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MATHEMATICAL MODELLING OF SILICA DEPOSITION

R.C.M. MALATE and M.J. O'SULLIVAN

Theoretical and Applied Mechanics, University of Auckland

Abstract

Various mathematical models describing the rate of depositon of silica from geothermal fluids are investigated. They are combined with the silica transport equation to predict the variation of silica deposition in experimental packed columns and near reinjection wells in geothermal reservoirs.

Introduction

One of the major constraints in maximizing the extraction of hot water from liquid dominated geothermal reservoirs is the inevitable deposition of chemicals as exploitation proceeds. Deposition or scaling occurs not only in the bores and surface equipment but also in the reservoir in the immediate vicinity of the wellbore (Cuellar, 1975; Rothbaum et al, 1979; Gudmundsson and Bott, 1979; Weres and Tsao, 1981; Horne,1982; Itoi et al, 1987a). Silica in amorphous form heads the list of the problem precipitates associated with the reinjection of waste water. Deposition of silica around the wellbore causes reduction in formation permeability, and subsequently the injectivity of the well (Hauksson and Gudmudsson, 1986; Itoi et al, 1987b).

As the geothermal fluid is extracted and steam is separated considerable supersaturation with respect to amorphous silica results in the reject water. Amorphous silica deposition may then occur, which appears to be governed by several factors such as degree of supersaturation, temperature, pH, presence of dissolved salts and foreign ions, availability of nucleating species, and fluid flow regime (Henley, 1983). Deposition is known to occur by direct deposition on solid surfaces (heterogeneous nucleation) or by polymerization followed by colloidal deposition (homogeneous nucleation).

Here a mathematical model is developed to describe silica transport in a porous medium which incorporates the changes in porosity and permeability resulting from deposition.

The governing equation is a standard chemical transport equation including a reaction term representing the deposition of silica. The chemical processes involved in silica deposition are complex and the mathematical equations considered here are approximate only. Uncertainty remains on how to best mathematically model silica deposition.

Several different forms of mathematical models representing silica deposition are considered here and analytic solutions are obtained for the idealised problem of constant rate injection into either a one-dimensional channel or into a uniform layer from a well, producing radially symmetric flow. The one-dimensional model is also appropriate for analysing experiments on deposition of silica in packed columns.

There are a number of experimental studies made on the kinetics of silica polymerization which have mostly measured the disappearance of monomeric silica during the course of the reaction (Rothbaum and Rohde, 1979; Makrides et al, 1980; Rimstidt and Barnes, 1980; Bohlmann et. al, 1980; Weres et al, 1982; Bird et al, 1986; Fleming, 1986). The kinetics of polymerization are so complex that the studies yielded various estimates of the order of reaction.

Rimstidt and Barnes (1980) performed their experiments in salt-free water and derived from theoretical considerations (Law of Microscopic Reversibility) the kinetics of silica dissolution-precipitation. Their rate equation can be expressed as

$$\frac{dC}{dt} = -k^{-}\frac{A}{M}(C - C_s) \qquad [1]$$

where k^{-} is the precipitation constant, A/M is the ratio of the surface area to the mass of water available, C is the silicic acid concentration (ppm), and C_s is the saturation concentration. This rate equation characterizes an apparent zero order dissolution and first order precipitation with respect to silicic acid. The rate constant has an Arrhenius temperature dependence.

Bohlmann et al (1980) studied molecular deposition from controlled synthetic solutions. They monitored the deposition of monosilicic acid flowing through a column packed with granular amorphous silica and other similar forms. They found that after the substrate was coated fully with amorphous silica the nature of the substrate had no effect on the rate of deposition. There is little effect of increasing salinity up to 1 molal NaCl and stronger effect at higher concentrations.

Bohlmann et al (1980) fitted their rate data in an empirical form:

$$\wedge = -k'A(C-C)^{2}_{s}OH^{\circ}-^{7} \qquad [2]$$

where OH is the hydroxide ion concentration calculated from solution pH, and C is the molal silica concentration. The rate constant k' is independent of temperature over the range 60 to 100°C. The apparent kinetic order for monomeric silica is about 2 and for the hydroxide ion is 0.7.

Rothbaum and Rohde (1979), Makrides et al (1980), and Weres et al (1982) employed nucleation theory to fit their rate data in their polymerization studies, and reported higher reaction orders.

Recently, Fleming (1986) studied silica polymerization (without nucleation) in an attempt to resolve conflicting results on the reaction order. His experiments were performed at 25 - 50°C, 0 -1 molal NaCl and $4 < pH < 8$ in unbuffered solutions. Based on the results of his differential rata data, he suggested that there are two different rate limiting regimes, and explained this behaviour by the enhanced solubility, C_x, of freshly deposited silica. The overall rate of both regimes appear directly proportional to the silica surface area, A_s. Polymerization is first order in both silicic acid concentration, C, and in the surface concentation of ionized hydroxyl group (surface charge) in amorphous silica, [SiO~].

His model rate equations are for high concentations:

for $C>C_x$ [3]

where $k_x = k\,A_s[SiO^{\sim}]$ and

$$k = A_o \exp\frac{-E}{RT} \exp\frac{A_{DH}\sqrt{I}}{1+\sqrt{I}}.$$

For lower concentration the equation is:

$$\frac{dC}{dt} = -k_2(C - C_s)^3 \qquad \text{for } C<C_x \qquad [4]$$

where

$$k_2 = k_{sr} A_s \Gamma_s^2 C_s^{-2}$$

and

$$k_{sr}\Gamma_s^2 = A_{sr}\exp\frac{-E_{sr}}{RT}.$$

The solution pH influences silica surface charge density but not the rate constant, while ionic strength (I) influences both the surface charge and the rate constant k. The complete model was also fitted to the polymerization rate data of Bohlmann et al (1980) and Weres et al (1982), and gave satisfactory results.

Basic Equations

The governing equations used in the model represent conservation of mass, momentum, energy and chemicals (Cheng, 1978).

The following general assumptions have been employed:

1) Darcy's law adequately describes the fluid movement.
2) The fluid and rock matrix are in local thermal equilibrium.
3) The porous medium is homogeneous.

For one-dimensional flow, assuming no sources or sinks, conservation of mass for a single phase fluid can be expressed as:

$$\frac{\partial(\phi\rho_\ell)}{\partial t} + \frac{\partial Q_m}{\partial x} = 0 \qquad [5]$$

where ρ_ℓ is the density of the fluid, ϕ is the porosity of the rock, and Q_m is the mass flux per unit area. Then Darcy's law gives for horizontal flow,

$$Q_m = \frac{-k}{\nu_\ell}\left(\frac{\partial p}{\partial x}\right) \qquad [6]$$

where ν_ℓ is the kinematic viscosity of the fluid and k is the permeability of the rock matrix.

The equation for conservation of energy is

$$\frac{\partial[\phi\rho_\ell c_\ell T + (1-\phi)\rho_r c_r T]}{\partial t} + \frac{\partial(Q_m c_\ell T)}{\partial x} = \frac{\partial}{\partial x} K \left(\frac{\partial T}{\partial x}\right) \qquad [7]$$

where c_ℓ is the specific heat of the fluid; ρ_r, c_r are the density and specific heat of the rock matrix, respectively; and K is the thermal conductivity.

Combining equations [5] and [7], and assuming c_ℓ, ρ_r, c_r and ϕ are approximately constant, then the conservation of energy becomes:

$$[\phi\rho_\ell c_\ell + (1-\phi)\rho_r c_r]\frac{\partial T}{\partial t} + Q_m c_\ell \frac{\partial T}{\partial x} = K \frac{\partial^2 T}{\partial x^2} \; . \qquad [8]$$

The silica mass balance equation can be expressed as

$$\frac{\partial(\phi\rho_\ell C)}{\partial t} + \frac{\partial Q_c}{\partial x} = -R_e \phi\rho_\ell \qquad [9]$$

where Q_c is the mass flux of silica given by

$$Q_c = C\,Q_m - D\frac{\partial C}{\partial x} \qquad [10]$$

and C is the mass fraction of silica in the fluid and D is the diffusion coefficient. The term R_e represents the net loss of dissolved silica per unit mass of fluid and is evaluated from one of the kinetic rate equations given above (1,2,3 or 4).

The corresponding differential mass balance for the rock is

$$\frac{\partial[(1-\phi)\rho_r Q_s]}{\partial t} = R_e\phi\rho_\ell \qquad [11]$$

where Q_s is the mass fraction of silica in the rock.

For single phase flow which is radially symmetric in a confined layer, mass conservation gives

$$\frac{\partial(\phi\rho_\ell)}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\,Q_m) = 0 \qquad [12]$$

where Q_m is the radial flux.

Also from Darcy's law,

$$Q_m = \frac{-k}{\nu_\ell}\frac{\partial p}{\partial r} \; . \qquad [13]$$

And the energy equation is

$$\left[\phi\rho_\ell c_\ell + (1-\phi)\rho_r c_r\right]\frac{\partial T}{\partial t} + \frac{Q_o c_\ell}{r}\frac{\partial T}{\partial r} = K\left(\frac{1}{r}\right)\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \qquad [14]$$

where $Q_o = rQ_m$ is a constant.

Conservation of silica is given by

$$\frac{\partial(\phi\rho_\ell C)}{\partial t} + \frac{1}{r}\frac{\partial(Q_o C)}{\partial r} = D\left(\frac{1}{r}\right)\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right) - R_e\phi\rho_\ell \qquad [15]$$

The rock mass balance equation is the same as for the one-dimensional flow model.

Method of Characteristics Solution

In the special case of a constant mass flow, and assuming that diffusion can be neglected in equation [9] then it reduces to a first order partial differential equation of hyperbolic type:

$$\frac{\partial C}{\partial t} + V\frac{\partial C}{\partial x} = -R_e \qquad [16]$$

Here $V = Q_m/\phi\rho_\ell$ is the particle velocity and therefore the velocity of movement of chemicals through the porous material.

For this equation it is possible to obtain an exact solution by using the method of characteristics. In particular, characteristic curves are established, along which simple relationships define the transport process.

By the chain rule, the derivative of C along some curve in the x-t plane can be written as

$$\frac{dC}{ds} = \frac{\partial C}{\partial t}\frac{dt}{ds} + \frac{\partial C}{\partial x}\frac{dx}{ds} \qquad [17]$$

where s is the arc length along the curve.

Comparing equations [16] and [17] the characteristic curves are defined by the following set of ordinary differential equations:

$$\frac{dt}{ds} = 1 \quad , \quad \frac{dx}{ds} = V \qquad [18]$$

or $x - Vt = x_o$ = constant (characteristic line).

Then [16] can be replaced by

$$\frac{dC}{ds} = -R_e \; . \qquad [19]$$

The solution of the ordinary differential equation [19] depends now on the form of the reaction term R_e. As discussed previously (eqns. [1] - [4]) different versions of this form will be considered:

(i) (Rimstidt and Barnes, 1980)

$$R_e = k'\frac{A}{M}(C - C_s) \qquad [20a]$$

(ii) (Bohlmann et al, 1980)

$$R_e = k'' A(C - C_s)^2\,\mathrm{OH}^{07} \qquad [20b]$$

(iii) (Fleming, 1986)

$$R_e = \begin{cases} k_1(C - C_x) + k_2(C_x - C_s)^3 & \text{for } C > C_x \\ k_2(C - C_s)^3 & \text{for } C < C_x \end{cases} \qquad [20c]$$

Note that the original version of Fleming's rate equation was not quite satisfactory and is rewritten here in a modified form to ensure continuity between the two kinetic regimes.

Using Rimstidt and Barnes[1] first order rate equation, equation [19] can be solved along the characteristic line $x = Vt + x_o$ in the form

$$\frac{dC}{dt} = -k(C - C_s) \qquad [21]$$

where $k = k' \frac{A}{M}$.

The pertinent initial and boundary conditions are

$$C(x,0) = C_s \qquad [22a]$$

and

$$C(0,t) = C_o \qquad [22b]$$

where C_o is the mass fraction of silica injected and C_s is the silica saturation concentration.

The solution to this equation is

$$C = C_s + (C_o - C_s)\exp\frac{-kx}{V} \quad \text{for } x < Vt \qquad [23]$$

and

$$C = C_s \quad \text{for } x > Vt\,. \qquad [24]$$

Since [23] is independent of t, it is also the steady state ($t \to \infty$) solution.

The two solutions are illustrated in Figure 1. Since V is constant, the characteristics are straight lines. It is clear that the type of solution depends upon the initial point (x_o, t_0) from which the characteristics emanate. In region I shown by the characteristic AA' (eqn [23]), the characteristic originates from the line x = 0, t > 0, and in region II shown by the characteristic BB' (eqn. [24]), the characteristic originates from the line t = 0, x > 0.

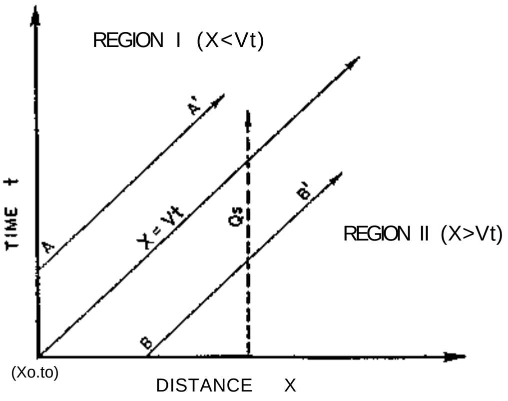

FIG. 1 TIME-DISTANCE DIAGRAM FOR THE SILICA CONCENTRATION.

The rate of silica deposition is evaluated from the rock mass balance equation [11]. Assuming ρ_r and ϕ are constant, equation [11] becomes

$$\frac{\partial Q_s}{\partial t} = \frac{\phi\rho_\ell}{(1-\phi_0)\rho_r}\, k(C-C_s) \qquad [25]$$

The porosity is given in terms of the amount of silica deposition (Itoi et al, 1987b);

$$\phi = \phi_o - \frac{(1-\phi_o)\rho_r\,(Q_s-Q_{so})}{(1-\phi_s)\rho_s} \qquad [26]$$

where ϕ_s and ρ_s are the porosity and density of deposited silica, and Q_{so} is the initial mass fraction of deposited silica in the rock. Substituting eqn. [26] into eqn. [25], and simplifying gives,

$$\frac{\partial Q_s'}{\partial t} = (\phi_o - \alpha Q_s')\beta(C - C_s) \qquad [27]$$

where

$$\alpha = \frac{(1-\phi_o)\rho_r}{(1-\phi_s)\rho_s}\,; \quad \beta = \frac{\rho_\ell k}{(1-\phi_o)\rho_r} \quad \text{and}$$

$$Q_s' = Q_s - Q_{so}\,.$$

Equation [27] can now be easily integrated (see Figure 1) as the two solutions for the silica concentration in regions I and II are both independent of t. The solution is

$$Q_s = Q_{so} \qquad x > Vt \qquad [28]$$

and

$$Q_s = Q_{so} + \frac{\phi_o}{\alpha}\left[1 - \exp(f(x,t))\right] \qquad x < Vt \qquad [29]$$

where

$$f(x,t) = \alpha\beta(C_o - C_s)\exp\left(-\frac{kx}{V}\right)\left(\frac{x}{V} - t\right)$$

The solution for C and Q_s using Bohlmann's and Fleming's rate equation can be determined by following the same procedure. These are summarized as:

(i) Bohlmann et al:

$$C(x,t) = C_s + \left[\frac{1}{(C_o - C_s)} + \frac{kx}{V}\right]^{-1}, \quad x < Vt, \qquad [30]$$

where $k = k' A\, OH^{0.7}$

and

$$C(x,t) = C_s\,, \quad x > Vt. \qquad [31]$$

Then [11] can be integrated to give

$$Q_S = Q_{SO}\,, \quad x > Vt \qquad [32]$$

and

$$Q_s = Q_{so} + \frac{\phi_o}{\alpha}\left[1 - \exp(f(x,t))\right] \qquad [33]$$

where

$$f(x,t) = \alpha\beta\left[\frac{1}{(C_o - C_s)} + \frac{kx}{V}\right]^{-2}\left(\frac{x}{V} - t\right)$$

(ii) Fleming

For $x < Vt$

$$C(x,t) = \lambda + (C_o - \lambda)\exp(-k_1 x/V)\,, \quad C > C_x \qquad [34]$$

and

$$C(x,t) = C_s + \left[\frac{1}{(C_x - C_s)^2} + \frac{2k_2}{V}(x - \bar{x})\right]^{-1/2}, \quad C < C_x\,. \qquad [35]$$

where

$$\lambda = C_x - k_2(C_x - C_s)^3/k_1$$

and

$$\bar{x} = \frac{V}{k_1}\ln\left[\frac{C_o - \lambda}{C_x - \lambda}\right].$$

Here $\bar{x}$ is the distance covered when the concentration C falls to C_x.

For $x > Vt$

$$C = C_s\,. \qquad [36]$$

Then as above Q_s is obtained by integrating [11]. For x < Vt:

$$Q_s = Q_{so} + \frac{\phi_o}{\alpha}\left[1 - \exp(f(x,t))\right] \qquad [37]$$

where for $C > C_y$,

$$f(x,t) = \alpha\beta\,(C_o - \lambda)\exp\left(\frac{k_1 x}{V}\right)\left(\frac{x}{V} - t\right) \qquad [38a]$$

and

$$P = \frac{\rho_\ell k_1}{(1-\phi_o)\rho m}\,.$$

For $C < C_\ell$:

$$f(x,t) = \alpha\beta' \left[\frac{1}{(C_x - C_s)^2} + \frac{2k_2}{V}(x-x) - \right]^{-3/2} \text{(H} \qquad [38b]$$

where

$$P^1 = \frac{\rho_\ell k_2}{(1-\phi_o)\rho m}\,.$$

Similarly for $x > Vt$

$$Q_s = Q_{so}\,. \qquad [39]$$

Radial Flow Model

By applying the method of characteristics to the radial flow model, equation [15] is transformed into

£•-». [40]

where s is the arc length along the characteristic curve

$$\frac{\text{f-i.}}{ds} \qquad \frac{dr}{ds} = \frac{V_r}{r}\,. \qquad [41]$$

Here V_r is the areal flow velocity in m^2/sec. The characteristic curves obtained by integrating [41] are parabolic:

$$r^2 - 2V_r t = r_0^2. \qquad [42]$$

Again the solution obtained by integrating [40] depends on the form of the kinetic equation. The results for the rate equation provided by Rimstidt and Barnes will be presented here. Results for the Bohlmann and Fleming equations can easily be obtained in forms analogous to (30) - (39)

$$C = C_s + (C_o - C_s)\exp\frac{-kr^2}{2V_r} \qquad \text{for } r^2 < 2V_r t \qquad [43]$$

and

$$C = C_S \qquad \text{for } r^2 > 2V_r t. \qquad [44]$$

Similarly, the deposition Q_s is derived for $r^2 < 2V_r t$ as

$$Q_s = Q_{so} + \frac{*>}{a}\,r \left[1 - \exp\left\{\alpha\beta(C_o - C_s)\exp\left(\frac{-kr^2}{2V_r}\right)\cdot\left(\frac{r^2}{2V_r} - t\right)\right\}\right].$$

$$\text{for } r^2 < 2V_r t \qquad [45]$$

and for $r^2 > 2V_r t$

$$Q_s = Q_{so} \qquad [46]$$

Model Validation

To test the validity of the equations derived, experimental results obtained by Bohlmann et al (1980) and Itoi et al (1984,1986) were simulated. The experiments were for flow along packed columns so that the one-dimensional flow model is appropriate.

(i) Bohlmann's Results (see Bohlmann et al, 1980)

The one-dimensional model using their rate equation was first used to match their observed results where outlet silica concentration was measured as a function of residence time in the column. Since their experiments were conducted at steady state conditions and constant temperature, equation [30] was used in the form

$$C = C_s + \left[\frac{1}{(C_o - C_s)} + kt\right]^{-1} \qquad [47]$$

where t is the residence time in the column. This is evaluated as

$$t = \frac{\text{Column Void Volume}}{\text{Flow Rate}}\,.$$

The parameters used in modelling their results are presented in Table 1. Equilibrium concentrations of amorphous silica (C_s) was calculated from Fournier, (1981).

Table 1. Data Used to Match Bohlmann's Experiments

Column Length	I.D.	C_o (molal)	pH	A (cm^2/kg)	ϕ	k^1
15.2cm	0.84 cm	0.02180	5.25	0.64E07	0.32	0.12
6.0cm	0.95cm	0.00914	6.76	0.90E07	0.30	0.12

Figure 2 shows the good agreement between the observed data and the results predicted by the model despite the fact that there is uncertainty in some parameter values (porosity for example).

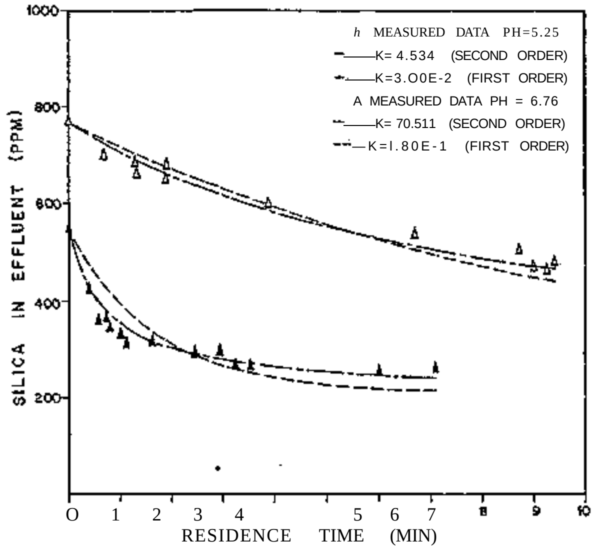

FIG. 2 SILICA CONCENTRATION VS. RESIDENCE TIME.

A first order rate equation (Rimstidt and Barnes, 1980) was also used in the model to match the observed data. The rate constant k~, as derived from Rimstidt and Barnes (1980), was changed since their value is several orders of magnitude less than that required to match the observed data.

The results are also shown in Figure 2. The model using Bohlmann's rate equation gave a better fit than the first order rate equation.

(ii) Modelling Itoi's results (see Itoi et al, 1984,1986)

Itoi et al (1984,1986) have also studied amorphous silica deposition in a porous column. In their experiments, a sample of the Otake hot water was introduced at constant temperature into the column packed with aluminium beads. The column was operated at constant pressure and changes in flow rate (due to deposition) were monitored. The amount of silica deposited in the column was then determined after drying the beads.

The result in one of their experiments (Run No. 37) was used to test the applicability of the one-dimensional model in simulating the rate of silica deposition. Here, first and second order rate equations were used, and the flowrate was assumed to be essentially constant throughout the experiment.

Since Itoi et al (1984) did not establish the initial surface area A available for deposition, this term as a first approximation, is represented by (Rimstidt and Barnes 1980);

$$A = \frac{8.55\ V_{sp}}{r}$$

where V_{sp} is the specific volume of water in m^3/kg. This assumes the column consists of closely packed uniform sphere of radius r, and is obviously an upper bound for the simulated conditions. The parameters used in modelling the results of Itoi et al (1986) are listed in Table 2.

Table 2. Data Used to Match Itoi's Experiments

Initial Concentration C_o = 475 ppm
Column Temperature T = 90°C
Initial Porosity (> = 0.37
Density of Rock p_r = 3300 kg/m^3
Density of Deposited Silica = 2040 kg/m^3
Porosity of Silica = 0.97
Surface Area = 8.86 m^2/kg
Length of Column = 50 cm
Diameter of the Column = 5 cm
Diameter of the Beads = 2 mm

The initial rate constants calculated from Bohlmann et al and Rimstidt and Barnes did not give the correct initial Q_s. Hence, several values were tried to match the initial deposition. The simulated results together with the observed data are presented in Figure 3. There is poor agreement between the experimental and model results away from the inlet. The model as shown suggests an almost constant deposition throughout the column.

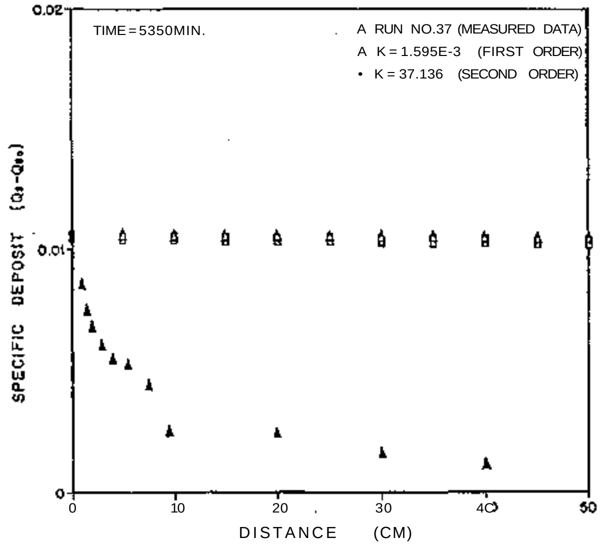

FIG.3 SPECIFIC DEPOSIT VS. DISTANCE.

Matching the distribution of Q_s proved to be difficult. Once k was selected to match Q_s at x = 0 then no other parameter remains to be selected and the decay away from x = 0 depends only on the initial concentration. Itoi and his co-workers also experienced this particular problem (personal communication). In several papers they investigated various alternative reaction models. They found that the only way to match the distribution Q_s is to use an artificially small initial supersaturation ($C-C_s$). Itoi et al (1984) justify this by claiming only some of the silica is available for deposition.

A very low effective initial supersaturation could be explained by polymerization of the monosilicic acid before introduction into the column, or in the early stages of the experiment. The presence of aluminium in Itoi's experiment may have also influenced the rate of polymerization. A small silica concentration decrease at the outlet of the column would support the claim by Itoi et al (1984) that only a small portion of the initial supersaturated silica is available to deposit in the column to form the observed Q_s distribution. However, Itoi does not provide data on concentration measurements at the outlet which could have possibly resolved these conjectures.

(iii) Results Using Fleming's Rate Equation

Fleming (1986), in his study, suggested two kinetic regimes for silica polymerization. The first regime which is a first order reaction with respect to supersaturation is controlled by the pseudo equilibrium concentation C_x, expressed in terms of temperature T by

$$\ln C_x = 9.74 - \ 2630$$

where R is the gas constant, 1.987 cal/mole-°K. As the monosilicic acid approaches C_x, a slower reaction follows which he considers as a surface rearrangement reaction of the silica molecule.

The one-dimensional model using Fleming's rate equation was also applied in matching the results of Bohlmann et al (1980) and Itoi et al (1986). Additional data used in modelling the experimental results of Bohlmann et al (1980) are listed in Table 3.

Table 3. Additional Data for Matching Bohlmann's Results

Ionic Strength M	pH	[Si0~] (nirr^2)	A_s (cm^2/cm^3)	k M'V^1	$k_{sr}r_s^2$ s^A,'^{1}
1.1	5.25	0.0359	6293	13.3	2.06E-5
1.1	6.76	0.3730	8850	13.3	2.06E-5

The silica surface charge [SiO^-] was based on the calculations made by Fleming (1986). The model parameters were slightly varied to match the observed data. The results of the model are presented in Figure 4. Clearly it can be seen that the model and experimental results quantitatively agree especially at lower initial silica concentrations.

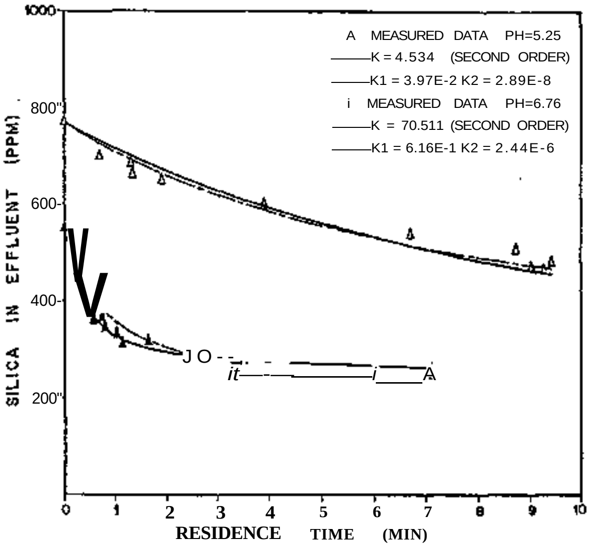

FIG.4 SILICA CONCENTRATION VS. RESIDENCE TIME.

The model was also applied to match the result of Itoi et al (Run No. 37). Again by varying the parameters, the model was able to match the initial Q_s but did not give the correct profile. Note that the effective supersaturation (C_o-C_x) in this case is 32 ppm, which is considerably lower than the previous models (> 100 ppm). However, even this value is not small enough to match the experimental results. There is considerable uncertainty in the calculation of C_x. To test the effect of varying C_x, changes were made by varying the temperature.

Since C_x is a strong function of temperature, an increased temperature increases C_x. It was increased by pushing up the temperature of the column to 98.85°C. This gives the initial "supersaturation" of less than 1 ppm, which is required to match the experimental data.

The parameters used are listed in Table 4 and the results are presented in Figure 5.

Table 4. Additional Data Used to Match Itoi's Experiment

Ionic Strength = 0.05 M
pH = 8.01
Surface Charge = 0.369 mrr^2
A_s = 85.52 cm^2/cm^3
k = 80.51 M'^{1} s''^{1}
$k_{ji}T_s^2$ = 1.57E-4 s''^{1} A_si

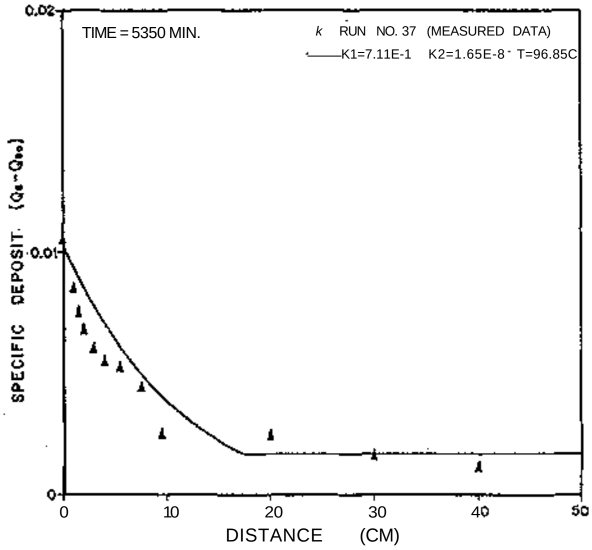

FIG. 5 SPECIFIC DEPOSIT VS. DISTANCE.

Deposition occurs near the inlet of the column which is modelled by the first kinetic regime, and reaches the transition at approximately 18 cm. After the transition, minimal deposition occurs in the part of the column which is modelled by the second kinetic regime.

Summary

The method of characteristics has been used to solve the combined silica transport and deposition equation, for several different deposition rate expressions. The most recent deposition rate equation proposed by Fleming (1986) accurately predicts the silica concentration observed in the column studies of Bohlmann et al (1980). Also the experimental studies of Itoi et al (1984, 1986) are explained qualitatively by Fleming's Theory.

The methods considered here can be extended to the non-isothermal case of injection of silica-rich fluid into a reservoir of a different temperature.

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