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PRESSURE TRANSIENT RESPONSE IN A FRACTURED MEDIUM

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ABSTRACT: This paper discusses tlie pressure transient response expected when a well intersects a single plane fissure. If boundary effects can be ignored then the long term response is described by a line-source (Tlieis) solution in a homogeneous medium; however the parameters in this solution must be given a different interpretation. For shorter times there are significant differences from the classical type curve.

INTRODUCTION:

It is generally recognized that pressure transient reponse curves in a fractured porous medium will show characteristic features which distinguish them from the type curves in a homogeneous porous medium. These curves not only signal tlie presence of fracturing, but may allow quantitative estimates of certain parameters describing tlie fracture system.

Some cliaracteristic features of the response curves relate to boundary effects, for example finite fracture length or finite block width. Our understanding of these influences can be improved if we study a simple system in which they are absent, eg a single infinite fissure in an infinite homogeneous porous medium. The long term pressure transient response of such a systein can be identified with tlie intermediate response in a bounded system, before tlie boundary effects have begun to play an important role.

A study of this nature was begun many years ago by Avdonin (1964), with reference to heat flow. We describe the results in terms of fluid flow. Solutions are obtained under the assumption that flow in tlie block is purely vertical. A flow pattern of this type (horizontal in tlie fissure, vertical in the block) has been called *bilinear* by Cinco-Ley and Sarnaniego-V (1981).

Tlie long term pressurc build-up for radial llow in a homogeneous porous medium is logarithmic in time. For bilinear flow, however, the restricted flow pattern leads to a quicker pressure buildup: the pressure increases like $t^{\frac{1}{4}}$. This characteristic pressure/time behaviour is the signature of bilinear flow.

The restriction to purely vertical flow in the block is obviously nonphysical, although it may often be a good approximation. A simple generalization of Avdonin's solution can be found which removes this restriction. The new solution will be described in this paper. It turns out that the strength of the horizontal pressure diffusion is controlled by a single parameter m^2 which is interpreted as the ratio of tlie horizontal diffusivity in the block to that in tlie fissure. When $m^2 = 0$ the flow in tlie block is purely vertical, and we have Avdonin's bilinear model.

The asymptotic beliaviour of the new solution is, however, quite different from bilinear flow. The additional degree of freedom resulting from the two dimensional block flow restores tlie line source character of tlie pressure build-up (though the axis of the line source is horizontal rather than vertical). In a sense tlie influence of tlre fracture disappears from the pressure response at long times. However, the paranieters of tlie line source solution now relate in part to tlie fracture properties rather than to the material properties of the homogeneous porous reservoir.

THE MODEL:

A schematic diagram of tlie model is shown in Figure 1. A single plane fissure is embedded in an infinite homogeneous porous medium. At the origin a well is connected to the fissure. We have shown a well intersecting a horizontal fissure, but another interpretation would be a well lying along the plane of a vertical fissure (eg a hydrofracture).

At time $t = 0$ the well is turned on (we consider injection, the results also apply to production). In a pressure build-up test the fluid is injected at a constant rate of Q kg/sec. We assume uniform initial conditions.

Fluid now enters the fracture and there is a pressure build-up along its length. Pressure transmission along tlie fissure is a diffusive process. This is

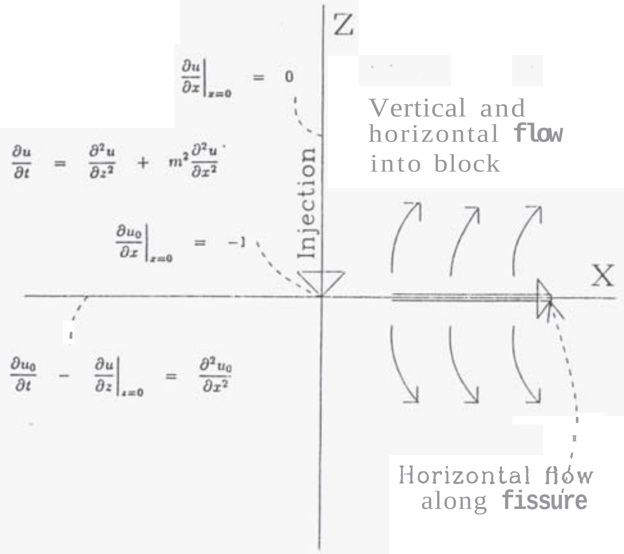
Time regime	*Early*	*Intermed*$_1$	*Intermed*$_2$	*Late*
Flow type	Linear	Bilinear fissure	Pseudo-radial	Linear formation
Dimensionality	1	$1\frac{1}{2}$	2	1
Characteristic fissure pressure (m = 0, vertical block How only)	$\frac{2}{\sqrt{\pi}}\, t^{\frac{1}{2}}$	$4\frac{\Gamma(3/4)}{\pi\sqrt{2}}\, t^{\frac{1}{4}}$		$\frac{t^{\frac{1}{2}}}{\sqrt{\pi(1+L)}}$
Characteristic fissure pressure ($m > 0$, horizontal block flow allowed)	$\frac{2}{\sqrt{\pi}}\, t^{\frac{1}{2}}$	$4\frac{\Gamma(3/4)}{\pi\sqrt{2}}\, t^{\frac{1}{4}}$	$\frac{1}{m\pi}\ \ln t$	$\frac{t^{\frac{1}{2}}}{\sqrt{\pi(1+L)(1+m^2L)}}$

Table 1: *Characteristic pressure/time behaviour in each time regime*

a consequence of the assumption that flow fracture system is controlled by tlie porous media equations with the appropriate fracture parameters.

The same equations also hold in the blocks, but with different coefficients (block parameters).

The two diffusion processes are linked by fluid flow across the block boundaries. This situation is illustrated in Figure 1.



Figurel: Schematic diagram of model with dimensionless equations and boundary conditions

Since the fracture aperture is so small compared with the block dimensions it is adequate to assume that flow from fissure to block can be represented by a *volume sink* term in tlie fracture equation. In the block, on tlie other hand, fissure/block flow is included as a *boundary condition.*

The diffusion equations under which tlie system operates are summarized in Figure 1 (see also Appendix). All quantities have been non-dimensionalized. In particular u is tlie dimensionless pressure in tlie block, and u_0 is the dimensionless pressure in tlie fissure. Boundary conditions include injection at the origin, and a no-flow condition along tlie z-axis (by symmetry). Tlie volume sink term in the fissure diffusion equation is tlie flow term $\left.\frac{\partial u}{\partial z}\right|_{z=0}$. Tlie strength of horizontal block flow is controlled by tlie diffusivity ratio m^2. When $m^2 = 0$ the flow is purely vertical (bilinear flow).

SOLUTION:

The equations are solved by taking a combined Laplace and Fourier cosine transform of the system. Transforming back to physical space the solution is obtained as an integral, and is given in the Appendix.

Of particular interest are the characteristic pressure/time behaviours during the flow periods shown in Figure 2. This information may be extracted (with some effort) from the solution given in the Appendix. The results are summarized in Table 1. Note how tlie the build-up is fastest in the beginning when the flow is restricted to one dimension, and slowest for large times during the period of 2-dimensional unconfined flow. Table 1 also shows a further period of linear flow due to interaction with tlre block boundary (dimensionless half-width t). This will not be be discussed further here.

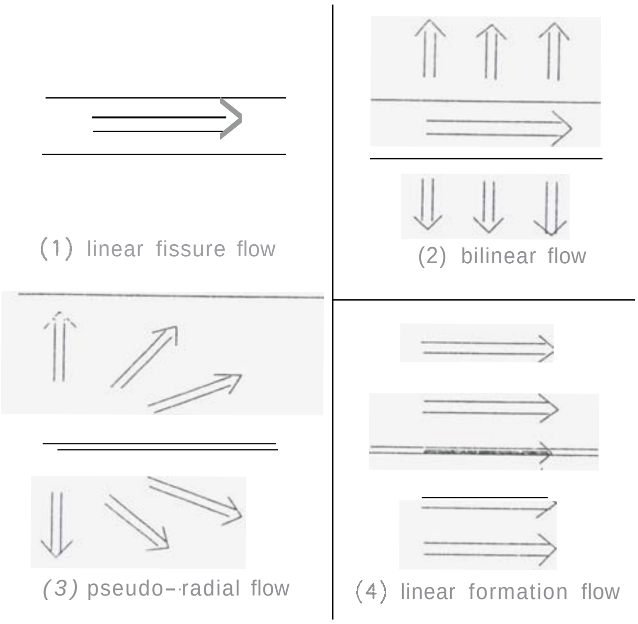


Figure 2: Schematic diagram of the characteristic flow periods for the model

A more detailed analysis of the asymptotic regime shows that for large times; and at sufficient distance from tlie origin, the dimensionless pressure has the form of an exponential integral

$$u \sim \frac{1}{\pi m} E_1\left(\frac{r^2}{4t}\right) \quad \frac{r^2}{4t} \ll 1,\ r \gg 1 \tag{1}$$

where $r \equiv \sqrt{x^2/m^2 + z^2}$ is the *effective mdius.* Transforming back to the physical variables (x^*, z^*, t') we find

$$P \sim P_0 + \frac{Q\mu}{4\pi\rho\hat{k}\hat{h}} E_1\left(\frac{\hat{r}^2}{4\hat{D}t^*}\right) \tag{2}$$

This is formally the classical line source solution (Theis (1935)) but the parameters with carets have new interpretations. The permeability $\hat{k}$ is tlie geometric mean $\hat{k} = \sqrt{k_x k_z}$ of the horizontal and vertical block permeabilities, and D is the diffusivity based on $\hat{k}$. The distance function $\hat{r}$ is given by

$$\hat{r}^2 = \sqrt{\frac{k_z}{k_x}}x^{*2} + \sqrt{\frac{k_x}{k_z}}z^{*2} \tag{3}$$

Also $\hat{h}$ is not the aquifer thickness but rather tlie *fissure width* along tlie y-axis, suppressed in Figure 1.

The line source solution is singular at the origin, and pressures in the source well are determined by evaluation at some finite well radius. By way of contrast, our fracture/block solution is finite at the origin. The long time beliaviour at tlie origin can be computed to be

$$u_0 \sim \frac{1}{m\pi}\left(\gamma + \ln(4m^4 t)\right) \tag{4}$$

where γ is Euler's constant. This should be compared with the beliaviour of the line source solution eqn (1) *near* the origin

$$u_0 \sim \frac{1}{m\pi}\left(-\gamma + \ln(4m^2 t) - 2\ln|x|\right) \tag{5}$$

which is singular at $|x| = 0$.

EXAMPLE:

Figure 3 and 4 show the time evolution of the pressure at the origin (source well) for different values of the diffusivity ratio. For small t Figure 3 shows that all the curves are tangent to a line with elope $\frac{1}{2}$. This is the linear fissure flow of Table 1. Later, if m^2 is sufficiently small ($m^2 < 0.01$), a period of bilinear flow commences and the curves are tangent to a line with slope $\frac{1}{4}$. If $m^2 > 0.01$ the bilinear flow period is absent.

For the late time asymptotic behaviour wc observe that if $m^2 = 0$ then the flow remains bilinear as $t \to \infty$ (Figure 3); but if $m^2 > 0$ all the curves eventually diverge from the bilinear flow regime. This is also seen in Figure 4 where the asymptotic pressure/time beliaviour is correctly described by eqn (4). It is referred to in Table 1 as tlie "pseudo-radial" flow period.

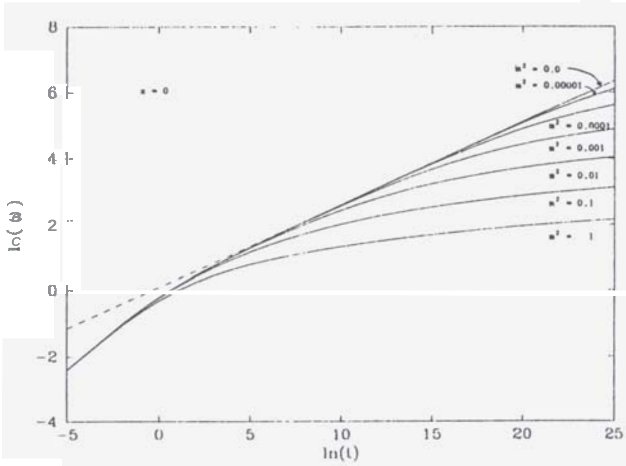


Figure 3: A log-log graph of dimensionless pressure against dimensionless time at $x = 0$ for $m^2 = 0$ (top), 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, 10^{0} (bottom). The dotted line has slope $\frac{1}{4}$.

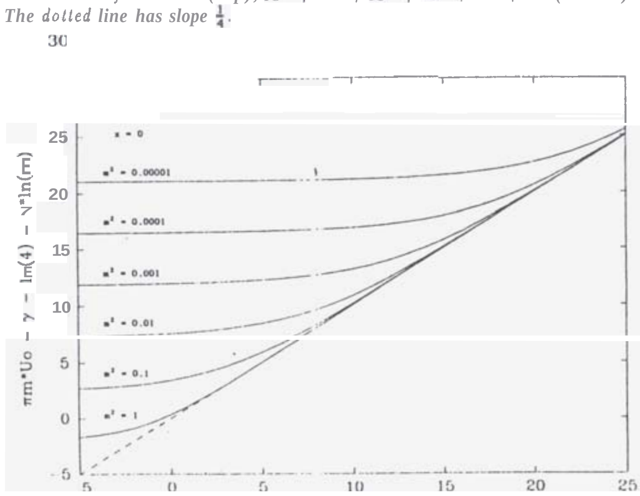


Figure 4: A semi-log plot of pressure against time at $x = 0$. We show for convenience the graph of $\pi m u_0 - \gamma - \ln(4m^4)$ against $\ln t$ for $m^2 = 10^{-5}$ (top), 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, 10^{0} (bottom).

CONCLUSIONS:

In a fissured region tlie interpretation of pressure transients must be approached with care. Even if a classical logt response is obtained over a certain time interval the usual interpretation may not be the most appropriate. Correct interpretation of the type curve may give new information about the reservoir parameters.

In a cylindrical geometry even more striking results are obtained. In tlie asymptotic regime the pressure response indicates an equivalent *point source* at the origin. In 3 dimensions this means that the pressure approaches a *steady state* as $t \to \infty$. The steady state profile can be obtained explicitly as a combination of Struve and Bessel functions. Experimentally, pressure transients would approach a *plateau* until boundary effects began to dominate.

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APPENDIX:

The fissure/block equations used in this paper are:

$$\phi_f c_f \frac{\partial P_0}{\partial t^*} - \frac{2}{\delta}\frac{k_{bz}}{\mu}\frac{\partial P}{\partial z^*}\bigg|_{z^*=0} = \frac{k_f}{\mu}\frac{\partial^2 P_0}{\partial x^{*2}} \quad (A.1a)$$

$$\phi_b c_b \frac{\partial P}{\partial t^*} = \frac{k_{bx}}{\mu}\frac{\partial^2 P}{\partial x^{*2}} + \frac{k_{bz}}{\mu}\frac{\partial^2 P}{\partial z^{*2}} \quad (A.1b)$$

for the pressure P_0 in the fissure (f) and the pressure P in the block (b) respectively. Here ϕ is the porosity, c the compressibility, μ the dynamic viscosity, 6 the fissure aperture; k_{bx}, k_{bz} are the horizontal and vertical permeabilities in the block, and k_f is the (horizontal) permeability in the fissure. If the horizontal permeability k_{bx} in the block is zero, then the first term on the right of eqn (A.1a) is dropped and the equations are of the Avdonin type. For further discussion of the fissure/block equations see eg McGuinness (1986).

The boundary condition at the fissure/block interface is:

$$P = P'', \; z^* = 0 \quad (A.1c)$$

while along the z^*-axis there is the 'no-flow' boundary condition:

$$\frac{\partial P}{\partial x^*} = 0, \; x^* = 0 \quad (A.1d)$$

The initial state is given by the constant pressure conditions:

$$P = P_0 = P_{init}, \; t^* = 0 \quad (A.1e)$$

Boundary conditions at infinity:

$$P, P_0 \rightarrow P_{init}, \; |x^*| \rightarrow \infty \quad (A.1f)$$

$$P \rightarrow P_{init}, \; |z^*| \rightarrow \infty \quad (A.1g)$$

The boundary condition at the origin:

$$-\frac{\rho k_f}{\mu}\frac{\partial P_0}{\partial x^*}\bigg|_{x^*=0} = \frac{Q}{2\delta \hat{h}} \quad \text{constant flow} \quad (A.1h)$$

Here ρ is the density, and Q is the mass flow into the fissure (width $\hat{h}$ and aperture 6) ($\frac{1}{2}Q$ in the $+x^*$ direction).

Non-dimensional variables are introduced according to $x^* = Xx = \left(\frac{\delta}{2}\sqrt{\frac{k_f\phi_f c_f}{k_{bz}\phi_b c_b}}\right)x$, $z^* = Zz = \left(\frac{\delta}{2}\frac{\phi_f c_f}{\phi_b c_b}\right)z$, $t^* = Tt = \left(\mu\frac{\delta^2}{4}\frac{\phi_f^2 c_f^2}{\phi_b c_b k_{bz}}\right)t$. The scaling of the dimensionless pressure is determined by the boundary condition-at the origin, eqn (A.1h). We choose $u = (P - P_{init})/\Pi$ where $\Pi = \frac{\mu}{\rho k_f}\frac{Q}{\hat{h}4}\sqrt{\frac{k_f\phi_f c_f}{k_{bz}\phi_b c_b}}$. This reduces the equations and boundary and initial conditions to the canonical form

$$\frac{\partial u_0}{\partial t} - \frac{\partial u}{\partial z}\bigg|_{z=0} = \frac{\partial^2 u_0}{\partial x^2} \quad (A.2a)$$

$$\frac{\partial u}{\partial t} = m^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \quad (A.2b)$$

$$u = u_0, \quad z = 0 \quad (A.2c)$$

$$\frac{\partial u}{\partial x} = 0, \quad x = 0 \quad (A.2d)$$

$$u = u_0 = 0, \quad t = 0 \quad (A.2e)$$

$$u, u_0 \rightarrow 0, \quad |x| \rightarrow \infty \quad (A.2f)$$

$$u \rightarrow 0, \quad |z| \rightarrow \infty \quad (A.2g)$$

$$\phi_0 \quad \frac{-}{\partial x} - t = 0 \text{ (constant Bow)} \quad (A.2h)$$

The dimensionless parameter m^2 is found to have the value $m^2 = k_{bx}\phi_f c_f / k_f \phi_b c_b$. In the case of finite block width eqn (A.2g) is replaced by $\partial u/\partial z = 0$, $z = L$ where $L^* = ZL$ is the half-width of the block.

The equations and initial and boundary conditions (A2) are then subjected to combined Laplace $(\bar{u}(s) = \int_0^\infty u(t)e^{-st}dt)$ and Fourier cosine $(\tilde{u}(\lambda) = \sqrt{\frac{2}{\pi}}\int_0^\infty \cos(\lambda x)u(x)dx)$ transforms. The solution of eqn (A2) in transform space is

$$\bar{\tilde{u}} = \bar{\tilde{u}}_0 \exp\left(-|z|\sqrt{s + m^2\lambda^2}\right) \quad \text{block} \quad (A.3a)$$

$$\bar{\tilde{u}}_0 = \frac{\bar{A}(s)}{s + \lambda^2 + \sqrt{s + m^2\lambda^2}} \quad \text{fissure} \quad (A.3b)$$

Here λ and s are the Fourier cosine and Laplace transform variables respectively and $\bar{A} = \$\sqrt{\frac{2}{\pi}}$ for constant flow. Using operation [37] on p.172 of Roberts and Kaufman (1966) and operation [11] on p.15 of Erdélyi et al (1954) the inverse transform for the pressure in the fissure is

$$u_0(x,t) = A(t) \star \frac{1}{2\sqrt{2\pi}}\int_0^1 q(1-q)^{-\frac{3}{2}}\exp\left(-\frac{q^2 t}{4(1-q)} - \frac{x^2}{4at}\right)\frac{1}{\sqrt{a}}\,dq \quad (A.4)$$

where $a(q) = q + (1 - q)m^2$. When $m = 0$ there is no lateral flow and the solution is of the Avdonin type.