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SOME SIMPLE MODELS OF TRACER TESTS.

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ABSTRACT.

Three simple models of fluid flow through'a porous rock matrix are used to match the results of tracer tests at Wairakei. It is found that a model which allows diffusion from fractures into the surrounding rock matrix gives a better match to test results than a model which allows only longitudinal dispersion or a double porosity model. Some tests require a model nhich includes two fractures to match the field data.

INTRODUCTION

During a tracer test a pulse of tracer is released at the injection well into the reservoir flow field and tracer concentration is then monitored at nearby observation wells. From these tracer returns it is hoped that the permeability structure of the surrounding region of the reservoir can be deduced. For geothermal reservoirs tracer tests may help in the prediction of thermal changes resulting from reinjection of cool waste water.

In an early study of dispersion Saffman (1959) models a uniform porous medium as a collection of randomly oriented straight pores or fractures. The dispersion of a tracer moving through such a highly and uniformly fractured medium results from the differential movement of the tracer labelled fluid through the differ-ent fractures. It is shown that provided the fracture lengths are small and the mean residence time is large the dispersion process can be represented by a uniform porous medium continuum model with an effective diffusivity depending on the average pore length and average velocity.

When the fracture lengths are not small the continuum model is not appropriate.

Neretnieks, Erikson and Tähtinen (1982) when analysing data from experiments on tracer movement in a natural fissure in a granite core (30cm long and 20cm in diameter) found that channelling and matrix diffusion (tracer moving between the fissure and the rock matrix) are more important. Schwartz, Smith and Crowe (1983) conclude that a continuum diffusional model does not adequately describe the dispersion observed in their numerical experiments on tracer flow in a discrete fracture network model consisting of an impermeable matrix with two sets of orthogonal fractures. Neretnieks (1983) shows analytically that for fluid flow in a fracture with tracer diffusion into the rock matrix (referred to in this work as the fracture band model) the effective longitudinal diffusion coefficient is dependent on the distance between injection well and observation point.

Fossum and Horne (1982) modelled tracer tests by considering flow along two fracture pathways with diffusion along the fractures. The formula they used for effective diffusivity was derived by Horne and Rodriquez (1983) assuming dispersion was due to the fluid velocity profile across the fracture and molecular diffusion across the fracture (Taylor dispersion).

Jensen and Horne (1983) considered flow along a single fracture with diffusion of the tracer into the neighbour-ing matrix. Both of the last two works discussed used Wairnkei data to test their models. The results obtained by Jensen and Horne (1983) are very similar to some of the results reported here.

In addition to the Eracture band model reported by Jensen and Horne (1983) the present work considers a uniform porous medium model with longitudinal dispersion similar to that of Saffmann (1959) and a pseudo-steady state double porosity model analogous to that derived for pressure responses in Barenblatt et al (1960). The uniform porous medium model is mathematically identical to the fracture flow model with longitudinal dispersion considered by Horne and Rodriguez (1981) and Fossum and Horne (1982).

For some of the tests conducted at Wairakei none of the above models can match the field results and a further double fracture band model is introduced.

THEORY

In each model it is assumed that there is a good connection between the injection and observation wells along a streamline s which is surrounded by a thin stream tube S of approximately constant cross section A and across which the tracer concentration is constant. Distance along the streamline is r, the mass of injected tracer entering S is m and thevolume flow rate through S is q.

(a) Uniform Porous Model

In this case it is assumed that the reservoir, including the stream tube S, contains uniformly distributed micro-fractures. Dispersion is due to differences in micro-fracture path length (see Saffman, 1959) and is modelled by a diffusion coefficient ν in the direction of flow. A typical control volume for a length Δr of the stream tube S is shown in figure 1.

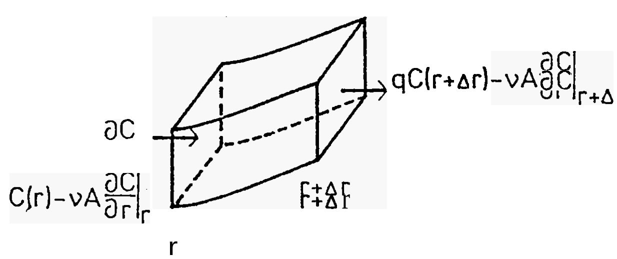

figure 1 : Control volume for the uniform porous model

The governing equation is

$$\nu \frac{\partial^2 c}{\partial r^2} - \frac{q}{A}\frac{\partial c}{\partial r} = \phi\frac{\partial c}{\partial t} \qquad (1)$$

where $C(r,t)$ is the tracer concentration and ϕ is the porosity of the reservoir. The pulse at the injection well is represented by the boundary condition.

$$C(0,t) = \frac{m\phi}{q}\delta(t) \qquad (2)$$

Here $\delta(t)$ is the Dirac delta function. These equations can be solved to give the concentration at the observation well $r=R$

$$C_{obs}(t) = C(R,t) = \frac{m\phi}{2q}\sqrt{\frac{t_m w}{\pi}}\frac{1}{t^{1.5}}\exp\left(-\frac{w}{4}\frac{(t-t_m)^2}{tt_m}\right) \qquad (3)$$

where $t_m = \phi RA/q$ is the mean arrival time and $w = qR/\nu A$ is a Peclet number corresponding to the ratio of tracer transport by convection to tracer transport by diffusion.

The physically dissimilar situation of fluid flow in a fracture with longitudinal diffusion (Fossum & Horne, 1982) yields the same governing equation because for the uniform porous model tracer does not leave the stream tube and for Fossum's model tracer stays in the fracture.

(b) Pseudo-Steady State Double Porosity

As in (a) the reservoir contains uniformly distributed micro-fractures. The micro-fractures divide the reservoir into 'blocks' that are assumed to have pores that are not swept for steady state fluid flow.

Tracer particles enter the blocks, by lateral movement (there is a small amount of fluid exchange) and molecular diffusion, stay for a while and then return to the micro-fractures. Dispersion due to differential movement of fluid in the micro-fractures (modelled in (a)) is ignored here in order to compare it with the above dispersion mechanism.

For this model, two uniform porous media - one with high permeability and low porosity (the micro-fracture network or the fractures) and the other with low permeability and high porosity (the blocks or the matrix) - are superimposed. There are two concentrations (fracture and matrix) at every point in the reservoir and the rate of tracer interchange per unit reservoir volume is assumed to be proportional to their difference. This is equivalent to assuming that the blocks are in a state of concentration equilibrium (the pseudo-steady state assumption). The wells are connected to the fracture only. A typical control volume is shown in figure 2.

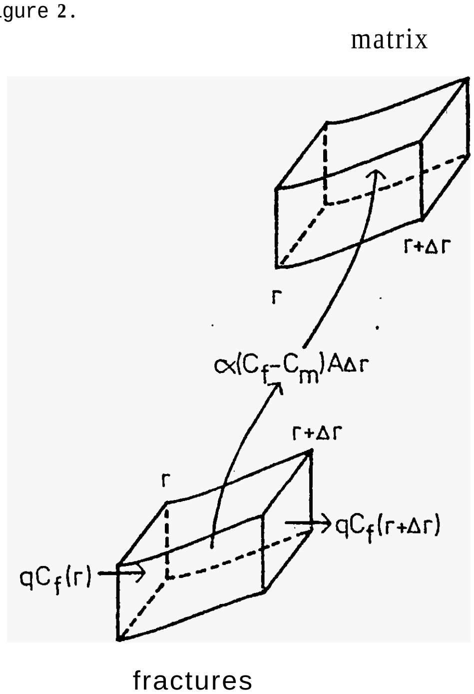

figure 2 : Control volumes for the double porosity model

The governing equation for flow in the fractures is

$$-\frac{q}{A}\frac{\partial C_f}{\partial r} - \alpha(C_f - C_m) = \phi_f\frac{\partial C_f}{\partial t} \qquad (4)$$

where C_f is the tracer concentration in the fracture and C_m is the tracer concentration in the matrix. The matrix equation is

$$\alpha(C_f - C_m) = \phi_m\frac{\partial C_m}{\partial t} \qquad (5)$$

The pulse of tracer at the injection well is represented by a boundary condition for C_f:

$$C_f(0,t) = \frac{m\phi_f}{q}\delta(t) \qquad (6)$$

The solution to 4,5,6 is:

$$C_{obs}(t) = \frac{m\phi_f}{q}\{U(t-t_b)\exp(-(\alpha_m t + (\alpha_f-\alpha_m)t_b))\sqrt{\frac{\alpha_f\alpha_m t_b}{t-t_b}}I_1(\gamma) + \exp(-\alpha_f t_b)\delta(t-t_b)\} \qquad (7)$$

where $\gamma = 2\sqrt{t_b\alpha_f\alpha_m(t-t_b)}$

Here $t_b = \frac{\phi_f RA}{q}$ is the time at which the pulse would reach the observation well if there was no dispersion,

$\alpha_f = \frac{\alpha}{\phi_f}$ is the rate of tracer interchange per unit fracture volume and

$\alpha_m = \frac{a}{\phi_m}$ is the rate of tracer interchange per unit matrix volume.

It should be noted that $\frac{m\phi_f}{q}\delta(t-t_b)e^{-\alpha_f t_b}$ is the pulse convected through without change in shape but with reduction in mass. This is not physically reasonable and restricts the validity of this model to those cases in which the proportion of mass carried to the observation well in this way $(me^{-\alpha_f t_b})$ is small.

(c) Fracture Band (arbitary orientation)

In this model there is a large plane fracture with micro-fracturing on either side. The dispersion mechanism is the same as in (b) with tracer leaving the main fracture and then returning, but the affect is different because the smaller fractures in (b) divide the rock matrix into small blocks that may be 'filled' with tracer, whereas here the matrix blocks are infinite.

A plane fracture (q is constant across the thickness, a) with diffusion (diffusivity ν) perpendicular to the fracture into an infinite uniform porous medium ($q = 0$) is used to model this rock geometry. Unlike the double porosity model, (b), there is no pseudo-steady state assumption about a linear concentration distribution in the block.

The streamtube S fills the thickness of the fracture. A typical control volume is shown in figure 3.

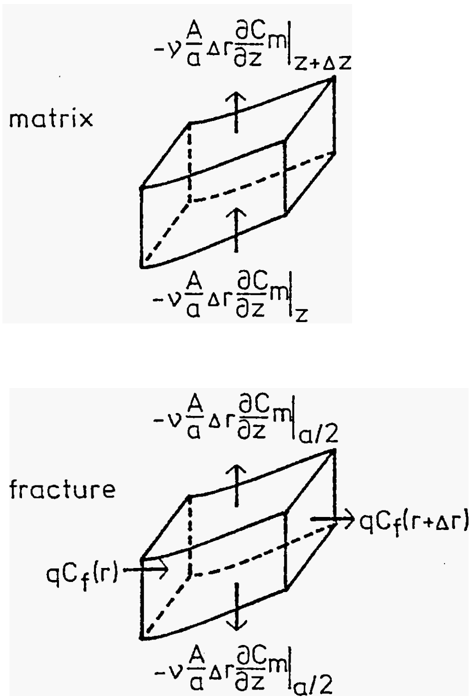

figure 3 : Control volumes for the fracture band model

The governing equation for flow in the fracture is

$$-q/A \frac{\partial C_f}{\partial r} + 2\nu/a \left.\frac{\partial C_m}{\partial z}\right|_{a/2} = \frac{\partial C_f}{\partial t} \qquad (8)$$

where $C_f(r,t)$ is the concentration of tracer in the fracture, $C_m(r,z,t)$ *is* the concentration of tracer in the matrix, **a** is the fracture thickness and z is the distance into the matrix. For the matrix, diffusion occurs according to

$$\nu \frac{\partial^2 C_m}{\partial z^2} = \phi_m \frac{\partial C_m}{\partial t} \qquad (9)$$

Here ϕ_m is the porosity of the matrix. At the boundary between the fracture and the matrix continuity of the tracer distribution is assumed

$$C_m(r,\ a/2,\ t) = C_f(r,t)\ .$$

It is assumed that all the tracer is injected into the fracture at r=0 ,

that is $C_f(0,t) = (m/q)\,\delta(t)$

The solution of (8) and (9) is

$$C_{obs}(t) = (m/q)\ U(t-t_b) \sqrt{\frac{wt_b}{\pi(t-t_b)^3}}\ \exp(-\frac{wt_b}{t-t_b}). \qquad (10)$$

Here $t_b = RA/q$ is the time at which the pulse would reach the observation well if there was no diffusion and $w = t_b\nu\phi_m/a^2$ is a measure of the ratio of the convective and diffusive velocities.

(d) Double Fracture Band Model

It is assumed there are two fracture bands that are sufficiently far apart over enough of their length that an insignificant amount of tracer is exchanged. Then the result in (10) can simply be added for two fractures:

$$C_{obs}(t) = \frac{q_1}{q_1+q_2} C_1(R,t) + \frac{q_2}{q_1+q_2} C_2(R,t)$$

$$= \frac{m_1+m_2}{q_1+q_2} \{ pU(t-t_{b1}) \sqrt{\frac{w_1 t_{b1}}{\pi(t-t_{b1})^3}} \exp(-\frac{w_1 t_{b1}}{t-t_{b1}})$$

$$+ (1-p)U(t-t_{b2}) \sqrt{\frac{w_2 t_{b2}}{\pi(t-t_{b2})^3}} \exp(\frac{w_2 t_{b2}}{t-t_{b2}}) \} \qquad (11)$$

where $p = m_1/(m_1+m_2)$, $t_{b1} = R_1A_1/q_1$,

$w_1 = R_1A_1\nu_1\phi_{m1}/q_1a_1^2$, $t_{b2} = R_2A_2/q_2$,

$w_2 = R_2A_2\nu_2\phi_{m2}/q_2a_2^2$.

MATCHING

In each case the parameters were chosen so that the model response curve had the same peak time (t_p) and response start time as the data curves. The peak time is calculated by solving

$$\frac{\partial C_{obs}}{\partial t} = 0 \qquad (12)$$

then the parameters are adjusted to match the response start time and shape of the response. The model response curves are automatically scaled to have the same maximum value as the data curve.

1. Uniform porous model

In this case condition (7) gives

$$w = 6\frac{t_p t_m}{t_m^2 - t_p^2} \qquad (13)$$

Then t_m is varied until the response start time matches the observed value with w being calculated using (13) in each case. No other parameters are available for matching and so the shape of the response curve cannot be further modified.

2.. Double porositv model

For matching purposes the spurious small pulse in (7) is ignored then (12) gives

$$\alpha_m = \sqrt{\frac{\alpha_f\alpha_m t_b}{t_p - t_b}}\ \frac{I_0(\gamma)}{I_1(\gamma)} - \frac{1}{t_p - t_b} , \qquad (14)$$

where $\gamma = 2\sqrt{\alpha_f\alpha_m t_b(t_p-t_b)}$. t_b is chosen to be the response start time and then $\alpha_f\alpha_m$ is varied until the shape of the early response curve (between t_b and t_p) matches. α_m is then calculated using (14).

3. Fracture band model

Forthismodel (12) gives

$$w = \frac{2}{3}\frac{t_b}{t_p - t_b} \qquad (15)$$

t_b is chosen **as** the response start time

4. Double fracture band model

In this case t_{b1} is chosen as the response start time. Then w_1 is calculated using (14) and the first peak time. The time t_{b2} is when the single fracture band response with parameters t_{b1} arid w_1 diverges from the data curve. A rough estimate of the second peak time . (t_{p2}) can be gained from the data curve. The parameters p and t_{p2} (calculating w_2 using (14)) are then varied until the second peak matches. Both the time and the height of the second peak, relative to the first peak, must agree.

RESULTS

Figure 4 shows the match of the single path models to the tracer measured in WK24 in response to injection in WK107 (McCabe et al, 1983). The fracture band model gives the best fit although the tail is too large. As shown in figure 5 this aspect of the fracture band model can be improved at the expense of the accuracy of the peak time or break-through time. However it is not possible to obtain a response with a large tail from either the uniform porous model or the double porosity model. Fossum and Horne (1983) used a double fracture model, corresponding to two of the uniform porous models considered here operating in parallel, to match tracer responses with large tails, but the parameters required to produce a good fit were not physically realistic.

For the 'best fit' fracture band model in figure *4* the diffusivity is given by

$$\nu_{fb} = \frac{w}{t_b}\frac{a^2}{\phi_m} = 2 \times 10^{-5} \times \frac{a^2}{\phi_m} .$$

Taking the matrix porosity as 10^{-2} and the fracture thickness as 10^{-2} m gives

$$\nu_{fb} = 2 \times 10^{-7} ,$$

which is reasonably close to 10^{-8}, the value obtained using the random walk model and experimental values in Saffman's 1959 paper.

Geological evidence supports the fracture band model as the Wairakei fault runs from near WK107 to near WK24 (McCabe et al 1983).

Similar results are shown in figure 6 for the response of WK101 to tracer injected at WK121 (McCabe et al, 1983). The actual response lies between the uniform porous model and the fracture band model. Calculation of the diffusivity gives, for the uniform porous model

$$\nu_{up} = \frac{R^2\phi}{wt_m} = 1 \times 10^{-7} R^2\phi$$

The distance from WK101 to WK107 is 500m and so the path length R is at least as large, giving

$$\nu_{up} \geqslant 3 \times 10^{-2} \times \phi$$

For the fracture band model

$$\nu_{fb} = \frac{w}{t_b}\frac{a^2}{\phi_m} = 5 \times 10^{-6} \times \frac{a^2}{\phi_m}$$

Again taking the porosity as 10^{-2} in each case and the fracture thickness as 10^{-2}m yields

$$\nu_{up} \geqslant 3 \times 10^{-4}$$

and

$$\nu_{fb} = 5 \times 10^{-8} .$$

Here ν_{fb} is much closer to the 10^{-8} estimate, suggesting that the fracture band is a better model of the matrix geometry. A possible compromise is to have either the injection or the production well not intersecting the fracture (combining uniform porous and fracture band models).

Using least squares matching, Jensen and Horne (1983) obtained fracture band matches similar to those shown in figures 4 and 6.

The response of well WK48 to injection at WK107, shown in figure 7, is more complicated - there are two peaks (McCabe et al, 1983). The two fracture band model gives a very good match. The best fit parameter values give

$$\nu_1 = \frac{w_1}{t_{b1}}\frac{a_1^2}{\phi_{m1}} = \frac{a_1^2}{\phi_{m1}} \times 2\text{x}10^{-5}$$

$$\nu_2 = \frac{w_2}{t_{b2}}\frac{a_2^2}{\phi_{m2}} = \frac{a_2^2}{\phi_{m2}} \times 4\text{x}10^{-6} .$$

Assuming that $\phi_{m_1} = \phi_{m2}$ and the mass of tracer entering a fracture is proportional to its thickness ($\Rightarrow \frac{a_1}{a_2} = \frac{p}{1-p}$) then

$$\nu_1 = 2\text{x}10^{-5} \text{x} \left(\frac{0.51}{1-0.51}\right)^2 \times \frac{a_2^2}{\phi_{m_2}} = 2 \times 10^{-5} \times \frac{a_2^2}{\phi_{m2}}$$

The closeness of ν_1 and ν_2 is supporting evidence for the two band model.

CONCLUSION

Matching to distinctive features (breakthrough time and peak time) of the response curve is straightforward and gives good matches for the fracture band model, which appears to be particularly useful for the fast responses modelled here.

The uniform porous and double porosity models produce matches that are almost the same, whereas the fracture band response, with its large tail, is quite different. This means that it is easy to distinguish when the fracture band model is appropriate. It also suggests that dispersion due to tracer being caught up in the blocks can be modelled by diffusion in the direction of flow when the blocks are small (double porosity) but not when they are large (fracture band).

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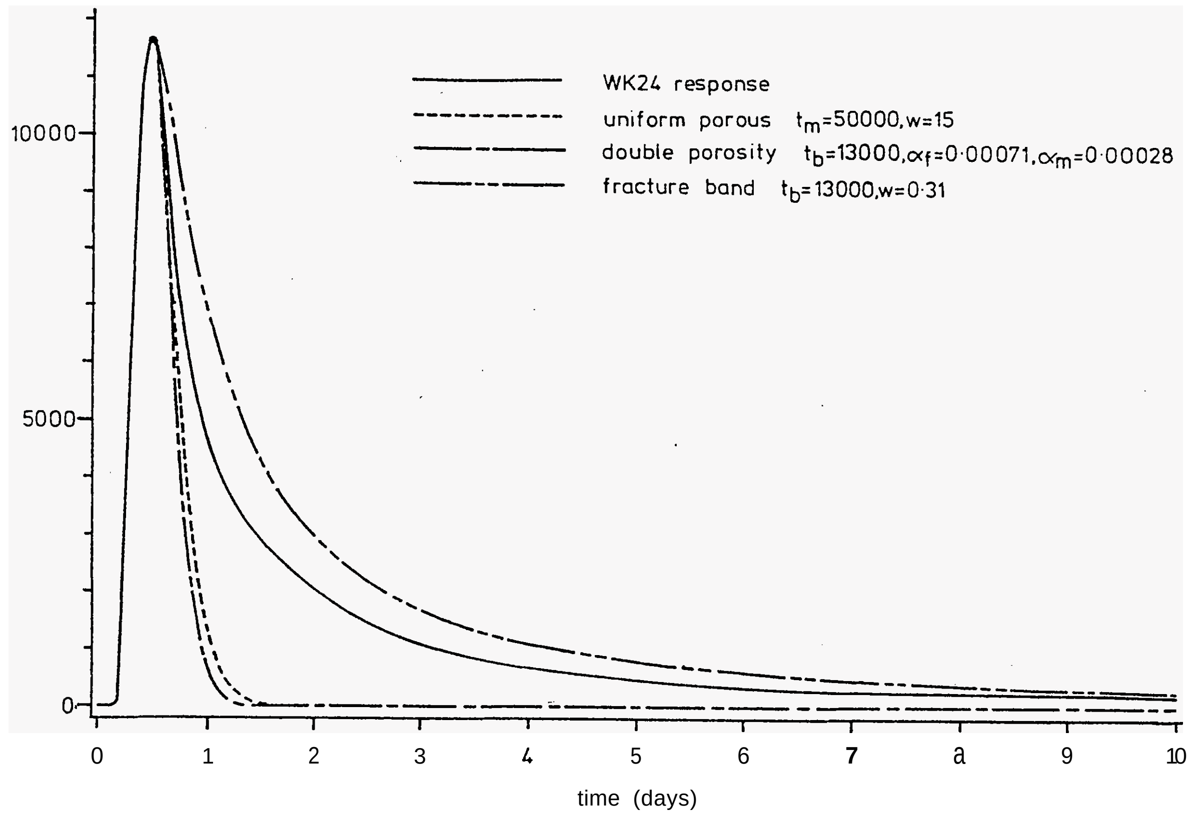

figure 4 : Matchs to the *WK24* response to injection *at* WK107. March 1979

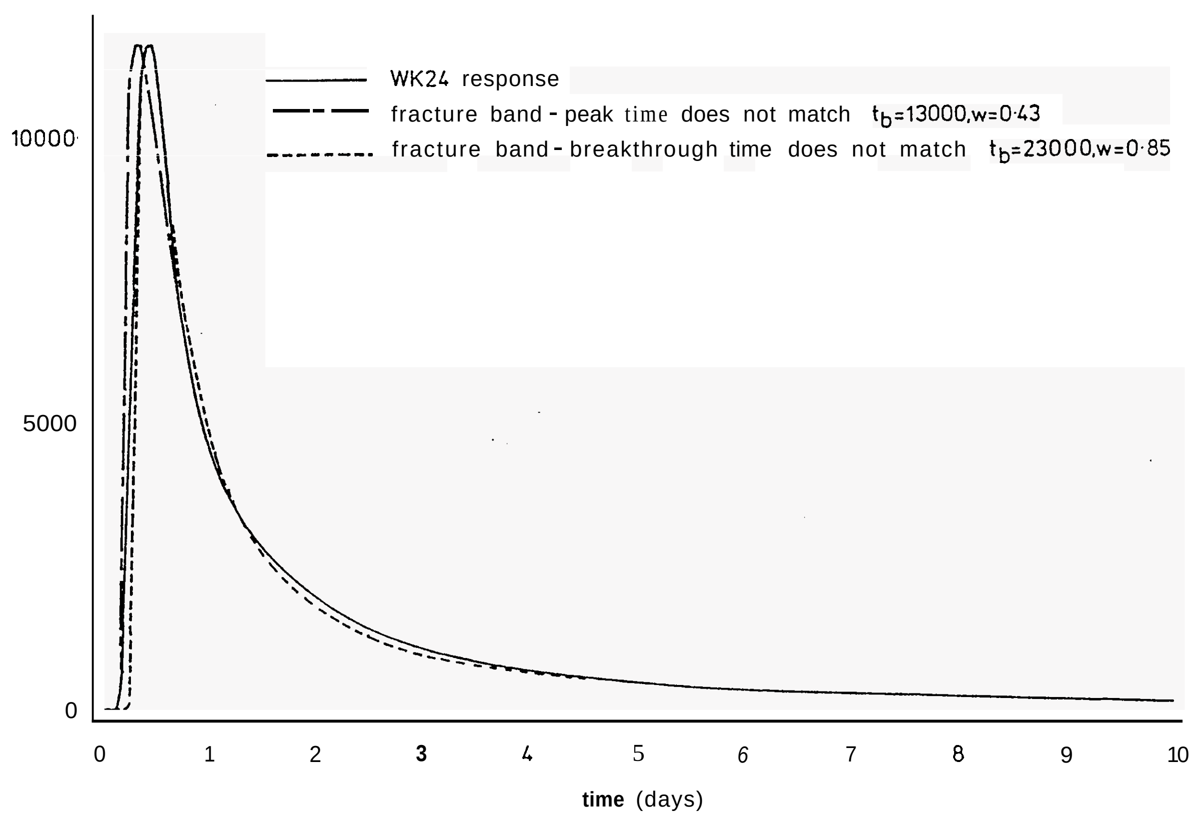

figure 5 : Sensitivity of the fracture band model match to the WK24 response

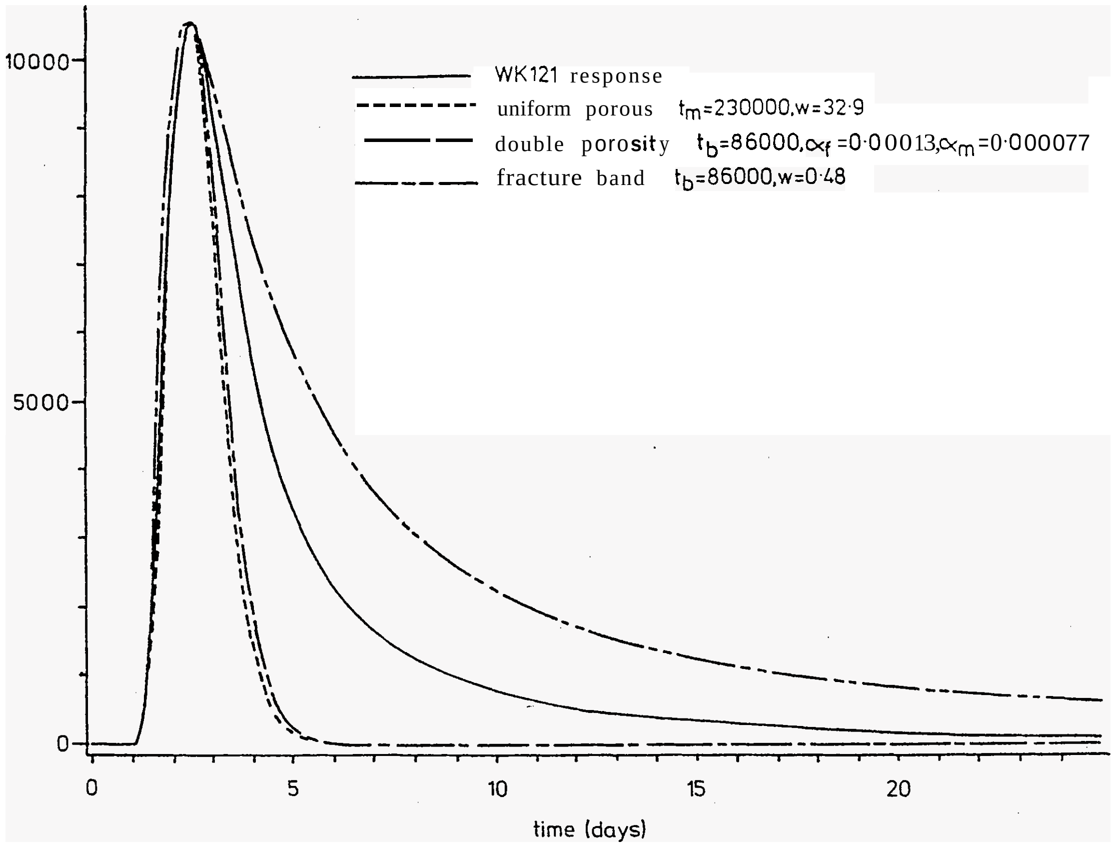

figure 6 : Matchs to the WK121 response to injection at WK101. July 1979

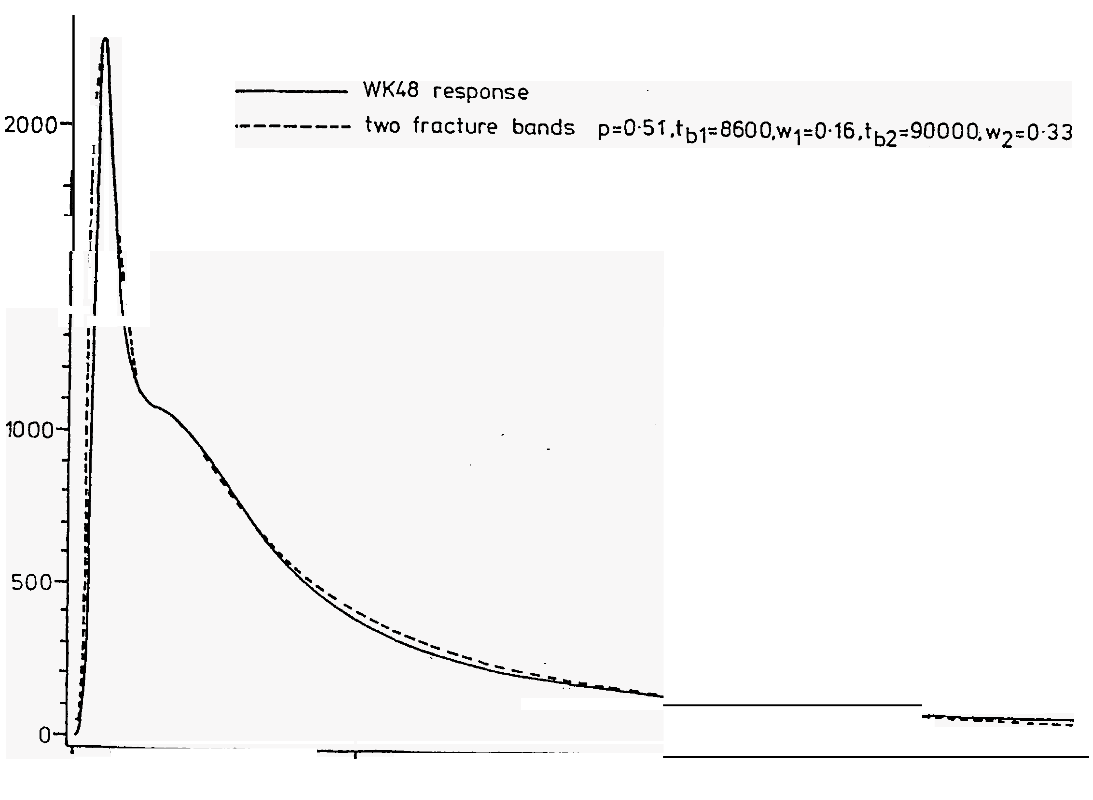

figure 7 : Two fracture band model match to the WK48 **response *to*** injection **at** WK107