Proc. 10th New Zealand Geothermal Workshop 1988

Tracer Modelling

Mark McGuinness and Michael Louie
Applied Mathematics Division, DSIR
Wellington, New Zealand

ABSTRACT

The matrix diffusion model for tracer flow in fractured media has enjoyed some success in geothermal reservoirs in New Zealand. The solution to this model is here examined in some detail, comments made on the effects and form of the parameters, and a way of quickly estimating parameters by hand is given.

INTRODUCTION

Tracer testing is an important tool for investigating geothermal reservoirs. Where separated fluid is required to be reinjected, rapid returns of a chemical tracer from reinjection wells to production wells provide a warning of later cooling of production wells, with a subsequent loss of production. Tracer movement, unlike pressure changes, indicates fluid flow paths within the reservoir.

A tracer test can take the form of injecting a slug of say I^{131} and monitoring production wells for the presence of this radioactive tracer. The time of first detection of tracer at a production well is a significant factor, but it is also useful to try to interpret from the shape of the curve of monitored concentration vs. time, something about the nature of the reservoir through which the tracer has moved.

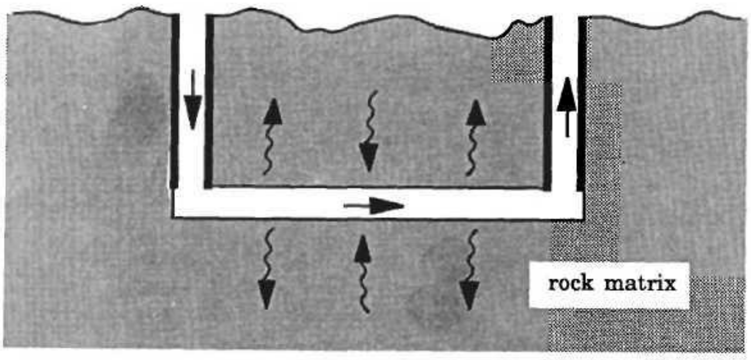
A variety of models have been used in the geothermal context, with varying degrees of success. These include simple diffusion models, two-dimensional potential flow models, channelling (Tsang and Tsang, 1987), matrix diffusion (Jensen and Home, 1983), and fast and slow flow paths in fractured reservoirs (A. McNabb, personal communication). The matrix diffusion model discussed here has enjoyed some success when applied to tracer returns at Wairakei (Jensen and Home, 1983). Most of the comments made here about fitting this model are also to be found in work done independently by D. Bullivant, and appear in his thesis.

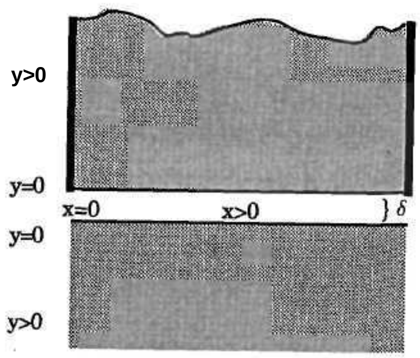
MATRIX DIFFUSION MODEL

This model for tracer flow was originally put forward by Neretnieks (1980) in the field of underground nuclear waste disposal, and was subsequently applied to tracer returns in the Wairakei geothermal reservoir by Jensen and Home (1983). It is assumed that there is a steady one-dimensional flow in an ideal smooth-walled planar fracture, from an injection well to a production well. The tracer is introduced at the injection well, flows along the fracture, diffuses into and out of the adjacent rock, and is detected at the production well (Figure 1).

This may be expressed mathematically as

$$R\frac{\partial C_f}{\partial t} + u_f\frac{\partial C}{\partial x} = \frac{2D_e}{\delta}\frac{\partial C_p}{\partial y}\bigg|_{y=0}, \qquad (1)$$





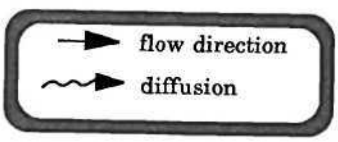


Figure 1. A sketch of the matrix diffusion model layout, x marks the distance along the fracture from the production well, while y marks the distance into the rock matrix from the fracture face.

in the fracture, while in the rock matrix

$$\frac{dC_p}{dt} = D_a \frac{d^*C_p}{dy^2} , \qquad (2)$$

The retardation factor R accounts for the slowing down of the tracer due to adsorption. For I^{131} adsorption is usually taken to be negligible, and R is 1. The two different diffusion coefficients D_a and D_e are explained in detail in Neretnieks (1980), and are related via a volumetric sorption coefficient. When R is 1, this coefficient is simply the porosity of the rock matrix.

When these equations are solved, for initial conditions with fracture and rock free of tracer, and a delta function input of tracer at time zero and x zero, the concentration observed at the production well is

$$C_i = \begin{cases} 0 & , t \le t_w R \\ \frac{E\alpha_1\alpha_2}{\sqrt{\pi}(\alpha_2 t - 1)^{1.5}} \exp\left(-\frac{\alpha_1^2}{\alpha_2 t - 1}\right) & , t > t_w R \end{cases} . \qquad (3)$$

This solution has two nonlinear parameters

$$\alpha_1 = \frac{(DJt_w)^{0.5}}{\delta}$$

and

$$a_2 \;-\; l/(t_w R) .$$

The scaling factor E reflects the experience that usually less than 20% of the total injected tracer is recovered from the production well, and is used to adjust the flow fraction to one. The water residence time t_w is the time taken to travel from injection to production well at the average velocity of the fluid in the fracture *uj.*

The typical shape of this solution is shown in Figure 2. When t is just greater than l/a_2, the exponential term leads to a rise in tracer concentration. This is eventually overtaken by the algebraic term, which leads to an algebraically slow decrease in concentration (or a *tail)* at larger times.

If this solution is fitted to data, the resulting values of *cti* and a_2 can be used together with estimates of D_e, <f> and R to obtain estimates of fracture width *8.*

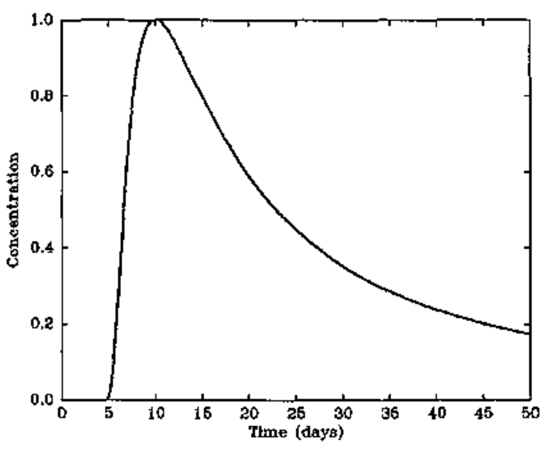


Figure 2. A typical tracer response curve for the matrix diffusion model. For this case, a_x is 1.5 and a_2 is 0.25 days^{-1}.

FITTING THE MODEL

When the solution in Equation 3 was taken as is, and a computer program written to fit it to tracer return data by nonlinear regression with a_x and a_2 as the nonlinear parameters, a problem was encountered. The data, from the Ohaaki reservoir, had a spread-out peak compared to older Wairakei data, similar to the fictitious data plotted in Figure 3. Fitting the solution involved finding the values of a_x and a_2 that give the minimum value of the sum of squared differences (F) between data and fit. For this data, the minimum value of F lay on a *line* in a_u a_2 space, with large values of a_2. There was no single value of (c*i, a_2) minimising F.

APPROPRIATE PARAMETERS

The cause of this problem lies in an understanding of what changing the parameters does to the solution, and in an appropriate choice of parameters. A clue lies in noting that $t_w R$ is the time that tracer would take to travel along the fracture if there was no matrix diffusion. Another clue lies in the observation that the peak of the tracer return curve corresponds to

$$- \frac{2a\backslash}{3\alpha_2} + \frac{1}{a_2} . \qquad (4)$$

Dividing through in Equation 3 by a_2, and making the replacements

$$t_b \equiv t_w R = l/a_2, \qquad (5)$$

$$7^2 = «i^2/<*2 , \qquad (6)$$

we obtain

$$C_f = \begin{cases} 0 & , t \le t_b \\ \frac{\wedge}{\sqrt{\pi}(t - t)^{1.5}} \exp\left(-A\right) & , <> h \end{cases} . \qquad (7)$$

where

$$7^2 = A\phi t_w^2 R/\delta^2 = D_e \phi t l/(S^2 R) , \qquad (8)$$

and

$$t_b = t_w R . \qquad (9)$$

The nonlinear parameters are now chosen to be 7 and t_b.

Then changing t_b in Equation 7 is clearly the same as translating the solution in time. That is, t_b only appears in the expression $t - t_b$, and changing t_b changes the time behaviour by a constant amount. Since the peak of the tracer concentration lies at

$$< = \frac{¥}{3} + *, \qquad \text{do)}$$

7^2 is a measure of the *spread* of the tracer return curve.

The problem encountered using c*i and a_2 was due to the optimum value of t_b being very close to zero. Then a_2 becomes very large, and even large changes in a_2 correspond to tiny translations of the solution curve, and make very little difference to F.

More appropriate parameters (than a_x and a_2) for matching or fitting are 7 and t_b. Changing t_b is like moving the solution back and forth along the t axis, to where it most closely matches the data. Changing 7 changes the spread of the solution, matching it to the spread of the data. When these parameters were used to fit the data illustrated in Figure 3, no problem was encountered.

The problem that was encountered when using a_x and a_2 does not occur with data that has a sharper peak and has t_b greater than 1 day.

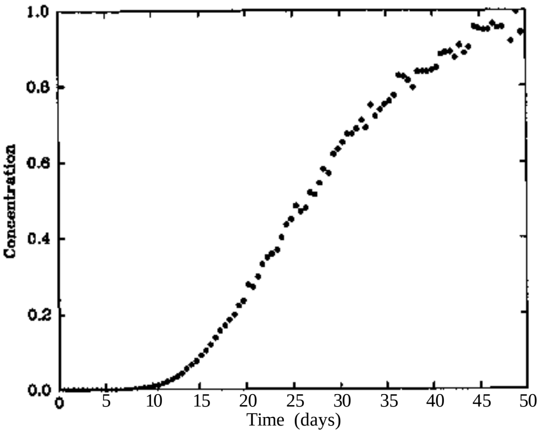


Figure 3. A fictitious tracer return data set, for which *ai* is about 30 and a_2 is 10 days"[1]. This set was generated by adding 10% white noise to the matrix diffusion model solution.

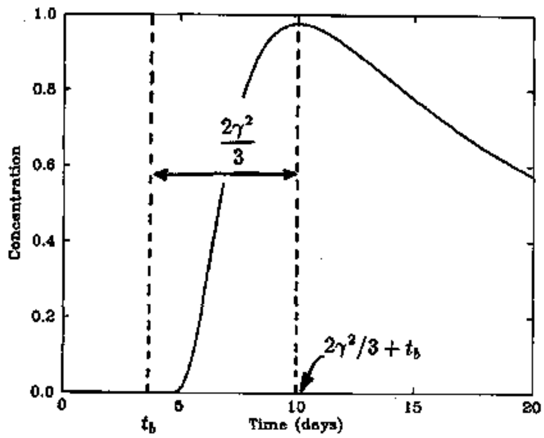


Figure 4. A tracer return curve for the matrix diffusion model, indicating how to estimate parameter values directly from the data plot.

QUICK PARAMETER ESTIMATION

The dependence of the solution in Equation 7 upon % and 7 is simple enough to be able to quickly estimate their values directly from the data.

The parameter 4 is the time the tracer would have taken to arrive at the production well in the absence of matrix diffusion. It lies between zero and the time of the first detectable tracer return.

The parameter 7 controls the *spread* of the tracer return curve. As indicated in Figure 4, the peak of the tracer return curve is located at

$$t = 2_7^2/3 + t_b \ ,$$

so that observation of this peak time and t_b gives an estimate of 7^2 directly and simply.

NOMENCLATURE

C_f	tracer concentration in fracture
C_p	tracer concentration in porous matrix
R	retardation factor
u_f	fluid velocity in fracture
D_e	effective diffusion coefficient
D_a	apparent diffusion coefficient
6	fracture width
x	distance along fracture
y	distance into rock matrix
E	a linear scaling factor
ϕ	rock porosity
t_w	water residence time
h	a timelike parameter
7	a nonlinear parameter
a	nonlinear parameters

CONCLUSIONS

More suitable parameters for fitting the matrix diffusion model for tracer return curves have been suggested. Changing the parameter t_b acts to translate the fitted curve, while changing the parameter 7 has the effect of spreading the fitted curve. A quick way of directly estimating the values of these parameters directly from the data has been given.

REFERENCES

Bullivant, D. (1988) *Tracer Modelling,* a thesis submitted for a Ph.d. at the Theoretical and Applied Mechanics Department, Auckland University.

Jensen, C.L. and Home, R.N. (1983) *Matrix Diffusion and its Effect on the Modelling of Tracer Returns from the Fractured Geothermal Reservoir at Wairakei, New Zealand,* in the Proceedings of the Stanford Geothermal Workshop, Stanford, California, December 1983.

Neretnieks, I. (1980) *Diffusion in the Rock Matrix: an Important Factor in Radionuclide Retardation?* J. Geophys. Res. 85 (B8) (August 10 1980) pp.4379-^397.

Tsang, Y.W. and Tsang, CF. (1987) *Channel Model of Flow Through Fractured Media,* Water Resour. Res. 23 Number 3 (March 1987) pp.467-479.