

MATHEMATICAL MODELLING OF SUBSIDENCE AT WAIRAKEI GEOTHERMAL FIELD

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**ABSTRACT**

A multi-dimensional model is required at Wairakei in order to account for the significant observed horizontal deformations. From the subsidence models and subsidence/compaction relationships examined, a finite element model was chosen for its potential to reproduce such deformations and for its ability to utilise either linear or nonlinear, nonhomogeneous material models. Using the ADINA finite element program, a simple two-layer model was developed incorporating the Pumice and Wairakei pumice breccia layers as the overburden, and the Huka formation as the reservoir. The deformation in the Wairakei formation was considered to be less significant and was neglected.

Using suitable estimates for the elastic properties of each layer (Young's modulus and Poisson's ratio) and by depressurising in the reservoir a zone whose lateral extent corresponds approximately to the point of inflection on the observed vertical deformation profile, a good fit to both the horizontal and vertical deformation profiles was obtained. An alternative model examined the effect of a lateral variation in Young's modulus, as opposed to a lateral variation in pressure decline. Again, a good fit was obtained, particularly to the vertical profile. Features of both models are apparent at Wairakei.

**1. Introduction**

A geothermal reservoir consists of one or more subsurface geological formations from which geothermal fluid (predominantly water and steam) is being withdrawn. If, as a result of such exploitation, nonsteady flow conditions arise in the reservoir (i.e. production rate exceeds the recharge rate) then a reduction in both the reservoir temperature and the pore pressure of the fluid in the reservoir formations will occur.

The formations above the reservoir from which no fluid is being withdrawn are known as the overburden. The weight of the overburden contributes to the state of effective stress in the reservoir. The reduction in pore pressure due to exploitation results in an increase in effective stress in the reservoir, which in turn leads to a reduction in the porosity of the porous reservoir formations. This process is known as pore collapse. In addition, the pressure drop also leads to the closing, in full or in part, of fluid-filled fractures which are generally present in geothermal reservoirs. Also, the reduction in temperature will cause thermal contraction of the reservoir formations.

The above three processes lead to a reduction in volume of the reservoir formations; this is known as compaction or consolidation. The deformations that occur at depth in the reservoir will be propagated through the overburden to the ground surface. The horizontal and vertical deformations that occur at the ground surface can be collectively termed as subsidence. The major physical process contributing to subsidence is pore collapse, although fracture closing may be significant in highly fractured geothermal systems, in which case it should be given

special consideration. Thermal contraction is generally the least significant physical process contributing to subsidence (Finnemore and Gillam, 1976).

The modelling of subsidence above geothermal reservoirs can be divided into two fundamental areas. The first of these involves determining the changes in pressure and temperature that occur within a geothermal reservoir as a result of exploitation. The second involves the modelling of the physical processes that lead to the actual deformation of the geological formations that comprise the reservoir and its overburden. The processes of reservoir compaction and the propagation of the deformations through to the ground surface together form the basis of a deformation model. For the purpose of verifying a deformation model a history of measured pressure and temperature changes can be used. Prediction of further subsidence requires a fluid mechanical model of the reservoir in order to provide an extrapolation in time of the pressure and temperature change data.

The compaction in the reservoir will reduce the formation permeabilities and will consequently affect subsequent changes in pressure and temperature. Therefore, a completely rigorous subsidence model would require the reservoir and deformation models to be fully coupled. In many cases, the degree of influence that reservoir compaction has on subsequent changes in pressure and temperatures is negligible; the reservoir and deformation models can be decoupled, thereby considerably reducing the computational complexity of the complete subsidence model.

If the exploitation of a geothermal reservoir (or indeed an oil or groundwater reservoir) leads to subsidence of the ground surface, the resulting deformations can have serious (if not disastrous) effects on the surface and subsurface installations directly related to the geothermal field. In the Long Beach-San Pedro area in California, the shear forces associated with differential horizontal and vertical deformations have resulted in the shear failure of oil well casings (McCaan and Wilts, 1951). At Wairakei, the steam pipelines, main drain and nearby highway are affected, as is the Wairakei stream (Stilwell et al., 1975). Consequently, the significance of predicting the magnitude and location of such surface deformations is apparent in both the design and placement of the installations and their environmental effect.

**2. Model Characteristics**

The extent to which a subsidence model can adequately represent the deformation processes within a geothermal system is dependent upon the complexity of the geometry and the assumptions made concerning the material model. Incorporating a reservoir model increases the computational complexity (especially if it is coupled to the deformation model) and adds to the problem of data acquisition.

The simplest subsidence calculations are one-dimensional, and in many cases they are adequate.

However, where the differential vertical deformation is high, it is likely that significant horizontal deformation will be apparent. Thus, a two-dimensional, axisymmetric or fully three-dimensional model geometry would be required in order to account for the observed deformations. The material model or constitutive relationship is the greatest cause of uncertainty in modelling the deformation processes. In models based on elastic or poro-elastic theory, a linear approximation is made to what is generally a non-linear relationship. Failure of the material is not often considered (particularly in tension as the material can no longer behave as a 'continuum'), although the partial irreversibility of plastic deformation can be easily accounted for by the use of additional parameters (indexes or moduli) for expansion or swelling. Additional simplifying assumptions are that the material is homogeneous and isotropic.

When large changes in temperature are occurring, thermal contraction may also be a cause of significant deformation. For some implementations, the effects due to temperature changes can be modelled as the result of an equivalent change in pressure (Miller et al., 1980). For a temperature change of  $\Delta T$ , the equivalent pressure change is  $3\alpha K \Delta T$ , where  $\alpha$  is the coefficient of linear thermal expansion and  $K$  is the bulk modulus.

The complexities of fluid flow add to the uncertainty created by the material model. Not only is there the possibility that the deformation and fluid flow processes are highly interactive (and therefore need to be coupled), there may also be a delayed response to the pressure changes. Other complications are provided if the flow is multi-phase, as the boundary between the liquid and vapour phases may not be stationary, and the saturation of the steam will generally be unknown. Thus, an assumption may be made that the flow is single-phase, and possibly isothermal.

It is therefore important that the accuracy of the model representing the physical processes of deformation and fluid flow be balanced against prohibitive computational complexity and the non-availability of sufficiently detailed and accurate input data.

### 3. Solution Techniques

#### One-dimensional consolidation

If the uniaxial compressibility is assumed to be independent of pressure and depth, then the vertical subsidence (compaction or consolidation) at a point on the surface is given by  $\Delta H = C_m \Delta p H$ , that is, the product of uniaxial compressibility, pressure change and depth of the compacting layer. If more than one layer is compacting, then the deformation of the surface is the combined compaction of all layers. The compressibility can be measured from core samples, under conditions matching those in situ. From poro-elastic theory,  $C_m = 1/(\lambda + 2G)$ .

The void ratio-effective stress relationship is nonlinear. However, the dependence of void ratio on the logarithm of effective stress is linear over a wide range. Thus, non-linear or Terzaghi type consolidation is expressed by

$$\Delta H = H/(1 + e_0) \cdot C_c \log(1 + \Delta p/p_0)$$

where  $e_0$  and  $p_0$  are the initial void ratio and pressure, and  $C_c$  is the compression index.

As the deformation process is inelastic, the deformations are not completely reversible. This is taken into account by additional co-efficients for expansion or swelling of the material. The same linear and non-linear (logarithmic) assumptions are made.

Repeating the calculations at a number of surface points enables contours of vertical subsidence to be established (Narasimhan and Goyal, 1979). However, if horizontal surface deformation is significant, then a subsidence/compaction relationship is required, such as a thin disc model (Geertsma, 1973) or a plate or beam analogy (Lee, 1969).

#### Nucleus-of-Strain methods

Nucleus-of-strain methods are based on the effect that isotropic contraction of a spherical inclusion in a semi-infinite elastic medium (or half-space) has in terms of displacement of the free surface of the medium. Subsurface displacements and shear stress are other important effects. At any point on the free surface, the displacements due to a zone of pressure decline within the half-space can be determined by integrating the effects of all the nuclei-of-strain that would fill the zone of pressure decline. A simplifying assumption is that a disturbance is occurring at a particular depth (the mean depth of the compacting layer), with the magnitude of the disturbing force dependent upon the magnitude of the pressure change and the thickness of the compacting layer at that point (McCaen & Wilts, 1951). The displacements at a point on the surface can then be determined by integrating the effects of all such disturbing forces at that depth. The integration can only be achieved by a discrete summation, which can be made at all points of interest on the surface. Also, as the medium is linearly elastic, the effects of disturbances at different depths can be summed, thus where the zone of pressure change is very thick, it can be divided into a number of suitably thin layers.

The nucleus-of-strain method provides a computationally simple technique for producing general three-dimensional displacements and shear stress distributions. In general, it is limited to homogeneous isotropic semi-infinite elastic mediums. However, this disadvantage was overcome by Borsetto and Carradori (1981), who used the finite element method to perform the integrations in a horizontally layered system.

#### Finite Element Methods

Where horizontal surface displacement is significant, three-dimensional finite element methods have the theoretical potential to produce the best results. They have the advantage of being able to model inhomogeneous anisotropic formations and non-linear constitutive laws, with fluid flow and deformation coupled. However, owing to the computational complexity and expense of operation, three-dimensional deformation models that are fully coupled to the reservoir model have not yet been successfully developed.

Models of lesser complexity are still capable of producing useful results. The finite element method has been applied successfully to the solution of two-dimensional, fully coupled fluid flow/deformation problems (Lewis and Schrefler, 1978; Sandhu, 1979) but with only single-phase, isothermal fluid flow and a linearly elastic constitutive relation for the materials. The two-dimensional problem could be extended to incorporate two-phase, non-isothermal flow but at the expense of considerable additional computational complexity. Lewis and Schrefler (1978) and Sandhu (1979) have both shown that decoupling the flow and deformation equations can lead to significant differences in the predicted results; therefore, a coupled formulation will be more reliable. Lippmann et al. (1976) have shown that temperature effects can also be significant.

#### Other Methods

Finite difference methods, the integrated finite difference method and boundary integral equation techniques are other methods available for solving the subsidence problem.

#### 4. ADINA Finite Element Program

The ADINA or "Automatic Dynamic Incremental Non-linear Analysis" finite element program has a variety of linear and non-linear material models available, including elastic-plastic and time-dependent models. The simplest are the isotropic and orthotropic linear elastic models. A large degree of orthotropic anisotropy can have a significant effect on the magnitude of both horizontal and vertical surface deformations.

Deformation can be induced by applying a pressure load on the boundary of a zone of pressure change. The isotropic thermo-elastic model enables temperature changes to be specified at element nodes, and using the relationship between temperature and pressure changes ( $\Delta P = 3\alpha K \Delta T$ ), pressure changes can also be applied as equivalent temperature changes. However, the isotropic thermo-elastic model is a non-linear implementation and therefore has a computational disadvantage over the linear elastic models.

The Curve Description model enables the elastic properties ( $E$ ,  $G$ ,  $\nu$ ) to be specified as piece-wise linear functions of volumetric strain, and incorporates depth-dependent tension cut-off or stress relief. The Von Mises elastic-plastic model incorporates either isotropic or kinematic strain hardening. The stress-strain relationship can be specified as a piece-wise linear function. A Mohr-Coulomb type failure criterion is provided by the Drucker-Prager plastic cap model. This differs from the other models in that the main parameters specified are the cohesion and angle of shearing resistance. This model also incorporates depth-dependent tension cut-off.

#### 5. Wairakei Case Study

The major advantages of the finite element method are its ability to use linear or nonlinear nonhomogeneous material models in a completely general (but finite) geometry. However, the more sophisticated the model, the more extensive the input data required. Drilling holes just for core samples is expensive. Indeed, analysis of such samples, using small-scale laboratory tests, cannot give a true indication of the macroscopic in situ material properties - this is particularly evident for highly fractured systems.

Since the rate of pressure decline in the steam zone and the rates of surface deformation (both horizontal and vertical) at Wairakei are approximately constant at present, a linear elastic material model was chosen for the MINA finite element program. Also, as the subsidence bowl at Wairakei is not axisymmetric, the deformations are essentially three-dimensional; however, since the major components of the horizontal surface movements are towards the centre of the bowl, a two-dimensional cross-section, analysed in plane strain, should adequately represent the structure.

##### Pressure decline

From depth-pressure relationships for the wells closest to the subsidence bowl, it can be deduced that most of the Steam Zone occurs in the Huka formation. The effect of groundwater extending into the Huka formation would result in the subsidence calculated here being an overestimate. Robertson (1984) included such influences; however, the extra information required on groundwater levels, etc., introduces complication with only marginal effects. In this study, therefore, the effect of changing groundwater levels was ignored. Also, the measured pressure decline in the hot water zone within the Maora formation is less than that in the steam zone or Huka formation.

From the subsidence models examined in Section 3 above, in particular the nucleus-of-strain models (Borsetto and Carradori, 1981; Mallon, 1984) and the

numerical experiments using ADINA, the point of inflection in the vertical deformation profile (which also corresponds to the maximum horizontal deformation) corresponds approximately to the edge of the depressurized zone. It was assumed, then, that the size of the current depressurized zone at Wairakei can be determined by examining the actual deformation profiles (Fig. 5.1).

The lateral extent of the zone on the minor axis of the ellipse is approximately 200-250 m, while on the major axis it is 350-400 m.

The well closest to the centre of the subsidence bowl, WK53, shows a 0.2 MPa pressure decline over the ten year period 1970 to 1980. Other wells, however, even as far away as the Western Borefield (e.g. WK65) show a pressure decline of 0.4 MPa over the same period. Averaging over wells closest to the subsidence bowl gives an annual pressure decline of 0.04 MPa per year in the steam zone. This is close to the value of 0.05 MPa per year cited by Allis and Barker (1982).

##### Material properties

For the linear elastic model being developed here, only the elastic moduli are of interest. Other properties such as density, porosity, permeability, etc., are relevant only if a reservoir model is to be coupled with the deformation model. Hendricksen (1976) measured the material properties of the formations at Wairakei from tests on representative core samples. Two methods, namely shear and plane wave tests and triaxial tests were used to measure the elastic moduli; there is a difference exceeding an order of magnitude between the results from each of the two methods.

Poisson's ratio can be assumed to be  $\nu = 0.3$  and could be justified by at least one of the test methods for each layer. However, the choice of  $\nu$  is not critical. The deformations are dependent mainly on the shear stiffness or shear modulus,  $G$ . For a linear elastic material, this depends on both Young's modulus  $E$  and Poisson's ratio ( $G = E/(1 + \nu)$ ).

Typical results for Young's modulus  $E$  are given in Table 5.1, and show the discrepancy between the different methods. The high relative strength of the Wairakei ignimbrite shows that this layer can be safely assumed to be incompressible.

TABLE 5.1: Results for Young's modulus, found by Hendrickson (1976). Confining pressures are in parentheses.

Formation	Shear and Plane Wave Tests		Triaxial Tests	
	(MPa)		(MPa)	
Pumice	2280	(3.45)	176	(1)
Huka	4770	(3.45)	154	(5)
Waiora	8060	(3.45)	0.9-2.9	(3.45)
Wairakei Ignimbrite	36800	(3.45)		

Consolidation tests on samples of Huka pumice breccia and Waiora pumice breccia (Allis and Barker, 1982) yield consolidation curves of classic shape, though each is distinct. The difference may be due to the fact that the Huka pumice breccia was deposited in water, unlike the Waiora pumice breccia. Also, the Huka pumice breccia may have been hydrothermally altered. The normally consolidated compressibilities

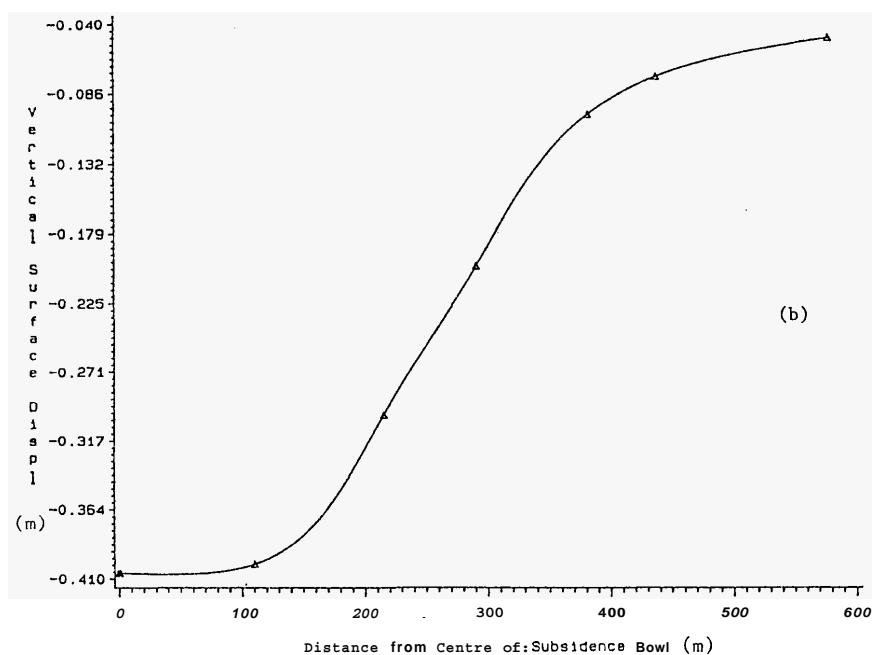
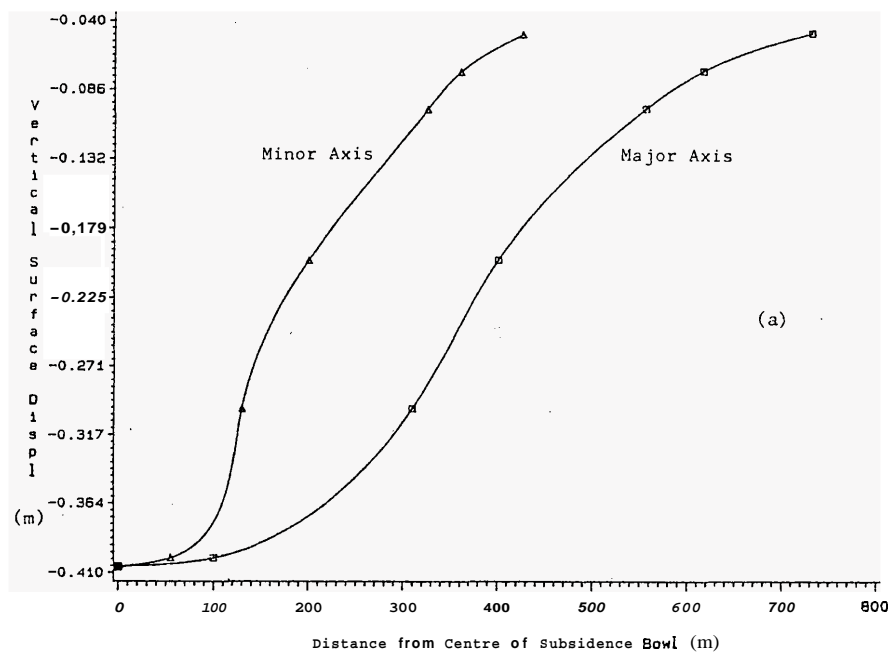


Figure 1: Vertical deformation profiles (a) minor and major axes of (elliptical) subsidence bowl; (b) South-east traverse.

for the Huka pumice breccia and Waiora pumice breccia were  $0.08 \text{ MPa}^{-1}$  and  $0.23 \text{ MPa}^{-1}$  ( $E = 10.4 \text{ MPa}$  and  $3.6 \text{ MPa}$ ) respectively. The preconsolidation pressure for both materials is about  $1 \text{ MPa}$ . From an idealised effective stress-depth profile, the Huka formation is most likely normally consolidated, while the overburden is still preconsolidated. As neither layer has been subjected to a significantly higher effective stress, there is no justification for an over-

consolidated overburden. Consequently, it is believed that the apparent transition from overconsolidation to normal consolidation is in fact a constitutive property of the Waiora pumice breccia. From calculations by Robertson (1984), the Young's modulus for the Waiora formation is about  $3500 \text{ MPa}$ ; there is thus very little contribution from the Waiora formation to total subsidence.

Hendrickson's (1976) results give no information on the Wairakei pumice breccia. It was assumed therefore that the Pumice and Wairakei pumice breccia layers have the same material properties. The two layers are combined to form a single overburden layer.

#### System geometry

The system is now reduced to two layers: the Huka formation represents the reservoir in which compaction is taking place due to decline in pore pressure, and the Pumice and Wairakei pumice breccia form the overburden. Suitable thicknesses are determined from core information at wells close to the subsidence bowl. A suitable first estimate for the depth of each layer is 100m for the overburden and 150 m for the reservoir.

#### Model One - lateral variation in pressure decline

The cross-section used, reduced to two layers, is shown in Figure 5.2. Displacements in the (y,z) directions are (v,w). Vertical displacements downwards are negative, while horizontal displacements towards the centre of the subsidence bowl are positive.

Using the assumptions outlined above, a central, localized region of pressure decline is defined. Compared with the nucleus-of-strain method, however, the finite element analysis has the advantage that each element can have its own set of material properties. In this case, it is assumed that the two layers (overburden and reservoir) are separately homogeneous.

There is, in reality, a finite pressure decline outside the zone of maximum subsidence; the pressure drop in the central region is thus taken to be that in excess of the decline in the surrounding reservoir. Also, symmetry has been exploited, with only one half of the cross-section being analysed. Non-symmetric cases are easily analysed, however. It was also found that, in order to minimize outer edge effects, the finite element model required a large width-to-depth ratio.

In Model One, the effect of a decline in pore pressure is obtained by a spread load, or two dimensional pressure load, around the boundary of the depressurized zone.

The primary assessment of the performance of a model can be made by comparing the calculated surface displacements with those measured by survey. After some experimentation, it was found that the overburden stiffness must be greater than that of the reservoir by at least an order of magnitude. The reservoir stiffness required to obtain vertical displacements of the correct magnitude is 10 MPa, determined from the measured normally consolidated compressibility of the Huka formation. The apparent normally consolidated compressibility of the Wairakei pumice breccia (in the overburden) corresponds to a stiffness of 3.6 MPa, which is less than the reservoir stiffness. It was therefore concluded that the overburden must be from the "overconsolidated" section of the curve in order to have stiffness greater than that of the reservoir. A suitable stiffness for the overburden, giving horizontal displacements of the correct magnitude, is 150 MPa.

Increasing the reservoir depth results in an increase in both the maximum vertical and horizontal displacements. With a reservoir depth of 150 m, overburden depth of 100 m, stiffnesses of 10 MPa and 150 MPa respectively, and a localized pressure decline of 0.04 MPa per year over a load width of 250m, vertical and horizontal displacements profiles of the correct magnitude and approximately correct shape were obtained (see Figure 5.3).

#### Model two - lateral variation in compressibility

Several features of the observed vertical deformation profile could not be accounted for by the results of model one. The curvature at the centre of the calculated profile is invariably too large, and the calculated vertical displacements tend to zero too quickly outside the region of maximum subsidence.

A more gradual transition from the zone of maximum pressure decline to where the pressure is constant would enable a better match to the top section of the observed vertical deformation profiles. However, at Wairakei, the pressure decline in the region of subsidence is laterally almost constant, therefore some other phenomenon must be contributing to the level of localized subsidence.

The second model examines the effect of a lateral variation in stiffness or compressibility. In Model One, discussed above, it was found that a normally consolidated stiffness of 10MPa in a 150m thick reservoir produced vertical displacements of the required magnitude, and an overconsolidated stiffness of 150 MPa in the 100 m thick overburden resulted in horizontal displacements of the required size. For Model Two, a reservoir stiffness of at least 10 MPa is still necessary near the centre of the subsidence bowl. If the pressure decline is taken to be the same throughout the reservoir, the lateral variation in stiffness must be considerable in order to produce such localized displacements. The most logical cause for such a variation is that the reservoir material in the subsidence bowl has been hydrothermally altered, while the surrounding reservoir has not. The "over-consolidated" stiffness of 150 MPa used for the overburden in Model One was used here for the outer reservoir zone in Model Two.

To model the pressure decline, a two-dimensional pressure load of 0.04 MPa was applied to the top of the reservoir; this corresponds to an annual pressure drop of 0.04 MPa throughout the entire reservoir (see Figure 5.4).

The analysis produced vertical displacement profiles that are too steep. During some experimentation, the width of the highly compressible zone was varied. The stiffness of this zone was adjusted so that the maximum vertical displacement was equal to the observed annual increase in subsidence at the centre of the subsidence bowl. By such adjustment in this model, vertical displacement profiles were found to be much closer to measured shapes. In all cases, the horizontal displacements were found to decay a little too quickly from the maximum value (see Figure 5.5).

#### 6. Summary

Both models are capable of producing a reasonable fit to the observed vertical and horizontal deformation profiles using a simple two-layer geometry with the material in each layer being homogeneous. However, there is no evidence to suggest there is any localized zone of greater pressure decline in the zone of maximum subsidence, therefore Model One is unlikely to represent the deformation process taking place at Wairakei. Therefore, it must be the depth-compressibility product which is the cause of the local subsidence. As the variation in thickness of the Huka formation is slight, a variation in compressibility such as in Model Two seems to be the most likely cause for the locality and extent at the subsidence at Wairakei.

The major advantage the finite element analysis has over a nucleus-at-strain analysis is the ability to account for the different layers and their individual properties. This advantage has enabled a good fit to the horizontal surface deformations as well as the vertical surface deformations.

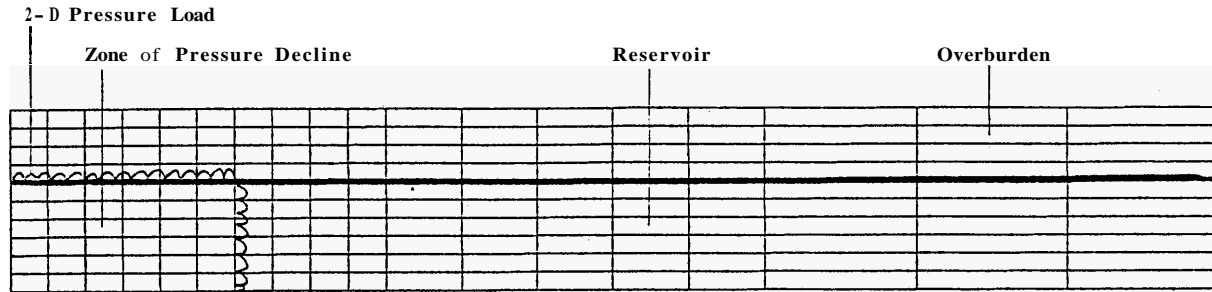


Figure 5.2.: Model One cross-section, including applied pressure load.

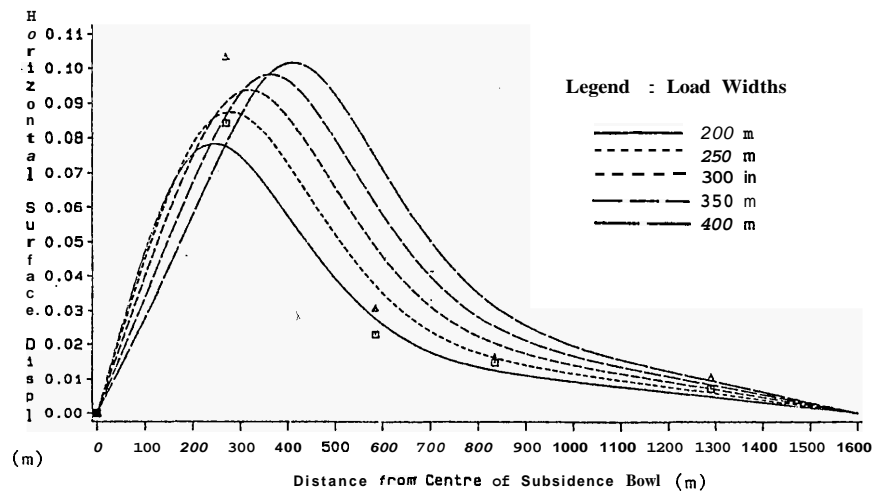
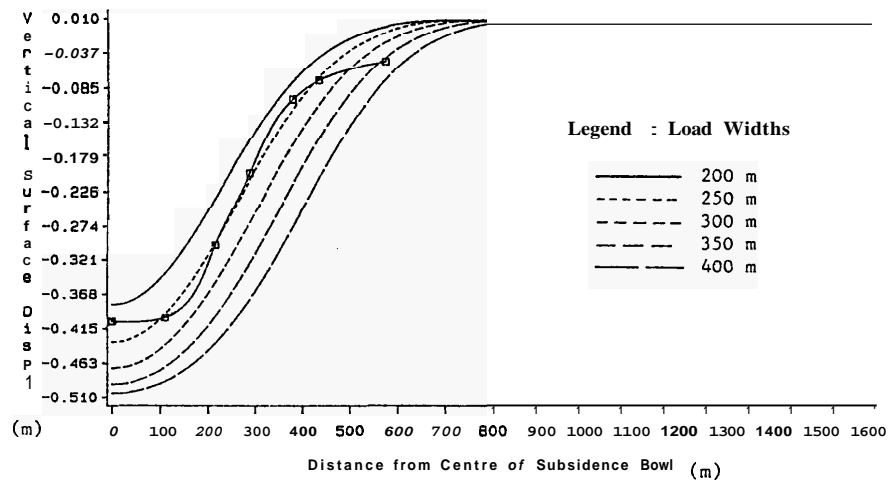


Figure 5.3: Vertical and horizontal displacements - Model One (150m reservoir depth).

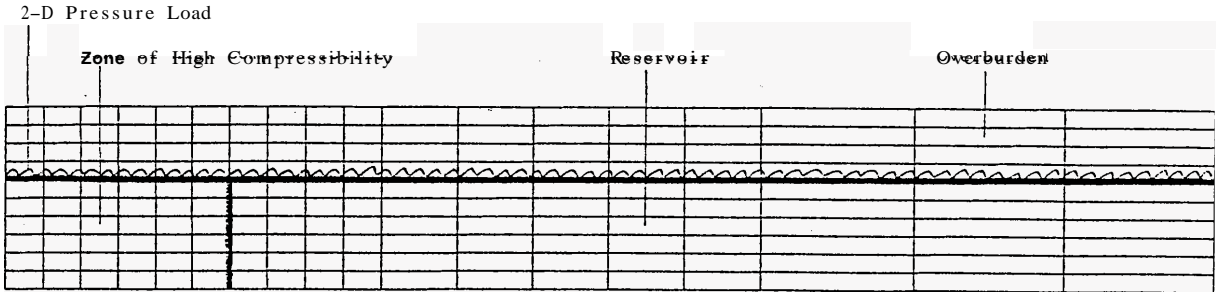


Figure 5.4 : Model Two cross-section, including applied pressure load.

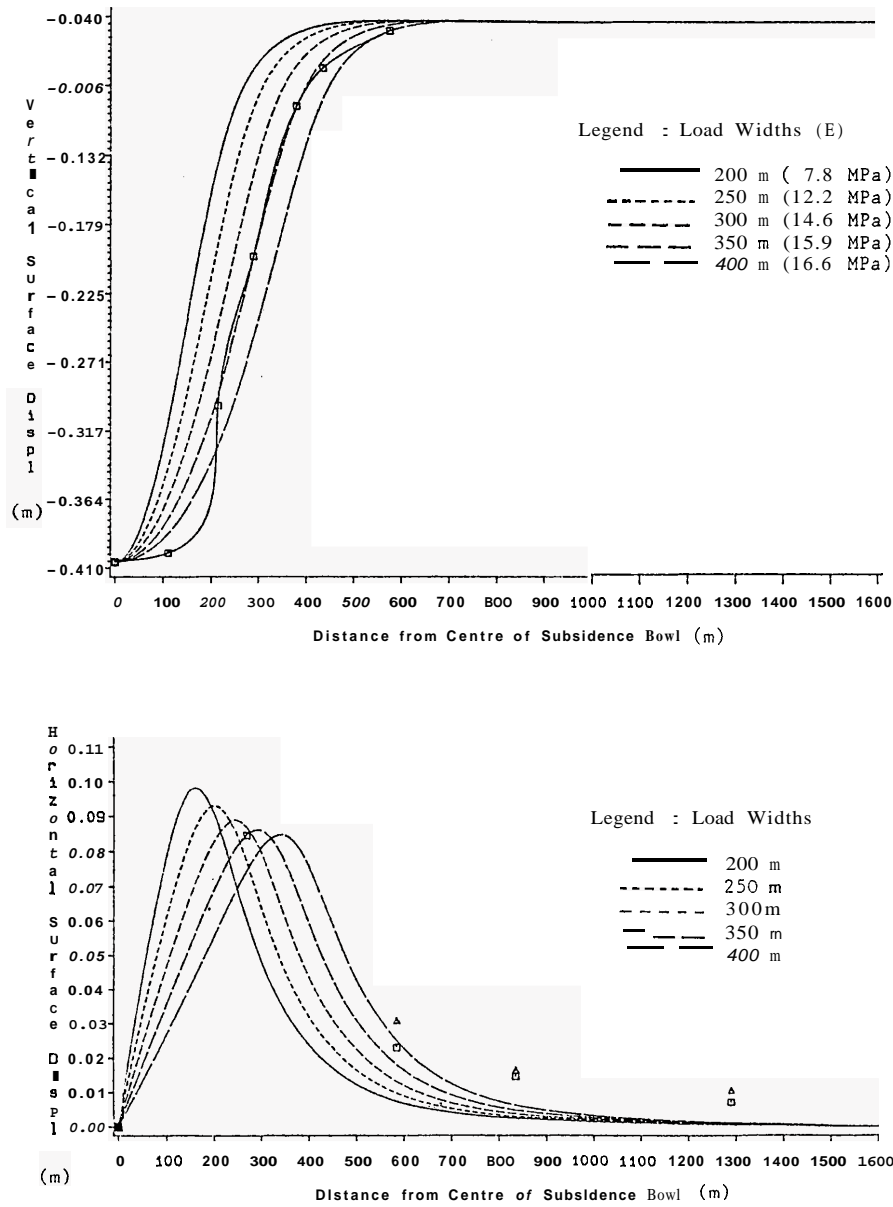


Figure 5.5: Vertical and horizontal displacements - Model Two (200m reservoir depth).

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