ON THE OPTIMUM DEVELOPMENT OF (GEOTHERMAL) RESOURCES

RICHARD C. LINDWALL
Unocal Corporation, Los Angeles, California, U.S.A.

SUMMARY - The optimum development plan for geothermal and other natural resources is shown to be dependent upon several critical factors. By applying the exponential decline equation to describe well producing capacity, formulae are derived for the optimum number of wells as functions of resource size, well performance, costs and price. The optimum is also shown to be dependent on the definition of optimum.

1. INTRODUCTION

The work described in this paper was prompted by the need for an algorithm for computing, in a consistent manner, production decline and makeup drilling schedules in a program for evaluating geothermal development project economics. This led to the derivation of formulae which show concisely the relationships of the principal factors which determine the optimum development of geothermal and other natural resources.

Because project economics are often prepared at a time when there is little or no field performance data available from the particular project, it is expedient in these instances to make a number of simplifying assumptions. The basis for computing well decline is key among these. In the present study, the assumption is made that resource production declines exponentially over time.

The performance of oil, gas and geothermal reservoirs *can* be characterized according to three graphical representations as shown in Figures 1,2 and 3. Each of these depicts the rate of production, Q, as a function of cumulative production, N. The quantity, N_o, for each curve is fixed by nature and represents the absolute maximum quantity of the resource that can be produced regardless of cost or the time required. Q, on the other hand, is a quantity over which people, rather than nature, have considerable control. One *can* elect to produce the resource at a relatively high rate or low rate, or anywhere in between.

If it is assumed for Figures 1, 2 and 3 that production is from wells, that all of the producing wells are drilled at the start of the project, and that no producing wells are subsequently added to or removed from the system, then the shape of each curve is suggestive of the drive mechanism of the corresponding reservoir. In each *case* the rate of production declines as the reservoir ages, where the rate of decline is related to the slope of the curve.

The curve in Figure 1 is referred to as hyperbolic decline. In this *case* the initial decline rate is relatively large and diminishes with time. Many oil fields have exhibited this type of decline as, for example, when the drive mechanism changes from gas cap or solution gas to gravity drainage. In a case such as this, the decline pattern due to gas alone or gravity drainage alone may or may not be characterized as hyperbolic, but the coexistence of the two drive mechanisms results in an overall production pattern approximated by a hyperbolic curve.

Geothermal reservoirs may also exhibit this type of production pattern. It is currently believed, for example, that The Geysers field in Northern California is exhibiting a production pattern which could be characterized as hyperbolic. In the *case* of geothermal systems, hyperbolic decline might be rationalized as the combination again of two concurrent production phenomena. First is the production from the major fracture systems in the reservoir, and this is later dominated by production from within the rock matrix. There may also be other reasons for

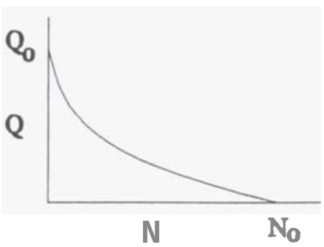

Figure 1 - Hyperbolic

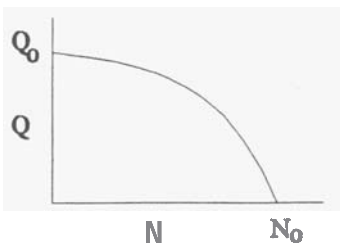

Figure 2 - Accelerating

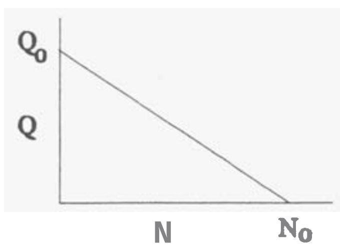

Figure 3 - Exponential

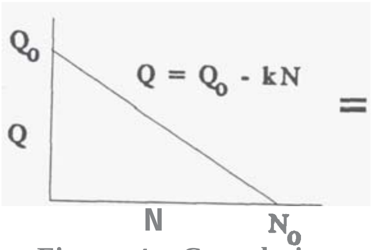

Figure 4 - Cumulative

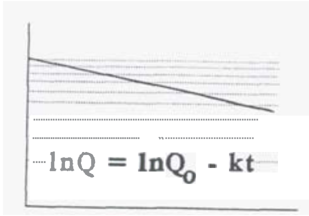

Figure 5 - Semi-Log

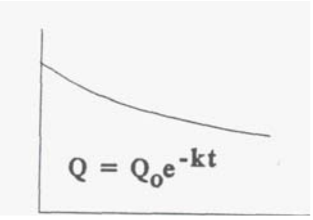

Figure 6 - Rectangular

anticipating hyperbolic decline from geothermal reservoirs.

The production curve in Figure 2 is characteristic of oil fields which have an active water drive producing mechanism. These resources enjoy a relatively constant rate of production until the reserves are nearly exhausted. In the case of oil fields, the accelerated decline near the end of the resource life is often due to water encroachment, or "watering-out" of the producing wells, rather than diminution of the driving force itself.

A boiling tea kettle exhibits this type of production performance and, at one time, there was speculation that The Geysers field might also. As recently perhaps as ten years ago, one theory which was discussed in connection with The Geysers was that the origin of the produced steam was a boiling "lake" of unknown size occurring at a depth beyond that of the deepest well drilled to date. Part of this theory was that, at some point the lake would boil itself dry leading to a production curve such as in Figure 2.

The fact that the shape of resource production curves can range from very concave to very convex offers a convenient excuse to consider in greater detail the implications of a rate-cumulative curve which is neither concave nor convex, but rather is a straight line, as shown in Figures 3 and 4. This curve describes what is known as "exponential" decline because it is mathematically equivalent to the curves in Figures 5 and 6.

2. DISCUSSION

2.1 Exponential Decline

The curve in Figure 6 is based on the following exponential equation in which t refers to time and k is a constant:

$$Q = Q_0 e^{-kt} \qquad (1)$$

Figure 5 shows the straight-line relationship between rate and time which results when rate is plotted in the traditional manner on semi-log graph paper. The equivalence between Figures 5 and 6 is immediately obvious by taking logarithms of equation (1). The equivalence of Figures 3 and 4 to Figure 6 results from noting that N is the integral of production over time, as follows:

$$N = \int Q(t)dt = \int Q_0 e^{-kt}dt \qquad (2)$$

$$= Q_0(e^{-kt} - 1)/-k \qquad (3)$$

$$N = (Q - Q_0)/-k \qquad (4)$$

$$Q = Q_0 - kN \qquad (5)$$

The existence of a "convenient excuse" is not the only justification for discussing exponential decline. Many resources, including geothermal resources, exhibit this pattern of decline over long periods of time. Figure 7 is a plot of the producing capacity of the Bulalo geothermal field in the Philippines. The time period from 1983 through 1988 is shown because during this time, production was fairly constant and no makeup production wells were drilled. The data points for this period plotted on semi-log graph paper follow a straight line, which is characteristic of exponential decline.

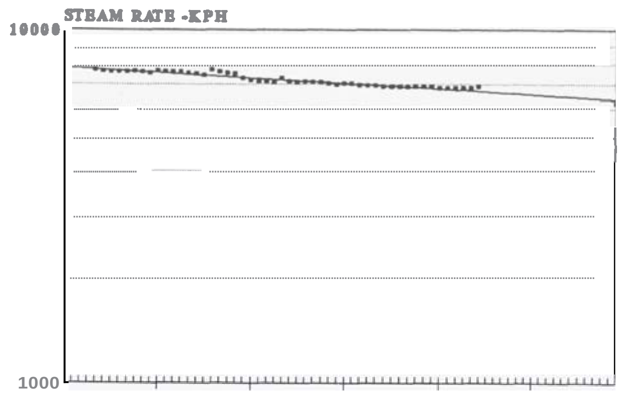

Figure 7 - Bulalo Steam Flow Capacity

In the discussion so far it has been assumed that no producing wells would be added to or removed from the system following first production. In practice, and especially in the case of geothermal resource developments, makeup wells are drilled during the course of a project with the objective of maintaining a relatively constant rate of production. At some point it is no longer practical to continue drilling makeup wells, so that from that point forward both production capacity and the rate of production decline. Figure 8 illustrates this in terms of rate vs. cumulative production based on exponential decline.

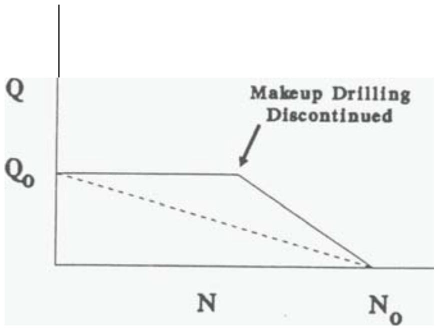

Figure 8 - Constant Production Prior to Exponential Decline

In this context, the optimum development of a resource reduces to answering two questions. Firstly, how many wells should be drilled prior to first production, and secondly, how many makeup wells should be drilled subsequently? To deal with this, several simplifying assumptions are made, important among these being the following:

- Decline is exponential.
- The resource is produced for infinite time.
- Revenue and operating costs are proportional to production, i.e., there is no economic limit due to fixed operating costs.
- All wells drilled prior to first production have identical costs, and all makeup wells have identical costs.
- At any point in time, all wells have identical producing characteristics.
- Wells are drilled in a continuous manner (mathematically their costs and production can be divided into increments which are infinitely small).
- Optimum means to maximize project present worth.

The last assumption mentioned above is central to most of what follows, and while it may seem to be an obvious choice, it should be pointed out that other criteria exist for defining "optimum" and each will lead to different answers. For example, if the objective is to maximize rate-of-return, the optimum development program for any particular resource will likely be very different from a plan based on maximizing present worth.

2.2 Present Worth

Present worth calculations are typically based on annual periods as a concession to tax and accounting conventions. For the present purpose, however, it is more convenient to calculate present worth on a continuous basis. It can be shown that corresponding (but not equal) expressions for present worth, P, of a single amount, A, on an annual basis and on a continuous basis are as follows:

Annual basis

$$P = A(1+i)^{-n} \quad (6)$$

Continuous basis

$$P = Ae^{-it} \quad (7)$$

The terms n and t refer to the number of years as an integer or as a continuous variable respectively, and i is an interest or discount rate. Likewise it can be shown that corresponding (but not equal) expressions for the present worth of a series of annual amounts, A_k, or of continuously varying amounts A(t), are given by the following:

Annual basis

$$P = \sum_k A_k(1+i)^{-k} \quad (8)$$

Continuous basis

$$P = \int Ae^{-it}dt \quad (9)$$

2.3 Optimum Initial Well Plan

To compute the present worth of production from a resource having a production pattern reflected by Figures 3 through 6, it is possible to combine equations (1) and (9) as follows:

$$P = \int EQ_0e^{-kt}e^{-it}dt \quad (10)$$

In equation (10), E is a measure of value on a unit of production basis and may correspond to price, cash flow or net profit after deducting operating costs and taxes.

The present worth of the project then results by subtracting from the present worth of production the cost of the wells and other facilities, which, for now, are assumed to all be installed prior to first production. In the equation below, the costs of wells and facilities are combined into an average cost per well, D, multiplied by the number of wells, W_0. The subscript with W_0 is included to emphasize that all of the well and facility costs are incurred prior to first production. In Equation (11) the term Q_0 is also replaced by W_0q_0, where q_0 refers to the initial producing capacity per well.

$$P = \int EW_0q_0e^{-kt}e^{-it}dt - W_0D \quad (11)$$

Integration of equation (11) between the limits of zero and infinity leads to the following expression for P:

$$P = (EW_0q_0)/(i+k) - W_0D \quad (12)$$

From equation (5) we note that when Q = 0, N = N_0 and k is given by the expression $k = W_0q_0/N_0$. Substituting this into equation (12) results in the following:

$$P = (EW_0q_0)/(i + W_0q_0/N_0) - W_0D \quad (13)$$

If we now assume that the terms D, E, q_0, R_0 and i are all fixed, it is then possible to find the maximum, or optimum, value of P as a function of the lone variable W_0, by taking the partial derivative of equation (13) with respect to W_0 and setting the differential equal to zero as follows:

$$\frac{\partial P}{\partial W_0} = \frac{iq_0 E}{(i + W_0 q_0/N_0)^2} - D = 0 \qquad (14)$$

On rearranging and making the substitution, C = D/E, equation (14) can be solved explicitly for W_0 to give the following:

$$W_0 = \left(\left(\frac{iq_0}{C}\right)^{1/2} - i\right) \times \frac{N_0}{q_0} \qquad (15)$$

Equation (15) gives explicitly the optimum number of wells, W_0 to be drilled prior to first production in order to maximize the present worth of the project. For equation (15) it is assumed that no makeup wells are to be drilled and that the following parameters are specified:

- i = Present worth discount rate, %/100
- q_0 = Initial well producing capacity (over a time period consistent with i, eg. annual)
- N_0 = Ultimate reserves, and
- C = D/E = Average initial investment per producing well divided by the net profit per unit of production excluding operating costs and taxes

Figure 9 illustrates results from equation (15) in graphical form based on a particular set of parameter values as follows:

- i = 10%
- q_0 = 5 MW or 10 MW per well (35,040 or 70,080 MWH/Year at 80% capacity)
- N_0 = 25 million MWH
- C = varied

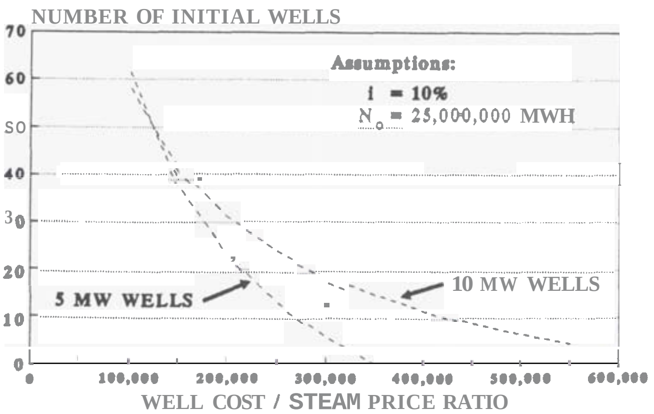

Figure 9 - Optimum Well Plans With No Makeup Wells

Prefered values for i and q_0 will depend upon the investing organization and upon the resource, but the reasonableness of any particular set of values is easily judged. The reasonableness of values for N_0 and C, however, is perhaps not so obvious. N_0 in Figure 9 is arbitrarily set at 25 million MWH. As a frame of reference, if a geothermal resource with such reserves was produced at a constant rate of 100 MW, it would have a producing life of about 28.5 years. Alternatively, if the same resource was produced at an initial rate of 100 MW declining exponentially, after 28.5 years it would still have a producing capacity of nearly 37 MW.

Several factors need to be considered in order to attach meaning to the term C. To illustrate with a specific example, consider a project where each producing well costs, say, $2,000,000 and for each producing well an injection well is required, also at a cost of $2,000,000. Assume further that the costs of pipelines and other production and injection facilities required at the start of operations are proportional to the number of initial producing and injection wells and that these add, say, another $2,000,000 to the cost of each well. Thus the incremental cost for each producing well drilled prior to first production is $8,000,000 in this example. Finally, assume that the resource price in this case is equivalent to $60/MWH, that operating costs are $10/MWH and the effective income tax rate is 36%. The result is that the operating profit per unit of production is $32/MWH as follows:

$32/MWH = ($60/MWH - $10/MWH) x (1 - 0.36)

The resulting cost/price ratio, C, would then equal 250,000 as follows:

C = 250,000 = $8,000,000/well / $32/MWH

Entering Figure 9 at 250,000 on the horizontal axis and finding the intercepts with the two curves shows, for example, that the optimum number of producing wells is either 13 or 24 depending on whether the initial well capacity is 5 MW or 10 MW per well. In other words, as may be seen in Figure 10, the optimum development plan calls for initial production rates ranging from 65 MW to 240 MW depending only upon the productivity of the wells.

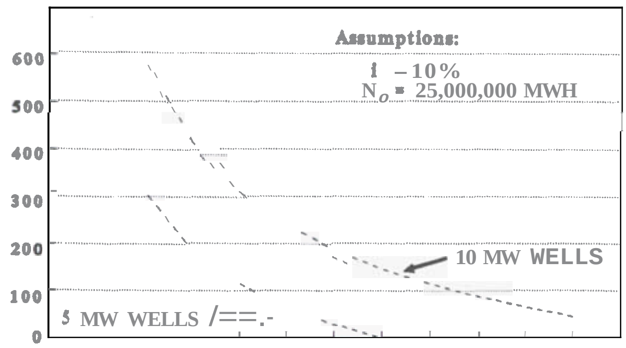

Figure 10 - Optimum Capacity With No Makeup Wells

2.4 Optimum Number of Makeup Wells

Given an initial development scenario, which may or may not be optimal, the next question to be answered is, "Should one drill makeup wells over time to keep the production facilities operating at full capacity, and if so, when should one discontinue drilling makeup wells?"

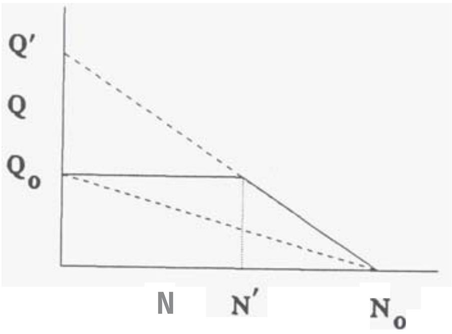

Figure 11 - Constant Production and Exponential Decline

Referring to Figure 11, the problem is equivalent to determining the optimum number of wells to produce the reserves increment between N' and N_0 where N' is defined below. We start by re-writing equation (15) with the subscripts omitted as follows:

$$W = ((iq/C)^{1/2} - i) \times N/q \qquad (16)$$

For the increment of production from N' to N_0, the corresponding formula would be:

$$W' = ((iq'/C')^{1/2} - i) \times (N_0 - N')/q' \qquad (17)$$

Where: Q' = the initial rate of production if the total number of wells, including makeup wells, had all been drilled at the outset.

W' = the total number of wells including initial and makeup wells.

N' = the cumulative production at the point when no additional makeup wells are drilled.

q' = the rate of production per well when makeup drilling is completed (N = N') such that $Q_0 = W'q' = W_0q_0$.

C' = D'/E'

D' = cost per makeup well

E' = price or profit per unit of production after N = N'

From the definitions above we can write:

$$q' = Q_0/W' = W_0q_0/W' \qquad (18)$$

and from Figure 11 we note the following relationships:

$$\frac{N_0 - N'}{N_0} = \frac{Q}{Q'} = \frac{W_0q_0}{W'q_0} = \frac{W_0}{W'} \qquad (19)$$

Therefore:

$$N_0 - N' = N_0W_0/W' \qquad (20)$$

Substituting equations (18) and (20) into equation (17) in order to eliminate q' and N', and on simplifying, there results:

$$W' = ((iq_0W_0/W'C')^{1/2} - i) \times N_0W_0/W'q'$$

$$= ((iq_0W_0/W'C')^{1/2} - i) \times N_0W_0/W_0q_0$$

$$W' = ((iq_0W_0/W'C')^{1/2} - i) \times N_0/q_0 \qquad (21)$$

This is an implicit relationship for the optimum value of W' where all of the other terms, including W_0, are assumed to be known. Even though W' is given by an implicit relationship, for convenience in calculating data points for plotting, it is possible to solve equation (21) explicitly for C' as a function of W and the other assumed known quantities as follows:

$$C' = iW_0q_0/(W'(W'q_0/N_0 + i)^2) \qquad (22)$$

In words, equations (21) and (22) give the relationship between the cost ratio C' and the optimum total number of wells to be drilled, given that W_0 wells are drilled prior to first production. To reiterate, W_0 in this formula may or may not be optimal.

2.5 The GEO123 Economics Evaluation Program

An analytical approach to finding simultaneously the optimum number of initial wells and the optimum number of makeup wells remains to be derived. In the absence of such an approach, a Unocal economics evaluation spreadsheet, known as GEO123, has been adapted to compute production decline schedules based on exponential decline. For specified preproduction drilling schedules, GEO123 computes the optimum makeup well drilling schedule adding wells only as needed to maintain a specified level of capacity. The number added is chosen so as to optimize some objective criterion (present worth, rate-of-return, etc.). GEO123 uses equation (21) to provide a first approximation to the optimum number of makeup wells required. The program then varies the total number of makeup wells until the objective function is optimized.

Using the same assumptions and parameters in GEO123 as were used in equation (15) for the preparation of Figure 9, leads to the results which are shown in Figure 12. The solid lines in Figure 12 are from GEO123 whereas the dashed curves are reproduced from Figure 9. Recall that the curves in Figure 9 are based on the assumption that all wells are drilled prior to first production, whereas the solid curves in Figure 12 show the optimum number of initial wells to be drilled given that the optimum number of makeup wells are also drilled.

An immediate conclusion from Figure 12 is that similar results are obtained from the two approaches, one approach being analytical and the other computer based. To the extent that the two methods do produce similar results, this suggests that the optimum number of initial wells is not critically influenced by the prospect of drilling makeup wells. The most significant result of Figure 12, however, is the implication that the optimum development plan can vary considerably depending on factors such as well performance, costs and price.

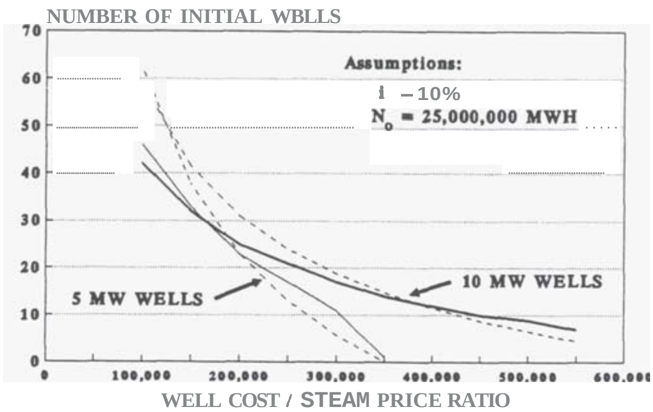

Figure 12 - Optimum Well Plan With Makeup Wells

It was mentioned above that the optimum development plan will also depend upon what is meant by optimum. The curves in Figure 13 are based on the same assumptions and parameters as in Figure 12, except that the objective function used in GEO123 was rate-of-return rather than present worth. As with the GEO123 results in Figure 12, the optimum number of makeup wells are calculated in the process of determining the optimum number of initial wells. It is immediately obvious from Figure 13 that far fewer wells are considered to be optimal under the rate-of-return criterion.

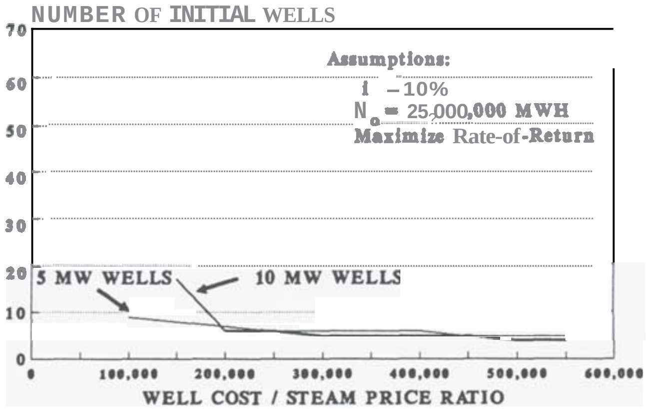

Figure 13 - Optimum Well Plan For Maximum ROR

3. CONCLUSIONS

Exponential decline can serve as a first approximation to the production performance of many natural resources, and specifically of geothermal resources. The simplicity of the exponential decline equation permits the derivation of concise algebraic equations for optimum initial and makeup well requirements.

A result of particular significance in applying these equations is that the optimum development plan for any particular resource is highly dependent upon several critical factors, of which resource size is only one. It has been shown that the optimal development plan is also dependent on the criterion for measuring optimality. This suggests that operations and management personnel need to have a clear, mutual understanding of the criterion to be used to be consistent with the organizations' investment strategy.

4. REFERENCES

Thompson, R.S. and Wright, J.D. (1984). Oil Property Evaluation. Thomas-Wright Associates, Golden, Colorado.