

## IDENTIFICATION OF DISTRIBUTED-PARAMETER SYSTEMS

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## ABSTRACT

A relatively new methodology for building distributed-parameter models, based on control-theoretical methods of data analysis, is described and compared to the standard approach to reservoir modelling which relies solely on computer simulators. A specific application to the analysis of the Eutt Valley - Port Nicholson groundwater reservoir data is discussed. The analysis indicates that additional wells should be drilled in certain areas before an adequate forecasting model can be identified. It shows, however, that even with the data available, some of the reservoir parameters can be estimated with an acceptable accuracy. It is suggested that a similar methodology could be used in modelling geothermal reservoirs.

## IDENTIFICATION OF DISTRIBUTED HYDROLOGICAL MODELS

Distributed models are commonly used in reservoir engineering for modelling reservoir performance. They are discretised versions of partial differential equations describing spatial and/or temporal variations of reservoir variables, such as fluid pressure, saturation, density, or enthalpy. They necessarily involve reservoir parameters, such as permeability (transmissivity in hydrology), porosity, or storage coefficient. Knowledge of the reservoir parameters is necessary for predicting reservoir performance under various modes of operation, but they cannot be measured directly. Their rough estimates can be obtained by engineers from geological considerations and using well tests. The estimates are customarily refined by employing so-called large simulators: computer programs designed to solve distributed-parameter equations for reservoir variables assuming reservoir parameters known. Computed values of the variables are then compared to measured values, say pressure history, and parameters are adjusted to give a better fit. This procedure presupposes that the available mathematical model is the best possible, and that parameter estimates giving a good fit must be good as well. Neither of these statements is necessarily true. The sensitivity analysis routinely used to establish the quality of obtained estimates can prove misleading as well.

An alternative approach to parameter estimation, which also can be used to pinpoint a better mathematical form of the involved equations, is known in the literature as System Identification (see e.g. Kubrusly, 1977; Bennett, 1977; Young, 1978; Fradkin, 1982). It is based on an extensive exploratory data analysis aimed at 1) the selection of the best among available forms of distributed parameter models, 2) finding the best possible reservoir Parameter estimates, and 3) validation of the adequacy of possible models for the purpose at hand. It treats identification as a multi-objective optimisation process, not

relying on history match as the only validation tool. A characteristic feature of the methodology is its use of various statistical parameter estimation algorithms conducted for diagnostic purposes, that is in order to assess the quality of the data a posteriori and not a priori to analysis. The methodology relies on the availability of a reasonable amount of data in the form of discrete measurements of reservoir variables and can be time-consuming, though not necessarily requiring too much computer time. Its application does not exclude the need for large simulators. On the contrary they are sometimes necessary in establishing the influence of various factors on model identifiability.

## EXPLORATORY ANALYSIS OF TEE BUTT VALLEY - PORT NICHOLSON UNDERGROUND WATER RESERVOIR

The above ideas were applied to a study of the Hutt Valley - Port Nicholson underground water reservoir, New Zealand, using the EXTISEA computer package (Fradkin, 1983). The reservoir has been modelled before by using the conventional simulator approach (Donaldson and Campbell, 1977). The purpose of the present study was to establish what additional information could be extracted from the data using the new methodology.

Only four months of data collected in 1973 have been analysed. These comprised hydraulic head measured once every 15 minutes in each of the 9 measurement wells (see Fig. 1), river head recorded just as frequently at the sites where the Hutt River is believed to recharge the groundwater reservoir, and withdrawals from each of the 22 withdrawal wells recorded on a daily basis. Rainfalls have been neglected as they had been shown to be insignificant by Donaldson and Campbell (1977). Since in identification problems the spatial grid is dictated by the location of measurement wells, the question of its adequacy is of primary importance. Another problem is the choice of appropriate time and spatial differencing schemes. It has to be tested as well whether all the significant terms are taken into account in the model. The analysis usually starts by considering different reservoir zones separately. Here we mainly concentrate on zone 4, centred at well 4

Some of the above questions have been answered simply by comparing results obtained using different versions of the basic difference equation. Others involved employing simulated data. On establishing the most reasonable form of the model, the properties of regression parameter estimates were studied. The estimates proved to be variable from day to day by up to 100%. By the process of elimination carried out in order to test the assumptions implicit in the regression procedure, it became clear that the linear flow model imposed on the system by the available grid could not be correct. The

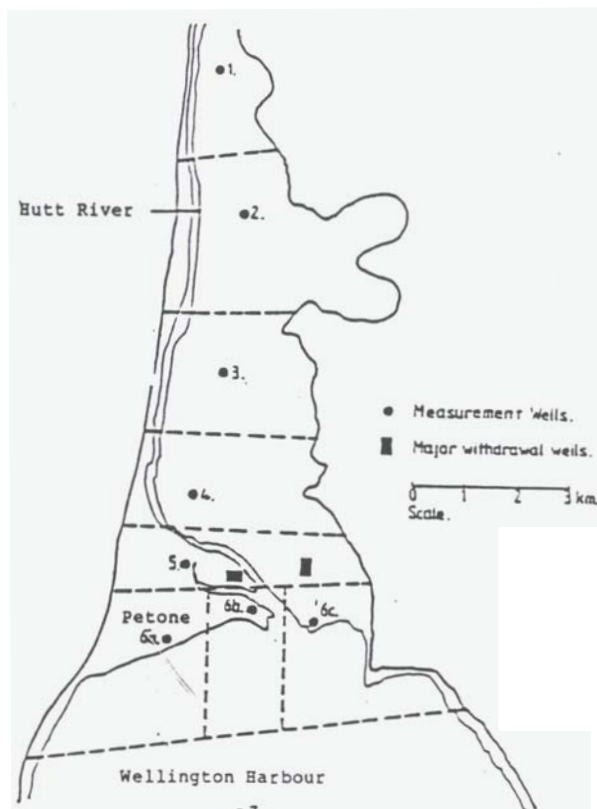


Fig. 1 Grid for the Butt Valley - Port Nicholson Groundwater Reservoir

possibility of a cross-flow in zone 4 has been tentatively raised by hydrologists before (Donaldson, pers. comm.), but has been positively established only in this work. Further experiments with simulated data showed that up to 100% biases in the principal reservoir parameter (transmissivity/storativity) lead to about 2% biases in hydraulic heads (pressures). This means that had a proper non-linear model been available the quality of obtained parameter estimates would be quite adequate for the purpose of predicting reservoir performance. Such a model could be constructed, however, only if additional measurement wells were drilled in zone 4 to account for cross-flows.

Other reservoir zones have been analysed in a similar manner, and it has been shown that none of them could be adequately modelled for the lack of the necessary minimum of measurement wells (Dokter, 1983). This would be hard to establish on the basis of a history match alone.

Thus the methodology met with considerable success in describing and helping to understand various features of the Hutt Valley - Port Nicholson underground water reservoir. It is felt that it can prove very useful in geothermal modelling as well.

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