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CORRECTION FOR FINITE DISTANCE BETWEEN POTENTIAL ELECTRODES IN SCHLUMBERGER RESISTIVITY SOUNDINGS

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ABSTRACT

Resistivity soundings made with an expanding Schlumberger array usually result in sounding curves comprising several offset segments, each corresponding to a different spacing of potential electrodes. The offsets are partly due to resistivity inhomogeneities near the potential electrodes and partly to the necessity of using finite separations between potential electrodes in order to obtain measurable signals.

By expanding the general theoretical expression for the Schlumberger apparent resistivity in terms of MN/AB (ratio of distances between potential and current electrodes) a power-series expression is obtained for the offset caused by finite potential electrode spacing. The power series is even, with the second order correction term being related to both the slope and curvature of the sounding curve.

Analysis of the corrections needed for sounding curves typical of geothermal regions allows the seriousness of the measurement problem to be assessed and leads to the development of a stepwise correction process which can be applied to existing field data and will result in more reliable layer interpretations.

INTRODUCTION

The Schlumberger vertical electrical sounding technique for determining the variation of electrical resistivity with depth is commonly used in geothermal investigations. With this technique it is usual to assume that the measured apparent resistivity is independent of the potential electrode spacing (MN in Fig. 1). However, when resistivity decreases rapidly with depth, as is frequently encountered in a geothermal environment, this assumption is not justified; measurements with different potential electrode spacings can result in markedly different values of apparent resistivity. In this paper we demonstrate the magnitude of this effect, and indicate how corrections can be made to the measurements.

A Schlumberger sounding (Fig. 2a) consists of a set of apparent resistivity measurements made at a single site with a series of increasing current electrode spacings (AB of Fig. 1). As the current electrode array is expanded, it is standard field practice to maintain fixed potential electrodes (MN, Fig. 1) until a reliable signal level can not longer be measured. At this stage, MN is increased and, after repeating one or more of the previous measurements, the expansion is continued. Often three or four different potential electrode spacings are used for a single sounding. Results are presented as a sounding curve, in the form of a plot of the log of apparent resistivity versus the log of AB/2, comprising a series of overlapping segments as shown in Fig. 2b. It is invariably found that these segments do not form a continuous curve but exhibit discontinuities at the overlaps. This raises the two problems; firstly, how to make field measurement with minimal discontinuities and, how to correct the disjointed sections to obtain a single smooth curve amenable to computer interpretation in terms of a horizontally layered earth.

The discontinuities at the overlaps arise from two main causes. One cause (Fig. 2c) is the effect of ground inhomogeneities near the potential electrodes. For a particular placement of MN, the apparent resistivities obtained in that segment will be influenced by approximately the same amount, while those from an adjacent segment will all have been amplified or attenuated by some other nearly constant factor. This results in parallel segments for which the appropriate correction is to move the segments vertically on the sounding curve until they line up and form a smooth curve.

This paper deals with the other main cause for the discontinuities, namely, the influence of MN on the measurement of apparent resistivity. Figure 2d shows that this affect results in the curves for different segments converging asymptotically with increasing AB. In addition, however, the fact that measurements are dependent on MN can also influence the interpretation of a sounding, as, implicit in matching of measured data to a theoretical curve, is the assumption of infinitesimally small MN which is made in the generation of any theoretical model. It is thus important to know in what circumstances the implicit assumptions are not valid.

PREVIOUS WORK

A discussion of the offsets caused by finite MN and how to correct for them was given by Companie Général de Géophysique (1955) in a paper which presented the first generally available set of Schlumberger master curves. They claim, without publishing their theoretical reasoning, that as long as the ratio MN/AB does not exceed 1/5, it suffices (in correcting for finite MN) to shift the points of the field curve to the left by between 0 and *6%*. While such a correction was well suited to the graphical interpretation techniques of the day, in which field data were compared with master curves, it would create difficulties when used with modern interpretation techniques which rely on regularly spaced AB/2 values. This will be discussed in more detail later in this paper.

The CGG (1955) criterion, requiring MN/AB $\leq$ 1/5 is often quoted as a rule of thumb for making

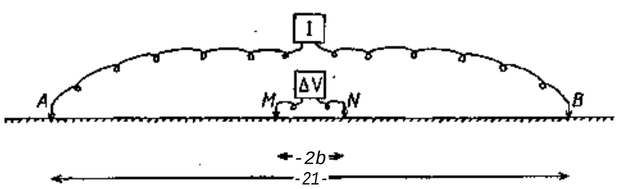

Figure Is Electrode arrangement for Schlumberger array.

electrical resistivity soundings. It is implied that, if this condition is obeyed, the finite nature of the potential array will have negligible influence on the magnitude of the apparent resistivity. Kunetz (1966), for example, states that the error would be less than *6%,* while Parasnis (1979), without discussion of the practicality of such a condition, requires that MN/AB be very much less than 1/5 for an apparent resistivity to be 'correct to within a couple of percent'. Not only can such statements be misleading, but failure to recognise this problem can be a source of significant error in the interpretation of sounding curves.

The finite MN problem is not addressed in the Geothermal Institute's "Introduction to Geothermal Prospecting" (Risk, 1982, pllO) where the condition MN/AB < 1/5 is stated as part of the definition of a Schlumberger array. The Schlumberger apparent resistivity pa is (accurately) defined by

$$\rho_a = \frac{\pi}{I}\left\{\left(\frac{AB}{2}\right)^2 - \left(\frac{MN}{2}\right)^2\right\}\frac{\Delta V}{MN} \qquad (1)$$

where I is the current injected between A and B, and AV is the voltage generated between M and N. It must be kept in mind that p^ is dependent on MN.

Some confusion may have arisen from the discussion in Keller and Fischknecht (1966, p96) where it is shown that the error in neglecting the $(MN/2)^2$ term in equation 1 is less than 5% provided MN/AB < 0.2175. However, this is a secondary issue related to the accuracy of p_a itself, and avoids the deeper problem of how to correct p_A for MN dependence.

THEORY

For an apparent resistivity measurement made using an ideal (MN -> 0) Schlumberger array, the apparent resistivity is given by

AH) - YՄl (2)

where £ is the resultant electric field strength midway between A and B.

The ideal array cannot be used in practice since the measurement of E requires that the voltage be measured between two collinear potential electrodes M and N, a finite distance apart. For this finite array the apparent resistivity p- (4>b) can be obtained from equation (1) by substituting ^ = AB/2 and b = MN/2. Thus

$$\rho_a(\ell,b) = \frac{\pi}{I}\left\{\ell^2 - b^2\right\}\frac{\Delta V}{2b}$$

$$= \frac{\pi}{I}\left\{\frac{\ell^2 - b^2}{2b}\right\}\int_{-b}^{b}\underset{\sim}{E}(\ell,\underset{\sim}{r}).\,d\underset{\sim}{r} \qquad (3)$$

where AV is the voltage between M and N, and E *(£,£)* is the electric field strength between M and N at~a point a distance r from the centre of the array.

In general, £ will not be a simple function but will depend on the resistivity distribution in the ground beneath the array. Regardless of the nature of this distribution, in the limit as b -> 0, equation (3) reduces to the idealised form - i.e.

$$\rho_a(\ell,0) = \rho_s(\ell) \qquad (4)$$

When considering a horizontally layered earth, as is generally assumed in electric soundings, it is possible to find a simple expression for the electric field vector £. For a single current electrode, the magnitude'*' E of the electric field vector, can be written in terms of the ideal Schlumberger apparent resistivity (Zohdy, 1978) as

$$E(R) = \frac{I\rho_s(R)}{2\pi R^2} \qquad (5)$$

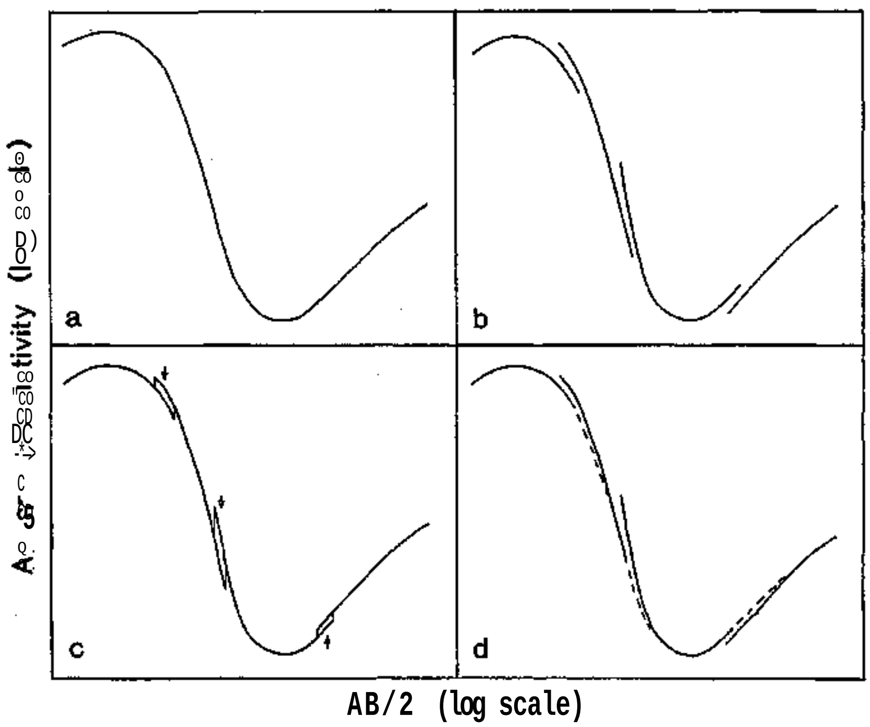

Figure 2: Typical Schlumberger sounding curve,

a) Theoretical curve.

b) Typical field curve measured in four segments.

c) The misalignment caused by local inhomoities displaces some sections vertically.

d) Misalignment due to finiteness of MN shows as asymptotically converging segments.

where R is the distance from the (single) current source of strength I. Using this expression to superimpose the contributions of electrodes A and B while noting that AB and MN are collinear, equation (3) can be written

$$\rho_a = \frac{l^2}{2b}\int_0^b \left\{ \frac{\rho_s(l+r)}{(l+r)^2} + \frac{\rho_s(l-r)}{(l-r)^2} \right\} dr = \frac{l^2}{2b}\int_{-b}^{b} \frac{\rho_s(l+r)}{(l+r)^2}\, dr \qquad (6)$$

An expression with the same form was derived by Depperman (1954) and has been used by several authors to derive approximate expressions for relating apparent resistivities made with finite MN (ρ_a) to the ideal Schlumberger values (ρ_s). Depperman (1954) evaluated the integral using Lagrangian interpolation to obtain $\rho_a(l,b)$ as a weighted sum of values obtained from the ideal curve ($b = 0$). Other approaches include approximating the sounding curve (on log-log paper) as straight line segments (Mundry, 1980), and using linear filter theory to formulate corrections (Koefoed, 1979; Das & Kumar, 1977). Emphasis was also placed, in these early works, on establishing the relationship between Wenner ($b = l/3$) and Schlumberger apparent resistivities.

A shortcoming of the above approaches is that none of them can be expressed in a way which allows the order of approximation to be made clear or the consequences of the simplifying assumptions to be spelt out. Thus, it is difficult to assess the accuracy of the corrections. The approach taken here is to formally expand equation 6 about b = 0 as a power series in the small quantity b/l which allows the size of neglected higher order terms to be assessed.

The integral in (6) can be expanded to any required order in powers of b/l; to third order the expansion is

$$\frac{l^2}{2b}\int_{-b}^{b}\frac{\rho_s(l+r)}{(l+r)^2}\,dr = \rho_s(l) + \frac{1}{6}\left(\frac{b}{l}\right)^2\left\{ l^2\frac{\partial^2\rho_s}{\partial l^2} - 4l\frac{\partial\rho_s}{\partial l} + 6\rho_s \right\} + O\left(\frac{b}{l}\right)^4$$

Thus

$$\rho_a(l,b) = \rho_s(l) + \frac{1}{6}\left(\frac{b}{l}\right)^2\left[l^2\frac{\partial^2\rho_s}{\partial l^2} - 4l\frac{\partial\rho_s}{\partial l} \right] + O\left(\frac{b}{l}\right)^4 \qquad (7)$$

Since the power series is even, it is rapidly convergent and the first neglected term is of order $(b/l)^4$. Even in the extreme case of a Wenner array for which $b/l = 1/3$, the error involved in neglecting the higher order terms is only about 1%. Since the second order term involves both first and second derivatives ($\partial\rho/\partial l$, $\partial^2\rho/\partial l^2$), the assumption made by Mundry (1980), that there is linearity of $\ln\rho$ vs $\ln l$, is not strictly correct as it would neglect the contribution from the second derivative.

Since sounding curves are invariably drawn as log-log plots, it is more practical to express (7) as

$$\rho_a(l,b) = \rho_s(l)\left[1 + \left(\frac{b}{l}\right)^2 F + O\left(\frac{b}{l}\right)^4\right] \qquad (8)$$

where

$$F = \frac{1}{6}\left[\frac{\partial^2 \ln\rho_s}{\partial \ln l^2} + \left(\frac{\partial \ln\rho_s}{\partial \ln l}\right)^2 - 5\frac{\partial \ln\rho_s}{\partial \ln l} \right] \qquad (9)$$

Thus, equation 9, allows the factor needed to correct an apparent resistivity measured with finite MN to be calculated from the gradients of the corresponding ideal sounding curve (equation 2).

ALTERNATIVE APPROACHES

An alternative approach which has been recommended by CGG (1955) is to make an adjustment to the spacing rather than to the apparent resistivity. Equation (6) can be interpreted as indicating that an apparent resistivity measured with a finite spacing is a weighted average of the theoretical Schlumberger resistivities covering the range of spacings between l-b and l+b, with the weighting strongly favouring the smaller spacing. Thus the use of a finite potential electrode spacing may be thought of as reducing the effective electrode spacing from l towards (l-b).

To determine an 'equivalent' spacing, we calculate the required fractional correction Δl to the spacing l such that

$$\rho_a(l,b) = \rho_s(l + \Delta l) \qquad (10)$$

By fitting the appropriate segment of the sounding curve to a quadratic, consistent with the terms in the expansion above, it can be shown, for (b/l) small, that

$$\frac{\Delta l}{l} \simeq F \Big/ \frac{\partial \ln\rho_s}{\partial \ln l} \qquad (11)$$

where F is given by (9) above. Hence, it is possible to 'correct' a measurement by estimating an equivalent spacing, and plotting the (uncorrected) measured value of apparent resistivity at this adjusted array spacing.

It is interesting to compare the two approaches discussed so far with that of making corrections using linear filter theory (Koefoed 1979) which requires making adjustments to both spacing and apparent resistivity to construct a modified sounding curve. As has been discussed earlier, there are practical advantages in making an adjustment to the apparent resistivity value alone, particularly when standard logrithmically spaced array sizes are being used.

EXAMPLE OF TWO LAYER CASE

For a two layered earth, the theoretical expression for $\rho_s(l)$ is simply expressed as a power series (Van Nostrand and Cook, 1966) from which the derivatives are easily computed. Using these, Figure 3b has been compiled to illustrate the size of the correction factor F as a function of resistivity contrast and the ratio of electrode spacing to thickness of the top layer. It is instructive to examine the size of the required correction to the finite MN apparent resistivity for the case where b/l has the threshold value mentioned in many textbooks, viz, $b/l = 1/5$; the scale on the right of Fig. 3b gives the size of the correction for this case. For ground resistivities decreasing in the ratio 1:1/40 a correction of nearly 17% is needed near the point of sharpest curvature.

The correction factor has relatively small values for cases where resistivity increases with depth and the maximum possible gradient is unity. However, for steeply descending curves, typical of soundings in a geothermal environment, the correction factor becomes very large and the departure from the theoretical curve can be well in excess of the error suggested in textbooks. During fieldwork, the problem can be compounded by the fact that, for steeply dipping curves, the signal level drops very rapidly, and the operator may be obliged to use larger values of MN than is desirable.

The term arising from curvature can make a significant contribution as the curve flattens out. The greatest gradients will occur in the H-type (or minimum) resistivity curve, a type of sounding curve often encountered in geothermal exploration. Hence, the corrections needed for adjusting measured resistivities in a geothermal situation can be large even when the standard criterion has been adhered to.

EFFECTING THE CORRECTION

If the sounding curves obtained for a particular survey contain sections of steep gradient or sharp curvature the need for making corrections can be avoided by restricting *bit* during fieldwork. However, this may result in a degradation of signal-to-noise ratio and poor quality data. A better approach is to ensure adequate signal levels by using a larger *b/£,* and to correct the measured apparent resistivity using equation 9. This raises several practical and computational considerations. The mismatch of two overlapping sections of a sounding curve will in general result from a combination of two causes: that due to inhomogeneities and that caused by the use of finite MN. In particular over-correction, due to confusion of the two effects, must be avoided. To ensure this, it is essential to correct for finite MN prior to any other adjustments.

The most practical method found so far for correcting for finite MN is to calculate the derivatives required to evaluate equation 9 directly from field data and to determine the derivatives separately for each segment of the field curve. The use of the field curves rather than the ideal Schlumberger curves does not introduce a major error as it can be shown that:

$$\frac{\partial \ln \rho_a(\ell,b)}{\partial \ln \ell} = \frac{\partial \ln \rho_s}{\partial \ln \ell}\left\{1 + O\left(\frac{b}{\ell}\right)^2\right\}$$

$$\frac{\partial^2 \ln \rho_a(\ell,b)}{\partial \ln \ell^2} = \frac{\partial^2 \ln \rho_s}{\partial \ln \ell^2}\left\{1 + O\left(\frac{b}{\ell}\right)^2\right\} \qquad (12)$$

Thus, making the substitution does not alter the order of approximation in calculating equation 8.

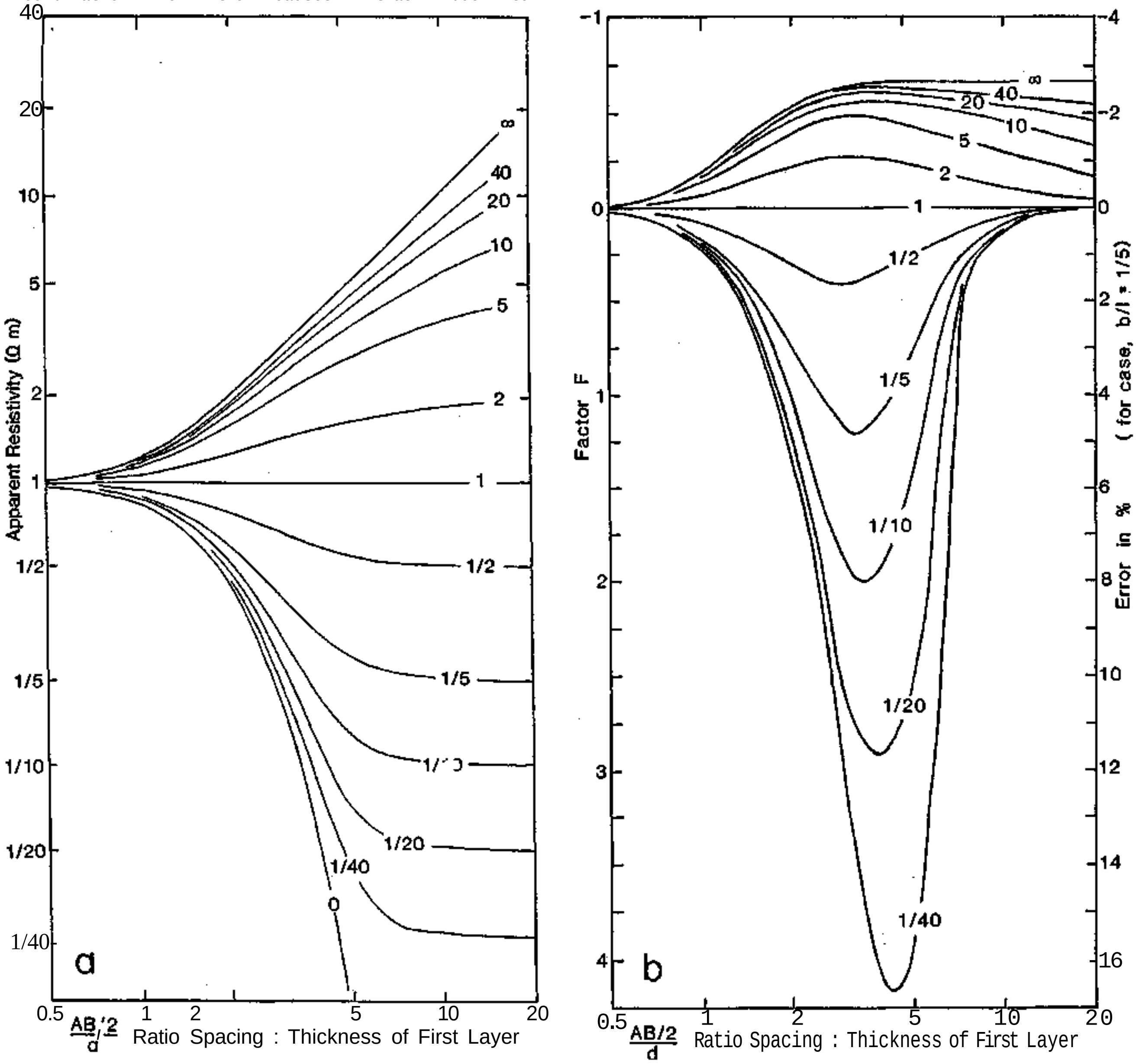

Figure 3: Theoretical resistivity curves for 2-layered earth plotted for a selection of resistivity contrasts between the upper and lower layer.

a) Apparent resistivity versus AB/2.

b) Correction factor F (in Equation 9) versus ratio of spacing to first layer thickness. The right hand axis gives the size of error incurred if the correction is not made, for the case where b/1 = 1/5.

Several interpolation schemes have been tested to calculate the gradients from field data spaced at 10 apparent resistivities per decade. The simplest scheme has been successfully applied to simulated field data, i.e. a set of smooth, regularly spaced apparent resistivities obtained from an exact computer model. For these data, a 5-point Lagrangian interpolation scheme was able to accurately calculate both first and second derivatives in equation 9, and hence the required correction. This approach is equivalent to doing an equally-weighted exact fitting of a 4th order polynomial to the nearest 5 data points straddling the point of interpolation.

This approach has several shortcomings when applied to real data. Scatter on the data makes it necessary to replace the exact fitting process with one incorporating some redundancy and smoothing by using a least squares fitting process. Divided differences formulae need to be used if the apparent resistivities are not regularly spaced. At points at either end of a segment of field data, the centred 5-point interpolation formulae can no longer be used and an asymmetric interpolation scheme must be applied.

As an example of the use of the correction scheme, a field sounding curve measured over a salt water intrusion in the South Island, New Zealand (Fig. 4, Table 1) has been chosen since the shape of the curve is similar to those often found in geothermal regions and the data is relatively smooth. The left segment begins with b/l = 2/5 (exceeding the usual guideline) while the other two begin with b/l = 1/5 (at the guideline threshold). For these data the corrections were obtained using a least-squares fitting process. Various schemes for weighting the data were tried. For the example shown, weights at, and on either side of, the interpolation point were: 2, 4, 7, 9, 7, 4, 2, for a 7-point interpolation scheme. A similar asymmetric scheme was used at the ends of the segments. Thus, for calculating the derivatives, the fitting had the effect of smoothing the data and weighting more strongly to points at the centre of the range.

Figure 4a shows the uncorrected apparent resistivities for the three segments which overlap by two points. The second overlap shows a discrepancy of 54% at AB/2 = 50 m where b/l = 1/5. The fractional corrections calculated from equation (9) for each point are shown in Fig. 5a and Table 1. The biggest corrections are required for the left hand three points of the third segment which corresponds to the part of the sounding curve at and

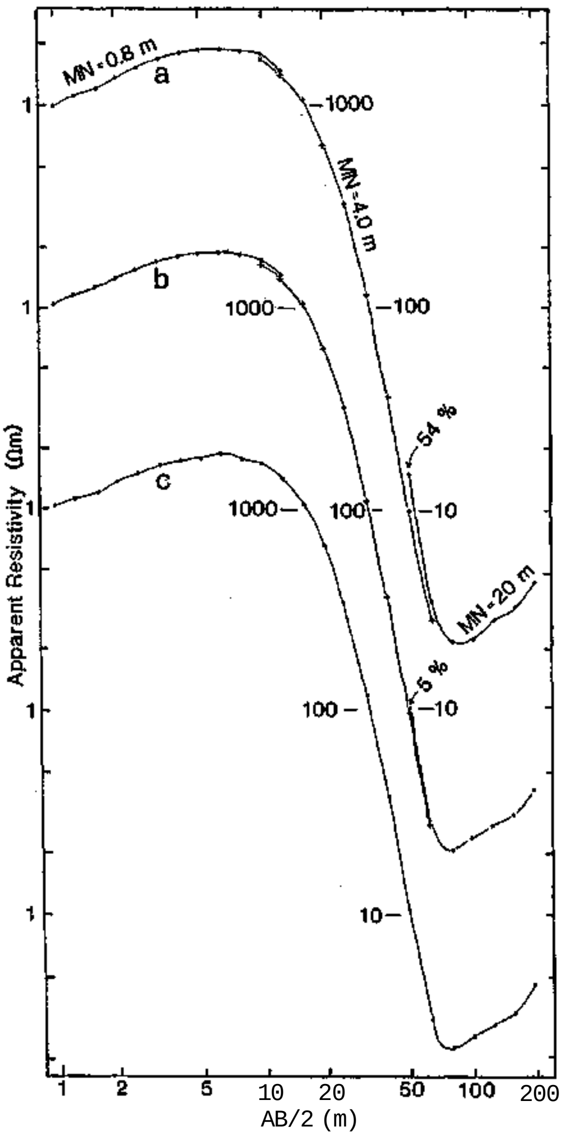

Figure 4: Field apparent resistivity curve measured over an aquifer of salt water in South Island, New Zealand. This curve has a similar shape to those often found in geothermal fields, but is smoother and, thus, the derivatives can be more easily calculated.

a) Uncorrected apparent resistivities.

b) Apparent resistivities corrected by the factor shown in Fig. 5(a).

c) Apparent resistivities derived from (b), further corrected to allow for the effects of lateral inhomogeneities. The right hand segment is unchanged; the middle section has been moved upwards by 4.2% and the left one downwards by 1.9% .

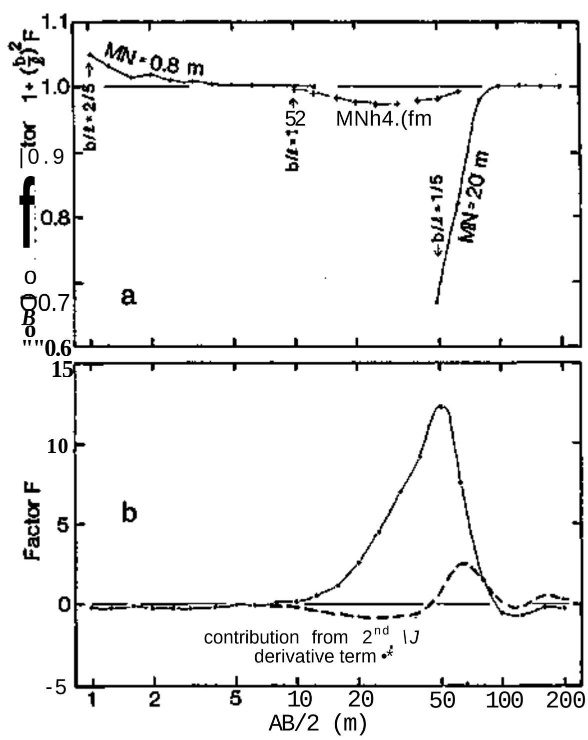

Figure 5: Corrections calculated from Equations 8 and 9 for the data shown in Fig. 4.

a) Total correction factor applied to each apparent resistivity in Fig 4a to obtain Fig. 4b.

b) Thin line gives factor F;
Thick, line gives contribution to F from the term in Equation 9 involving the second derivative.

Table 1: Details of the steps taken in correcting the right-hand two segments of data shown in Fig. 3. First the correction for finite MN is applied to both segments using factor f = 1 + F(b/1) . Then the first segment is adjusted vertically by the factor f = 1.0425 to correct for local inhomogeneities.

AB/2 (m)	F	Appar. resist. uncorr-ected (0m)	t	Appar. resist. MN-corrected (Om)	f	Appar. resist. corr-ected (Om)	Appar. resist. uncorr-ected (Om)	f	Appar. resist. corr-ected (Om)
		<------ MN = 4 m ------>					<--- MN = 20 m --->		
10	0.167	1613	0.993	1602	1.0425	1670			
12.5	0.518	1382	0.987	1364	1.0425	1422			
16	1.280	1030	0.980	1009	1.0425	1052			
20	2.561	632.9	0.975	617.1	1.0425	643.3			
25	4.506	321.9	0.972	312.9	1.0425	326.2			
32	7.062	110.6	0.973	107.6	1.0425	112.2			
40	9.050	35.3	0.978	34.5	1.0425	36.0			
50	12.370	9.48	0.981	9.30	1.0425	9.70	14.56	0.669	9.74
63	7.520	2.68	0.992	2.66	1.0425	2.77	3.28	0.841	2.76
80	1.468						2.07	0.978	2.02
100	-0.472						2.31	1.005	2.32
125	-0.589						2.65	1.004	2.66
160	-0.134						3.05	1.001	3.05
200	-0.517						4.08	1.001	4.08

just to the left of the minimum. After applying these corrections to the (uncorrected) curve in Fig. 4a, the one in Fig. 4b is obtained. The left hand overlap is altered only slightly but a big improvement is made to the second overlap where the discrepancy has been reduced to only 5%. The residual discrepancies are now presumed to be due to local inhomogeneities and vertical adjustments by factors of 1.06 and 1.04 for the left and right overlaps, respectively, produces the nearly smooth curve in Fig. 4c which is then ready for layered interpretation. Details of the correction process for the vicinity of the second overlap are given in Table 1.

The difference between our approach and that of some of the earlier workers, particularly Mundry (1980), who neglected the contribution from the second derivative, can be gauged by considering Figure 5b. The solid curve gives the value of F while the dotted curve gives the contribution to it of the part involving the second derivative in equation 9. This component is usually rather small, sometimes augmenting, and sometimes opposing the main component, but it becomes very important at the minimum (and the earlier maximum) where the contribution from the first derivative vanishes.

CONCLUSION

The theory presented here shows that the accuracy obtainable from layered interpretations of Schlumberger resistivity sounding data can be improved by correcting for the finiteness of the potential electrode spacing MN using the correction factors in equations 8 and 9. The need for making the correction can be avoided by restricting the ratio b/ ; however, where steeply dipping or sharply curving soundings are involved, b/ must always be much smaller than the value 1/5 often quoted as a maximum threshold value.

Application of the correction process is at present cumbersome and in need of stream-lining. A reasonably accurate scheme has been worked out for relatively smooth sounding data but, at present, this cannot be readily applied to noisy data such as is often encountered in geothermal exploration.

There is a need to develop a practical scheme which can be used routinely for making corrections for both the finiteness of MN and the effects of local inhomogeneities. At present, these two corrections are done sequentially, after which the corrected data are inverted using a further computer process. It would be more satisfactory to integrate these three processes so that the raw field data can be inverted with a non-linear optimisation process that simultaneously makes the required corrections and determines the best fitting layered interpretation.

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