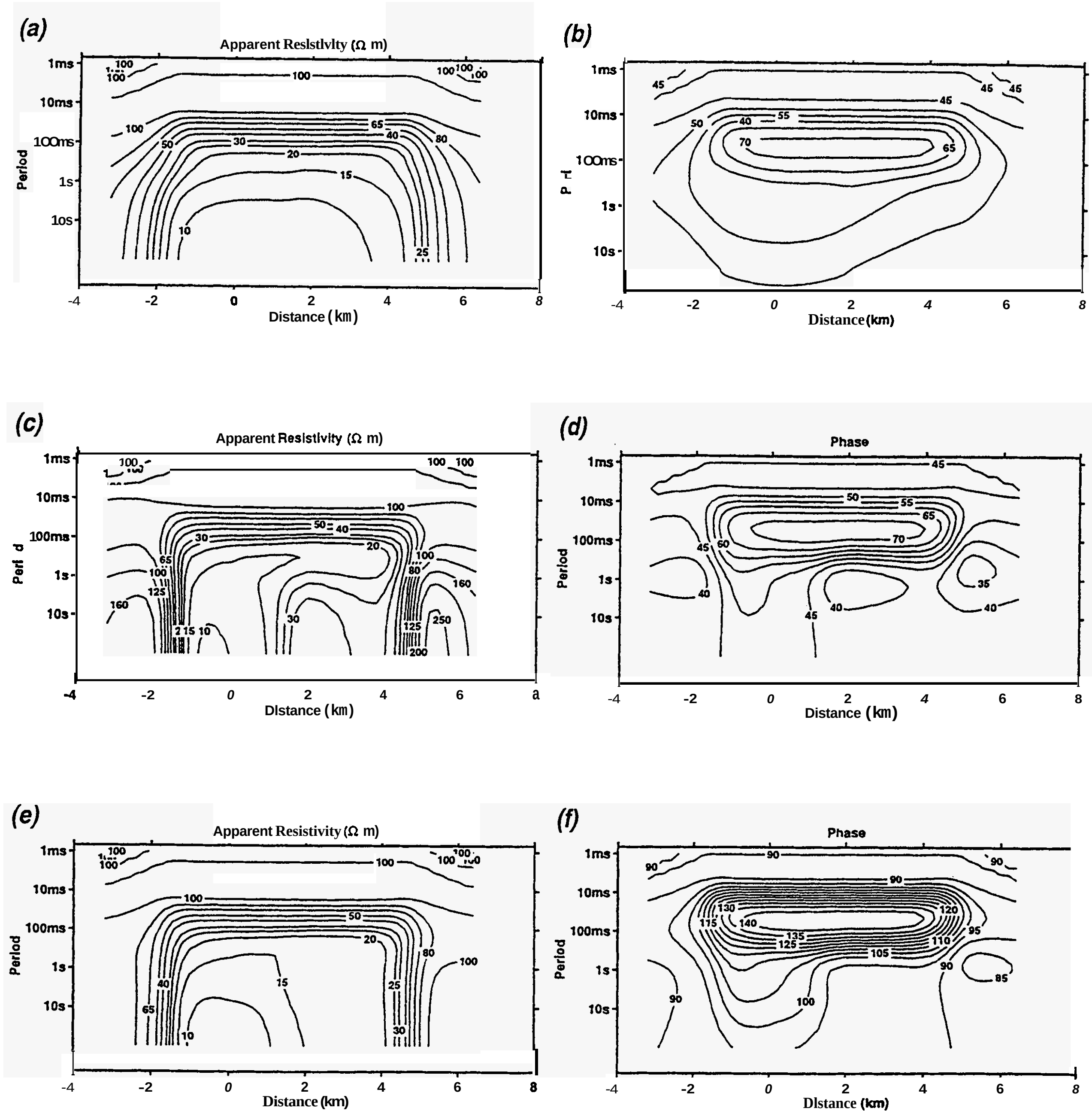


Figure 2: Illustrations of the traditional representation of MT data as pseudo-sections along profile AA '(Fig. 1.) For this profile along the axis of symmetry the data can be split into T_E and T_M modes.
a), b) Contoiirs of apparent resistivity and phase angle for the T_E mode (magneticfield parallel to the profile).
c), d) Contoiirs of apparent resistivity and phase angle for the T_M mode (electricfield parallel to the profile).
e), f). Apparent resistivity and phase angle based on the determinant of the impedance tensor (an invariant). Note that the phase is approximately twice the usual phase angles.

In DC resistivity, the second invariant of the apparent resistivity tensor $P_2=[\det \rho_{ij}]^{1/2}$ is a good indicator of anomalous structure. Fig. 3a, b shows this parameter for both the apparent resistivity and apparent permittivity tensors. Not surprisingly, P_2 shows similar characteristics as the determinant of the impedance tensor (Fig. 2e), as both represent mean values of the apparent resistivity. The apparent permittivity, however, shows very clear characteristics that arc related to the structures in the model.

Figs 3c, d and 3e, f show the extreme values of the tensors. The maximum values of apparent resistivity would be expected, in this instance, to closely approximate the traditional T_M mode, as is confirmed from a comparison of Fig. 2c and 3c. Similarly, the minimum value produces similar characteristics to the T_E mode. The apparent permittivity tensor, being related to the complex part of Z, would bc expected to exhibit some of the characteristics of the phase pseudo-sections. In all the examples shown, the

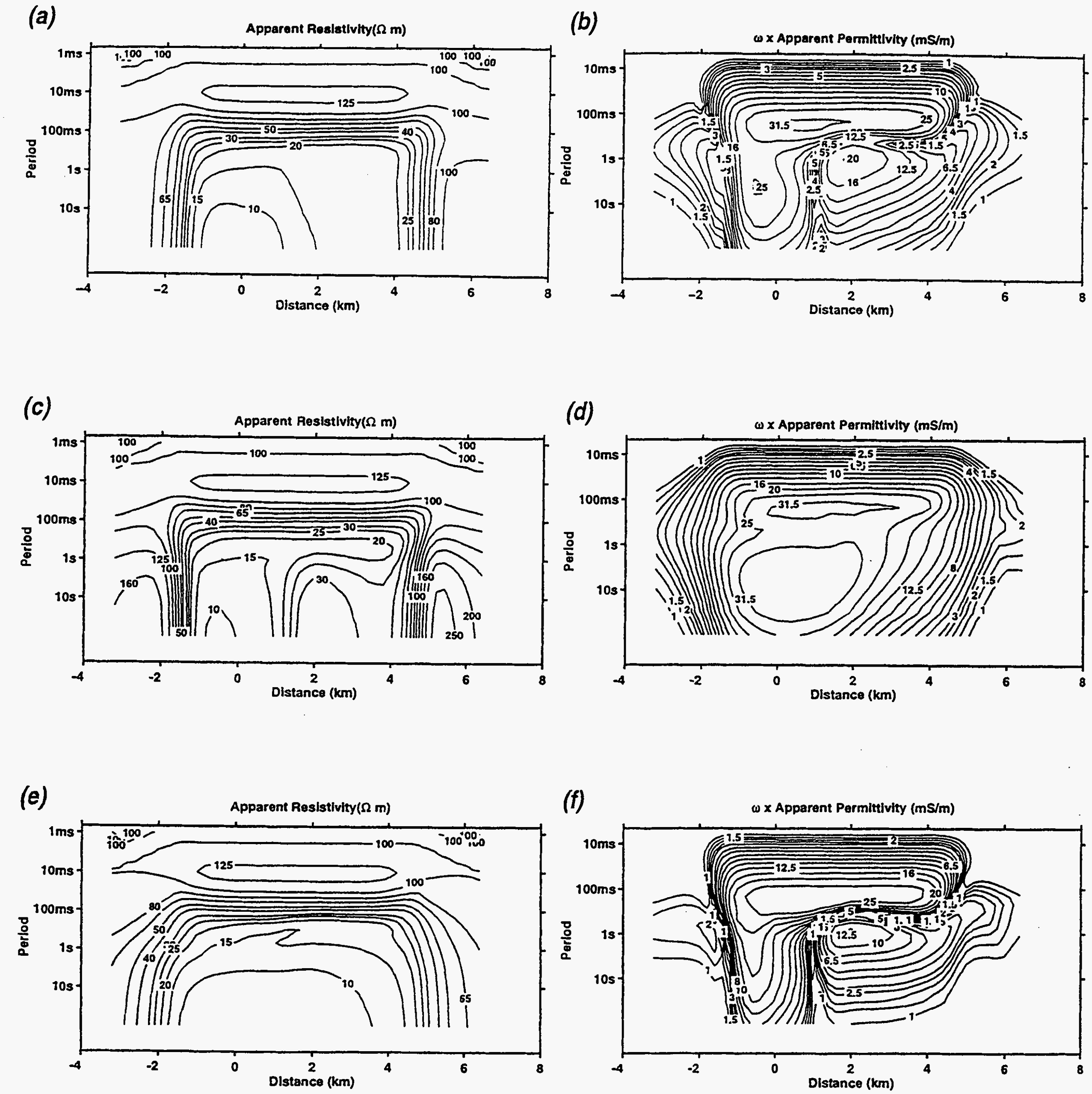


Figure 3. Three forms of representation of the apparent resistivity tensor and apparent permittivity tensor derived from the Reilly decomposition of the MT data along profile AA'. For presentation of the apparent permittivity, the tensor $\omega\varepsilon_{ij}$ is used. This has the dimension of conductivity.
a), b) The second invariants (square root of the determinants) of the apparent resistivity and normalised apparent pemittivity tensor respectively.
c), d) Maximum of the tensors.
e), f) Minimum of the tensors.

apparent permittivity appears to outline the features of the model extremely well.

Of the three tensor invariants shown, the minimum value of apparent permittivity (Fig. 3e) gives very rapid gradients which seem to reflect the changes of resistivity in the model, and thus provides an image of the disturbing body. For horizontally layered structures, the phase and apparent resistivity are related, so that to a first approximation, the departure of the phase from background (45°) is related to the gradient of the local sounding curve ($\delta(\log\rho_a)/\delta(\log T)$, Jiracek et al., 1995). Inspection of the contours of Fig. 3 suggests a similar relationship may exist between the gradient of apparent resistivity and the apparent permittivity. In particular, the highest values of apparent permittivity in Fig. 3f correspond to the greatest gradients of the maximum apparent resistivity (Fig. 3c).

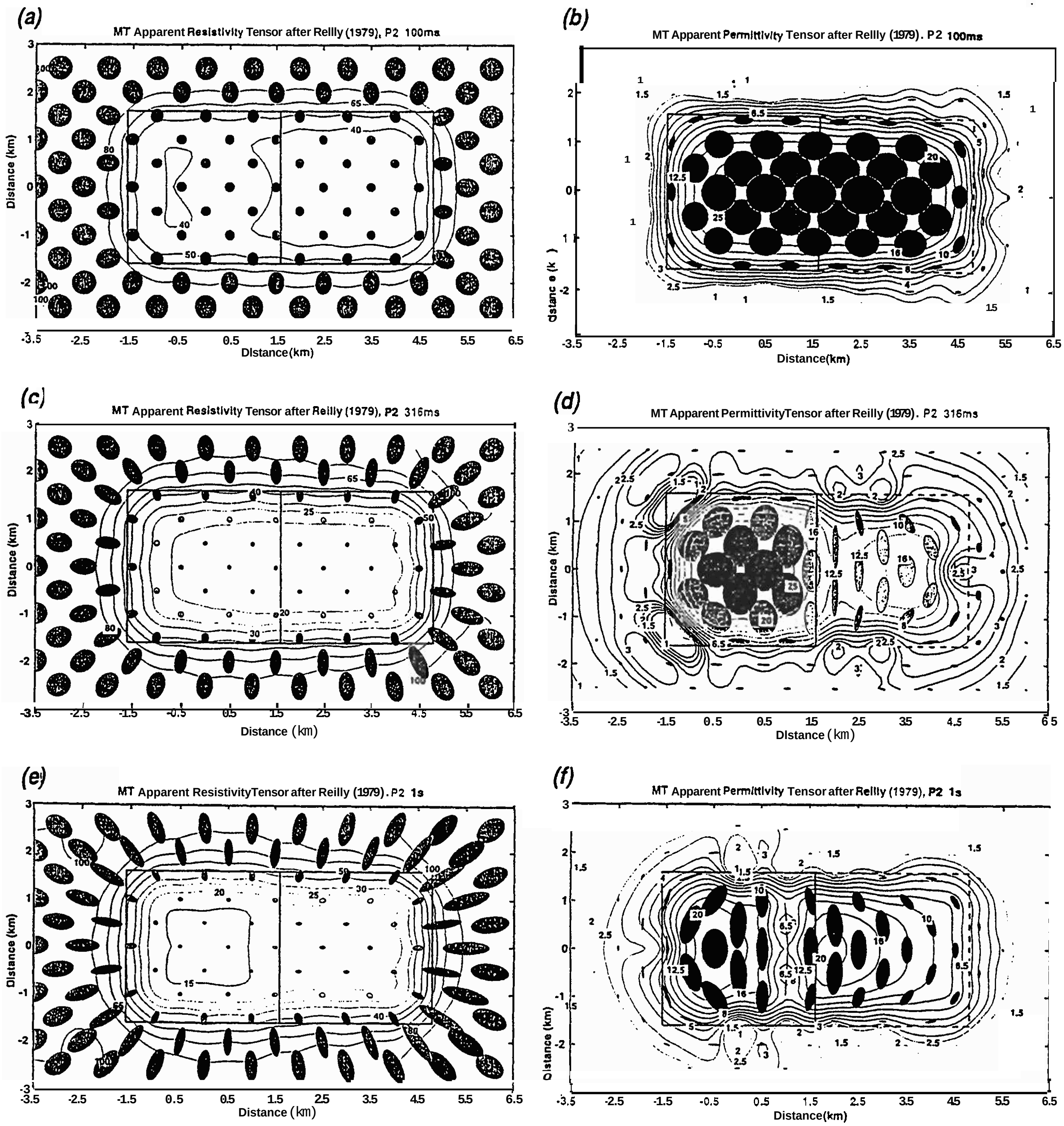


Figure 4. Elliptical representation of the apparent resistivity and apparent permittivity ($\omega\varepsilon_{ij}$) tensors at three periods together with contours of the second invariant (P_2), which is equal in value to the mean radius of the ellipses. Periods shown are 100 ms (a, b) 316 ms (c, d) and 1 s (e, f).

The other means of pictorially representing the tensor quantities is as apparent resistivity and apparent permittivity ellipses. These are shown for three periods (100 ms, **316** ms and 1s) in Fig. 4. As penetration increases with increasing period, these can be regarded as sections of increasing depth.

The apparent resistivity tensor behaves in a similar manner to MT polar diagrams. When passing from a resistive to conductive material the major axis of the ellipse will be oriented close to perpendicular to the boundary between the materials. At 100 ms, (Fig. 4a) the upper surface of the geothermal system is just beginning to be observed. By 316 ms (Fig. 4c), the upper layer is clearly shown, and the boundaries are clearly indicated by the size and orientation of the ellipses. By 1 s, the first influence of the deeper structure is becoming apparent (Fig. 4e) with lower values of P_2 apparent resistivity (contours) occurring on the left side.

The apparent permittivity shows highly diagnostic variations. While, at 100 ms, the apparent resistivity is only just beginning to sense the geothermal system, the apparent permittivity shows high values in the centre and clearly delineates the body. Note that the permittivity ellipses are greatest size inside the conductor, and they are aligned parallel to the boundary (that is the minimum is approximately parallel to the maximum of the apparent resistivity tensor). By 316 ms (Fig. 4d) two distinct regions are delineated, and the plume structure at depth is clearly shown. Again, the orientations of the ellipses are diagnostic, with strong ellipticity indicating the boundary zone. At 1 s, the two regions can be distinguished although the contrast is not **as** great. At greater periods (not shown here), the shape of the plume structure becomes clearer. **Thus,** the apparent permittivity appears to reflect changes in structures at earlier times than the apparent resistivity, and provides patterns that are highly diagnostic for interpretation.

DISCUSSION AND CONCLUSIONS

Although the method of decomposition of the MT impedance tensor was proposed in 1979 (Reilly 1979), the lack of three-dimensional modelling algorithms at that time meant that no previous study has been made to demonstrate the advantages (or otherwise) of such a decomposition. The Reilly technique recognises some of the complexity of the impedance tensor that tends to be ignored in traditional MT analysis. In full generality, the impedance tensor has eight independently measured parameters. Reilly (1979) represents these as parameters that have physical analogues. Furthermore, this technique is best suited to analysis of three-dimensional structures of which geothermal systems are probably the best known example, and among the most difficult to investigate.

Apparent resistivity tensor analysis is used for direct current measurements (Bibby, 1986), and one of the parameters used here is the MT equivalent of the DC apparent resistivity tensor. The use of an apparent permittivity tensor is **a** device which allows the observed behaviour to be explained in terms of properties of an anisotropic earth, although it does not necessarily correspond to realistic properties. It can be thought of as an analogue to a reactance produced by a capacitance in circuit theory. Despite the tenuous link to real earth properties, the apparent permittivity has been shown to be a particularly useful parameter for use alongside the apparent resistivity. It appears to be highly diagnostic of resistivity boundaries in three dimensions.

As yet this analysis has not been applied to measured MT data. It is now important to assess the ability of this technique to deal with the complications found in real data. In particular, it is important to assess the influence of static shift on the tensors, and determine whether such effects are easily recognised.

This paper has served to introduce an alternative approach to the analysis of MT. Further testing is still required to establish its viability and its effectiveness in geothermal applications.

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