

1-D GEOTHERMAL MODELS - THE GOOD, THE BAD AND THE UNLIKELY

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SUMMARY - Recent results in the analysis of geothermal heat pipes are used to assess conceptual steady-state models of two-phase geothermal reservoirs. It is found that some models are good (possible and likely), some are bad (impossible), and some are unlikely, when permeability is uniform and flow is 1D. Shocks or boundary layers in saturation play an important role in this analysis. In particular, the popular model of liquid condensate overlying a vapor-dominated heat pipe is unlikely, highlighting the importance of permeability changes in actual reservoirs.

1 INTRODUCTION

There have been significant recent advances in the understanding of steady counterflow of steam and liquid in geothermal reservoirs. The term *heat pipe* is used here to mean the mass flows of liquid and steam are roughly equal, and in opposite directions. This allows efficient heat flow with little or no net mass flow (eg [5, 13, 7]). Two complementary approaches are considered here. The first is one in which the differential equations governing the time-varying flow are re-arranged to take advantage of the method of characteristics for dealing with shocks in the liquid saturation. No capillary pressure effects are accounted for. The second is one in which the steady-state equations are integrated once, and rewritten as two coupled first-order ordinary differential equations. With capillary pressure present, saturation shocks are resolved as diffusive boundary layers.

The method using shock theory finds its natural expression in terms of a *flow plane*, with axes proportional to the through-flow of mass and heat.

In the capillary boundary layer method, solutions are plotted in a temperature-saturation *phase plane*, with fixed mass and heat through-flow.

Both methods indicate aspects of steady-state solutions of 1D two-phase reservoirs. Permeability is taken to be constant everywhere. We expect the effects of varying permeability to be quite significant, and this will be the subject of future work.

In particular we examine

the popular model of liquid condensate overlying a vapor-dominated region. This is impossible for the usual choices of heat flow and permeability in a 1D uniform reservoir. It is possible but highly unlikely for larger values of heat flow. This model is an important part of the seminal paper by White *et al* [14] on vapor-dominated systems. Straus and Schubert [9] use this model for The Geysers in northern California and Kawah Kamojang in West Java. It is also the basis of a pivotal study of the

gravitational stability of liquid over a vapor-dominated region ([10]).

- the model of a liquid-dominated heat pipe above a vapor-dominated heat pipe. Such a steady situation is impossible in a uniform 1D reservoir.
- the model with a deep brine underlying a vapor-dominated region. This is also part of the models of White *et al* [14] and Truesdell and White [12]. This model is only possible for small heat flows, or for small enough permeability that conductive effects are important. Recent computer studies by Lai *et al* [2] have an example of a 2D convecting deep liquid under a vapor-dominated heat pipe with flow that is 1D.
- good models, including pure vapor under a vapor-dominated region, as postulated for the Geysers by Truesdell [13] and supported by the results in Pestov [6], and pure liquid overlying a liquid-dominated region.

In the following sections, we briefly review theoretical results, then discuss their implications for the models above.

2 GOVERNING STEADY STATE EQUATIONS

In a single-phase state, mass and energy flows are of the form

$$J_M = J_\alpha, \quad J_E = h_\alpha J_\alpha - XVT, \quad \alpha = w, s \quad (1)$$

for liquid (water, or *w*) and vapor (steam, or *s*) respectively; while for a two-phase state

$$J_M = J_w + J_s, \quad J_E = h_w J_w + h_s J_s - XVT \quad (2)$$

For one-dimensional steady flow J_E and J_M are constant in both time and space.

In the two-phase case the Darcy flows of liquid and vapor are given by (*z*-axis vertically downward)

$$J_\alpha = -\frac{kk_{r\alpha}}{\nu_\alpha}(\nabla P_\alpha - \rho_\alpha g) \quad \alpha = w, s \quad (3)$$

In the single phase case the relative permeability factor $k_{r\alpha}$ is omitted and the suffix can be removed from the pressure

P_α . The relative permeabilities are assumed to be monotonic functions of saturation. Their forms are generally not known. In this paper we shall assume, for convenience, that they satisfy a commonly adopted normalizing condition $|k| = k_{rw} + k_{rs} = 1$ (see eg [9]). For numerical computations we employ the linear functions with zero residual saturations, and thermal conductivity λ is taken to be $1 \text{ W/m}^\circ\text{C}$. Various constant values of heat flow and permeability are considered, over ranges of interest in the geothermal context.

Capillary pressure is taken to be

$$P_c(S, T) = P_s - P_w, \quad (4)$$

and the particular form for P_c is kept general at this point. Vapor-pressure lowering (the Kelvin effect) is also represented in a general way as (after [1])

$$P_s = f_{vpl}(T, S) P_{sat}(T) \quad (5)$$

where the vapor-pressure lowering factor is approximated by

$$f_{vpl} = \exp \left\{ \frac{m_w P_c(S, T)}{\rho_w R(T + 273.15)} \right\}, \quad (6)$$

and where P_{sat} is the saturated vapor pressure of bulk liquid, obeying the Clausius-Clapeyron relation

$$\frac{dP_{sat}}{dT} = \frac{P_w \rho_s h_{sw}}{(T + 273.15)(\rho_w - \rho_s)}. \quad (7)$$

See also [8, 4, 5] for these.

3 THREE DIAGRAMS: THE FLOW PLANE, THE CHARACTERISTIC DIAGRAM, AND THE PT DIAGRAM

In this section we aim to show why certain phase transitions are allowed in one-dimensional steady-state flow and others not. The arguments have a purely local application: this means in particular that they hold irrespective of whether permeability, conductivity etc vary with depth. This paper is concerned primarily with the case of zero net mass flux (heat pipe), however in this section it is useful to broaden the argument to include non-zero mass flow. We concentrate on a particular example of a phase transition, liquid water over two-phase fluid. Other cases will be considered in the next section, and further detail will be found in [16].

Further, we assume in this section that capillary pressure can be ignored so that $P_w = P_s = P$. The pressure gradient may then be eliminated between equations (2) and (3) yielding an equation connecting the relative permeability function $k_{rw} \equiv x$, the mass and energy flows J_E, J_M and pressure:

$$x(1-x) = (1-x)w - xs \quad (8)$$

where the dimension-less coefficients w, s are linear in J_E, J_M and are given by

$$w = \frac{-\nu_w}{g\Delta h\Delta\rho} \left[\left(\frac{J_E}{k} \right) - h_s \left(\frac{J_M}{k} \right) - [(J_M/k)\nu_s - \rho_w g] \left(\frac{\lambda}{k} \right) \frac{dT_{sat}}{dP} \right] \quad (9)$$

$$s = \frac{-\nu_s}{g\Delta h\Delta\rho} \left[\left(\frac{J_E}{k} \right) - h_w \left(\frac{J_M}{k} \right) - [(J_M/k)\nu_w - \rho_w g] \left(\frac{\lambda}{k} \right) \frac{dT_{sat}}{dP} \right] \quad (10)$$

and $Ah = h, -h_w, Ap = \rho_w - \rho_s$. Note that w, s are pressure dependent through the thermodynamic functions, but that temperature dependence can be eliminated by using the Clausius-Clapeyron relation (7).

3.1 The flow-plane

The flow-plane (Fig. 1) is the plane of the coordinates w, s . From equation (8) the constant saturation contours are represented by straight lines in this plane. It may be shown ([15]) that these lines can be characterized as the line envelope of a certain curve Γ lying in the fourth quadrant of the flow-plane between the points $W(0,1)$ and $S(0,-1)$. Two-phase flow is only possible for mass-energy states which can be accessed by a saturation line, that is, the first quadrant Q_1 , the third quadrant Q_3 , and the region A in the fourth quadrant.

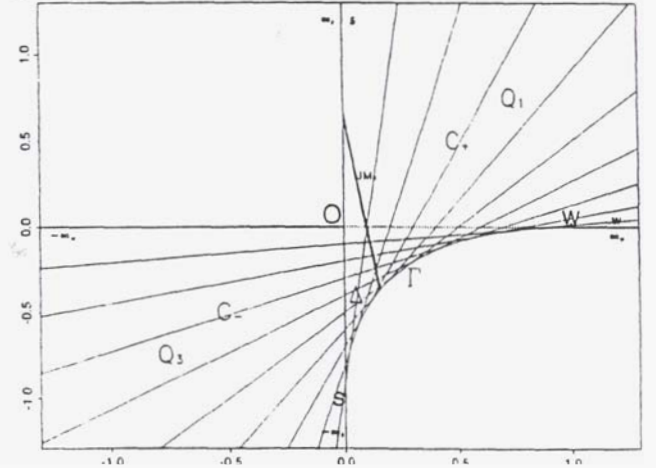


Figure 1 – The flow-plane, ruled with lines of constant saturation.

3.2 The characteristic diagram

Another effect of ignoring capillarity in the dynamic situation is that the saturation variable satisfies a hyperbolic equation rather than a parabolic one, and in particular, a saturation wave speed C vector can be then identified. Furthermore, this wave speed is still defined in the steady-state. Fig. 2 shows the saturation characteristic diagram with z positive (downwards) to the left. The figure shows two characteristics intersecting a boundary. The sign of the wave speed is the same as the sign of the characteristic slope. Suppose saturation data is assigned at the boundary. Then the characteristic for which $C < 0$ will carry (different) saturation information into the boundary, and there will in general be a saturation discontinuity (a shock) there. On the other hand, the characteristic for which $C > 0$ will carry information smoothly away from the boundary, and there will be no shock. Later in this section we will interpret the boundary as a phase boundary between

a two-phase and a single-phase region.

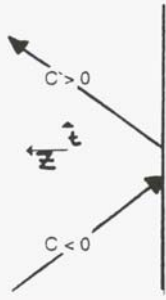


Figure 2 – The characteristic diagram for saturation, showing the possibilities at a boundary.

3.3 Back to the flow-plane

The exact expression for C is given in [15]. It can be shown that $C = 0$ along the curve Γ in the flow-plane. Hence each saturation line is divided into two parts, one with $C < 0$, the other with $C > 0$, and $C = 0$ at the point of tangency with Γ . This suggests that the two-phase part of the plane can be described naturally in terms of two overlapping sheets: on the upper sheet $C_+ = Q1 \cup A$ the wave speed is positive, while on the lower sheet $C_- = Q3 \cup A$ it is negative. The two sheets overlap in A and are in contact along Γ . Each mass-energy flow state in the overlap region has two saturations, corresponding to the two tangents which can be drawn to Γ . In the absence of conduction A corresponds to the counterflow region of the flow-plane, whereas in $Q1$ ($Q3$) both phases flow downwards (upwards). When conduction cannot be ignored this picture becomes more complicated ([15]), but the same general considerations apply. From equation (8) we can see that $x = 1$ along the w -axis, and $x = 0$ along the x -axis. This corresponds to liquid (vapor) conditions respectively. Note, however, that liquid conditions are obtained on the lower sheet for the left-half line $(-\infty_w, 1)$ as far as W , and on the upper sheet for the right half-line $(1, \infty_w)$. Similarly there are vapor states on the lower sheet in the range $(-\infty_s, -1)$, and on the upper sheet in the range $(-1, \infty_s)$.

Note that states with small mass and energy flows near the flow-plane origin pick out high liquid saturations if they are on the lower sheet, and high vapor saturations if they are on the upper sheet. For this reason the lower sheet can be referred to as **liquid dominated** (or wet) and the upper sheet as **vapor dominated** (or dry). This is only strictly true for small flow rates. For large flow rates low liquid saturations may be encountered on the wet sheet, and low vapor saturations may be encountered on the dry sheet.

From equation (10) if mass flow is held constant while energy flow is allowed to vary, then a straight line in the flow-plane is obtained. Fig. 1 shows the locus $J_M = 0$. A heat pipe is represented by a series of points along this line.

3.4 Saturation shocks in the flow-plane

Since mass and energy flow (and pressure) are continuous across a steady-state shock (these are sometimes referred to

as the Rankine-Hugoniot conditions) it follows that a shock can be represented in the flow-plane as a jump from one state to another which retains the same flow coordinates. Jumps to single-phase states are possible from the quadrants $Q1$ and $Q4$, and within A a jump between C_+ and C_- is also possible. Note that if there is a transition to single-phase at a boundary state, eg along the boundary $(1, \infty_w)$ on the upper sheet then this must count as a smooth **transition** to single-phase liquid since $x = 1$ there. On the other hand transitions to single-phase from the interior of the two-phase region must be counted as **saturation** shocks.

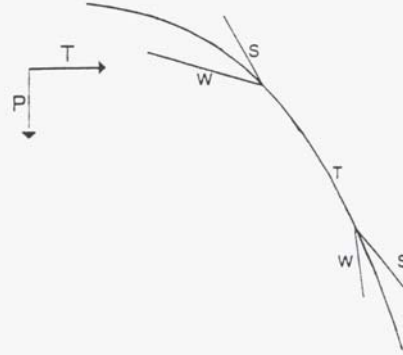


Figure 3 – Possible intersections of the Clausius-Clapeyron curve and single-phase pressure-temperature lines.

3.5 The PT diagram

Fig. 3 shows the Clausius-Clapeyron curve $P = P_{sat}(T)$, and some single-phase PT curves which may intersect it. For a single-phase liquid

$$\left(\frac{dP}{dT}\right)_w = \frac{\lambda(-\frac{\nu_w}{k} J_M + \rho_w g)}{h_w J_M - J_E} \quad (11)$$

In the case of a phase transition involving liquid (W) over two-phase (T) we must have

$$\left(\frac{dP}{dT}\right)_w \geq \frac{dP_{sat}}{dT} \quad (12)$$

3.6 All 3 diagrams together

If this thermodynamic inequality is examined using equations (8) and (10), we find that in terms of the flow-plane it is equivalent to

$$s \leq 0 \quad (13)$$

This means that the two-phase state immediately below the phase boundary must lie either on the lower sheet $C < 0$ of the flow-plane, or it could lie in the region A of the upper sheet $C > 0$. If we re-examine the characteristic diagram we see that transitions of the type WT with $C > 0$ must be smooth, hence on the upper sheet the only allowable states for the phase transition must lie along the line $(1, \infty_w)$. These smooth transitions correspond to positive mass and energy flows (downflows). In particular for zero net mass flow (a heat-pipe) there can be no phase transition from the upper sheet. Thus the only WT transitions for a heat pipe are those from the lower sheet. For small flow-rates this implies a jump transition from a single-phase liquid to a liquid-dominated two-phase state.

A jump transition from the liquid to a vapor-dominated two-phase state is possible, but only for large values of the heat flow. At 250°C the minimum liquid saturation obtainable (assuming piecewise linear relative permeabilities with zero residual saturations) is about .3, and with a permeability of 10 md the corresponding heat flux is 70 W/m². The natural state heat flux at the vapor-dominated geothermal reservoir The Geysers was about 2 W/m². At 250°C the (lower sheet $C < 0$) liquid saturation corresponding to this steady-state heat flux is .99, or liquid-dominated. Thus the steady-state at The Geysers cannot be modeled as a one-dimensional heat pipe of liquid water over two-phase fluid (as was done in [9]).

4 THE PHASE PLANE

In this section, an alternative and equivalent view is taken, in which shocks are resolved as capillary boundary layers. As in [3], the steady-state equations (2) may be rewritten in the form

$$\frac{\partial P_s}{\partial z} = -\frac{\mathcal{H}(P_s, S)}{\mathcal{F}(P_s, S)}, \quad (14)$$

$$\epsilon \frac{\partial S}{\partial z} = \frac{\mathcal{G}(P_s, S)}{\mathcal{F}(P_s, S)} \quad (15)$$

where pressure has been scaled to the range 0–1. For k/λ values greater than $10^{-15} \text{ } ^\circ\text{C m}^3/\text{W}$, convective effects dominate conduction, the sizes of terms in \mathcal{H} and \mathcal{G} are comparable, and ϵ is much less than one. Solutions may be plotted as T against S , tracing out trajectories in the (S, T) phase plane; or separately against depth z in the usual way. Solutions have two parts, an outer solution which takes up most of the heat pipe, and an inner boundary layer that is narrow, in which capillary effects are important. There may be more than one boundary layer present in a given solution trajectory. The boundary layer resolves the shocks of the previous section, and an analysis of these boundary layers is equivalent to an analysis of shocks using the Rankine-Hugoniot conditions.

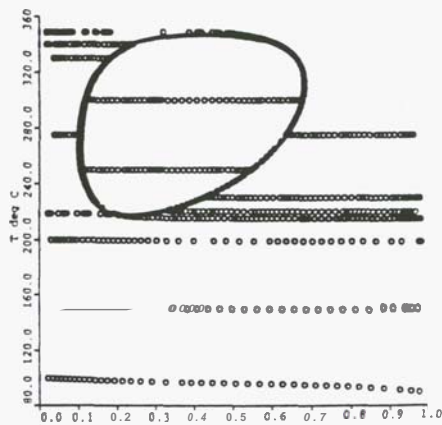


Figure 4 – Steady heat pipe solutions in phase space, for $Q/k = 6 \text{ kW/dm}^2$.

Outer solutions are given by the algebraic relationship $\mathcal{G} = 0$. This is equivalent to the flow-plane equation (8), but the coordinates in which solutions are viewed are different. For a given value of Q/k (and λ), this leads to a curve or curves in the (S, T) phase plane. Solutions move rapidly (in z) through boundary layers, then track close to the $\mathcal{G} = 0$ curves, as illus-

trated in Fig. 4 for $Q/k = 6 \text{ kW/dm}^2$. The $\mathcal{G} = 0$ relationship has been visualized as a surface in three dimensions ($S, T, Q/k$) in Fig. 5, for various values of k , and $\lambda = 1 \text{ W/m}^\circ\text{C}$. Each surface, corresponding to a different permeability, has level contours which are the $\mathcal{G} = 0$ curves for a given Q/k value. These visualizations were performed with the package Iris Explorer, bundled with the Silicon Graphics Iris Indigo workstation, which allows interactive real-time manipulation of these images in 3D.

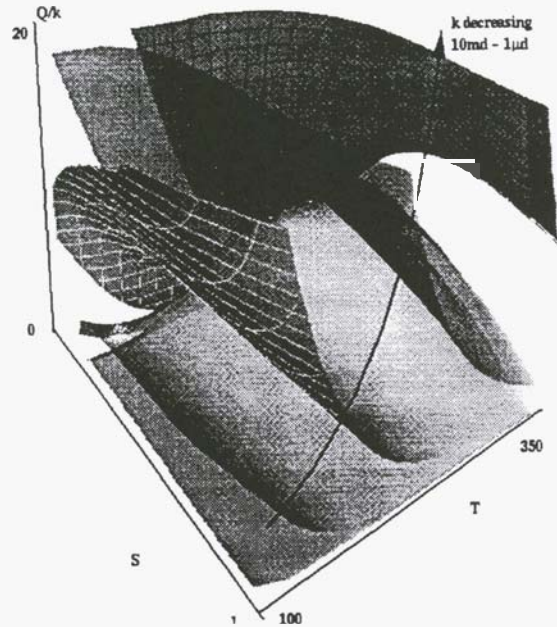


Figure 5 – The $\mathcal{G} = 0$ relationship viewed as a surface, for various permeabilities.

A particular steady heat pipe solution lives on a horizontal slice of constant heat flow in this figure, with a trajectory that depends on initial conditions, and moves through narrow boundary layers to track the $\mathcal{G} = 0$ curve that corresponds to slicing through the surface with the appropriate value of permeability. Hence this figure, and blown up versions of it as required, allow immediate and general conclusions about which steady solutions are possible and which are not. General conclusions are independent of particular choices of relative permeability curves. Particular conclusions about a given numerical simulation depend strongly on such choices, and on initial conditions.

Solutions in the phase plane are unique, since the right-hand sides of the differential equations (14) and (15) are continuously differentiable with respect to the thermodynamic variables P_v (or T) and S . Hence solution trajectories cannot cross each other in the phase plane.

5 DISCONTINUOUS SOLUTIONS?

One critical point of the analysis here is the possibility that there is a jump in saturation between a two-phase region and a single-phase region. At such a point, the governing differential equations change. We argue that such a jump must obey the two-phase restrictions of (equivalently) the Rankine-Hugoniot two-phase shock conditions, or the directionality

imposed by two-phase capillary boundary layers.

A contact between a two-phase and a single-phase region must have continuous pressure and temperature through the contact point. Discontinuities are not supported, as they diffuse away due to the second derivative in pressure, and due to Fickian diffusion in temperature, in the governing differential equations. Hence the state of the fluid at the contact point must satisfy two-phase conditions, as the liquid phase is in thermodynamic equilibrium with the steam present at the interface.

This means that the saturation value (0 or 1) at the interface must exist in the two-phase region, so that any analysis of solutions inside the two-phase region applies right up to and including the interface with single-phase fluid. In particular, solutions in the phase plane must reach either $S = 0$ or $S = 1$ at the interface.

6 MODELS

The model with liquid condensate overlying a vapor-dominated heat pipe has been popular. The phase plane solutions must, in this model, reach $S = 1$ at the cooler end of the heat pipe. Fig. 6 shows solution trajectories for a variety of values of heat flow and permeability.

For large heat flow values ($Q/k > 1 \text{ kW/dm}^2$) and $k > 1 \text{ md}$, there is only one special value of temperature at the interface for which a solution trajectory reaches from $S = 1$ to the vapor-dominated branch, illustrated in Fig. 6(a) for several values of Q by the larger symbols. There is another restriction on temperature; given the heat flow through a quiescent liquid layer with a surface temperature of say 20°C , as illustrated in Fig. 7 for various values of Q , there is a specific temperature at which the conductive temperature gradient meets the Clausius-Clapeyron curve. This must happen at the interface between liquid and two-phase regions. Both of these temperatures must agree. They are plotted against heat flow for the case $k = 1 \text{ d}$ in Fig. 8. They intersect at a heat flow between 1 and 2 kW/m^2 . Only for this single value of heat flow is this configuration possible, given the temperature 20°C at the surface where liquid meets atmosphere. Such a configuration is highly unstable to perturbations of heat flow or boundary temperatures and pressures or model length.

These comments are invalid when a 2D flow commences in the overlying liquid condensate region. Then it will be easy to match up a 20°C temperature (say) at the surface with a particular temperature at the interface. Nevertheless, this can only happen for heat flow values larger than typically experienced in vapor-dominated reservoirs.

For smaller values of heat flow ($Q/k < 1 \text{ kW/dm}^2$) and $k > 1 \text{ md}$, there are no phase plane trajectories that cross from the vapor-dominated branch to $S = 1$ at the cooler end. For very small values of heat flow, the liquid-dominated branch disappears, and there is a boundary layer joining the vapor-dominated branch with $S = 1$, but the direction of the boundary layer requires it to be at the hotter (deeper) end of the heat pipe.

For intermediate values of heat flow, the liquid-dominated

branch appears between the vapor-dominated branch and $S = 1$, getting in the way (solutions cannot cross in the phase plane) and preventing any boundary layer from connecting the two regions.

A heat pipe with a liquid-dominated section underlying (at higher temperatures than) a vapor-dominated section is clearly possible, from the solution trajectories in Figs. 4 and 6. However, given the heat flow, such a trajectory has a special relationship between temperature and saturation at its endpoints, and will be difficult to obtain if boundary conditions are to fix these at one or both ends.

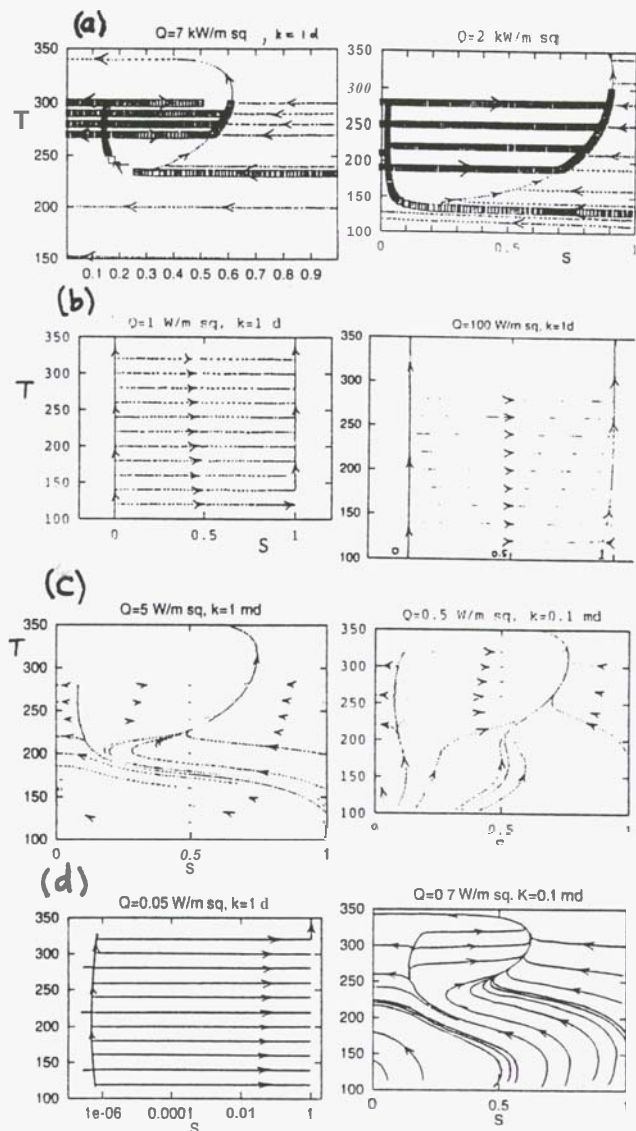


Figure 6 – Steady solution trajectories in the temperature-saturation phase plane, for a variety of values of heat flow and permeability. Larger symbols show solutions that do reach from the vapor-dominated heat pipe to $S = 1$.

Arrows indicate the direction of increasing depth z along trajectories.

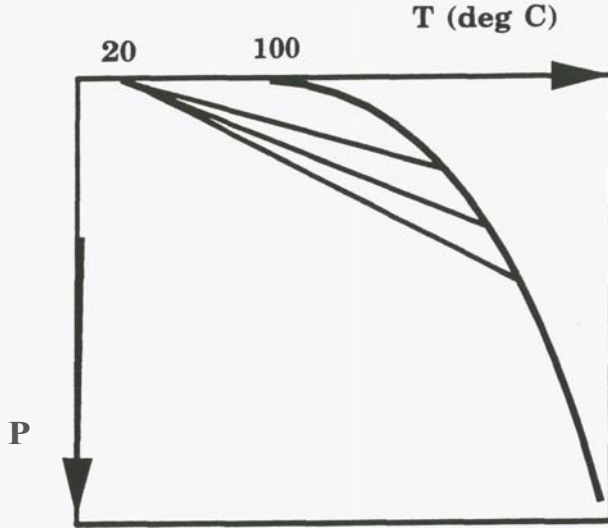


Figure 7 - A sketch of possible intersections of the pressure-temperature curve for a liquid condensate layer with the Clausius-Clapeyron curve for an underlying two-phase region.

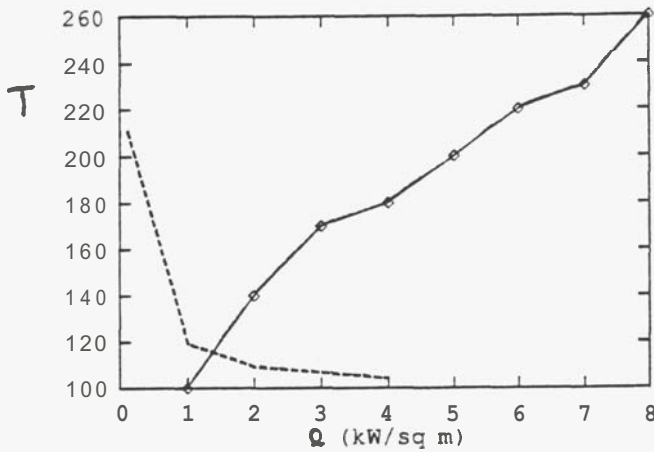


Figure 8 - A plot of the intersection temperatures in Fig. 7. (dashed line) and the temperatures at which vapor-dominated solution trajectories leave the $G = 0$ contour to cross to $S = 1$ (solid line), against heat flow $Q \equiv J_E$ for $k = 1$ d.

An easy way to obtain such a configuration would be to fix a constant temperature and saturation in the center of the internal boundary layer between the two sections.

It is **also** clear from Figs. 4 and 6 that many trajectories have a hot end with a large variety of values of saturation, eg $S = 0$, and that most of these track the vapor-dominated branch. Many trajectories have a cooler end at $S = 1$, and most of these track the liquid-dominated branch. These correspond to what we call **good** models.

7 CONCLUSIONS

Two complementary analytic approaches to the governing equations for 1D single-phase and two-phase fluid flow in a uniform porous medium have been used to investigate which steady-state models are good, bad or unlikely. In particular, the popular and significant model of liquid condensate over a vapor-dominated counterflowing region is not likely.

We believe that the weakest point of these 1D models is the assumption of uniform permeability. In particular, it is clear that

if permeability was to be reduced at the cooler end of a vapor-dominated heat pipe, a transition to pure liquid is possible. In the phase plane, this change in permeability could cause the $G = 0$ contour to suddenly terminate (meeting with the other liquid-dominated branch), allowing the solution trajectory to move to $S = 1$.

8 ACKNOWLEDGEMENTS

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9 NOMENCLATURE

d	unit of permeability, 1 darcy = 10^{-12}m^2
h	specific enthalpy
$h_{s,w}$	latent heat of vaporization
g	gravitational acceleration
J	J_M, J_E, J_s, J_v flows of mass , energy, liquid, vapor
J	the Leverett J-function, $1.417(1 - S) - 2.12(1 - S)^2 + 1.263(1 - S)^3$
k	permeability
k_r	relative permeability
md	millidarcy, 10^{-3} darcy
m_w	molecular weight of liquid water (kg/kmol)
u	mass flux density ($\text{kg s}^{-1} \text{m}^{-2}$)
P	pressure
P_c	capillary pressure
Q	$\equiv J_E$ energy flux (W/m^2)
\bar{R}	gas constant ($\text{m}^3 \text{Pa K}^{-1} \text{mol}^{-1}$)
s	flow coordinate
S	liquid saturation
T	temperature ($^{\circ}\text{C}$)
w	flow coordinate
x	$\equiv k_{rw}$ relative permeability
z	vertical distance (m)

Greek symbols

β	ν_w / ν_s
γ	$\frac{dT}{dP}(\nu_s) / (h_{s,w} \Delta \rho)$
λ	thermal conductivity
λ_w	liquid mobility, $k k_{rw} / \nu_w$
λ_s	vapor mobility, $k k_{rs} / \nu_s$
μ	dynamic viscosity
ν	kinematic viscosity
ω	dimensionless heat flow ($Q \nu_s / (k g h_{s,w} \Delta \rho)$)
ϕ	porosity
ρ	specific density
σ	surface tension (Kg/s^2)

Subscripts

w liquid water s steam or vapour phase

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