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COMPUTERIZED WELL TEST ANALYSIS TO UTILIZE SIMULTANEOUS PRESSURE AND FLOWRATE MEASUREMENTS

Roland N. Horne, Alain Guillot and Adalberta Rosa *

Stanford University

ABSTRACT

The development of reliable spinner tools may help avoid much of the ambiguity which often accompanies well testa in geothermal wells, due to interlayer flows through the well bore. However. the use of both pressure and flow rate changes requires new methods of well test interpretation. The Stanford Geothermal Program has been developing microcomputer-based techniques for the simultaneous analysis of pressure and flow rate measurements. There are two key steps in the procedure. Firstly. the non-linear regression is achieved by calculating the gradients of the response (with respect to the unknown reservoir parameters) in Laplace space, and inverting numerically. Secondly, the variable flow rate is represented in terms of a superposition of many step changes - this was found to work better than a spline fit to the data. One problem was encountered when attempting to analyze data in which the spinner "stalled, causing a jump to zero bow rate.

The method shows great promise in that the degrees of freedom on the interpretation are greatly reduced, the well bore storage effect disappears. and inter-feed flows do not affect the results.

INTRODUCTION

Well test analysis is often hampered by complicating factors such as wellbore storage (afterflow) and multi-phase dow. As a result, great care is required in associating any portion of data with the "correct semilog straight line". The difficulties are often greater in the case of geothermal wells since the wellbore storage effect is usually substantiaf, and the additional problem of multiple feed points in the same well makes the analysis ambiguous. One solution to these various problems is the use of a spinner to measure downhole flow rate during a test. Provided the feed point is properly located, and the downhole pressure and dow rate measured at an appropriate place. it then no longer matters whether that feed *zone* produces (or accepts) all or only some of the test fluid; nor does it matter if the feed zone produces (or accepts) fluid to (or from) other feed zones in the same well rather than to (or from) the surface—the spinner will always monitor the actual reservoir flow rate. At present stages of technological development. spinners can operate in geothermal wellbore environments (although with safety mainly in injection tests), and combined tools with simultaneous pressure, flow rate and temperature measurements have been wed (Benson, Goranson, Solbur and Blocca, 1981). At the present time however. conventional well test analysis techniques cannot easily use these new measurements. except when they serve only to show that the flow rate is constant. New interpretation techniques are required.

Stanford University's Department of Petroleum Engineering has been developing computerized pressure transient techniques, using non-linear regression to fit field data to a reservoir model. The first problem to overcome was the difficulty in describing the reservoir model in a closed analytic form, since many conventional type curves have been developed using numerical techniques. Rosa and Horne (1983) showed a way of formulating the reservoir model in Laplace space (in which most solutions exist in closed form). and then numerically inverting the transforms of the solution and its derivatives with respect to the unknown reservoir parameters. The numerical inversion was performed using an algorithm by Stehfest (1970). Experimentation with non-linear regression methods indicated that variable projection (Kaufman, 1975) or Gauss-Yarquardt with penalty functions (Bard. 1974) would be the preferred methods to use.

Following the successful development of a set of methods capable of fitting a wide range of common type curves using a computer, the next step was to extend the work to encompass reservoir models that cannot be represented graphically in a type curve. Included in these is the case where flow rate is not constant. This work by Guillot (1983), examined the advantages of representing the variable flow rate in terms of a superposition of step functions, or in a rpline approximation.

This paper will summarize the results of both works, discussing first the advantages in performing the interpretation using the computer, and second the specific case of variable flow rate tests.

Computerized 'Type Curve' Matching

Well test interpretation in the conventional manner usually requires the separate analysis of one, two. three or more different portions of the pressure-time response. Commonly the early time response is used to estimate wellbore storage and skin effects, and also to correctly locate the start of the "correct semilog straight line" which is the second (intermediate time) response. The semilog straight line occurs when the fluid flows radially

*Present Address Petrobrás, Brazil

through the reservoir and no external boundary effects have been reached. Recognition of the Correct straight line is one of the more frequent sources of error in well test interpretation since several different semilog straight lines may be apparent in some real data. The slope of the straight line is used to estimate permeability-thickness product (kh). The third region that may be analyzed is the late time response, which indicates the presence of and distance to reservoir boundaries.

Interpretation of a well test may be difficult or impossible if one or more of the separate responses is absent. For example, a well with a large storage volume. close to a fault or boundary, may show a response which changes from storage-dominated to boundary-dominated without ever showing a semilog straight line. Worse still a spurious straight line may be apparent in the data, resulting in an erroneous interpretation. For this, and other reasons, it is also possible to obtain different estimates for the reservoir parameters from different parts of the response.

Many of these problems may be overcome with computerized curve matching. Using non-linear regression algorithms, a computer can fit curves as easily as straight lines and is therefore not dependent on the sometimes subjective choice of "the semilog straight line". The algorithm also does not obtain its estimates from a sub-region of the data, thus it cannot obtain different parameter estimates from different sub-regions. The procedure fits the entire pressure response at one time, regardless of its straightness or otherwise, and obtains a set of parameter estimates consistent with the whole data set. These estimates may still be in error if the model used was inappropriate, but on the whole the number of degrees of freedom (i.e. possibilities for error) are reduced relative to conventional graphical methods.

The analysis method proceeds as follows:

Define:

$\left[\Delta p_{wf}\right]_{t=t_i}$ = theoretical value of pressure drop at the wellbore at time $t=t_i$, as defined by the reservoir model

Δp_{wf_i} = recorded value of that pressure drop

The least squares procedure calls for minimization of the sum of squares of the residuals, defined by:

$$SSR(\vec{\theta}) = \sum_{i=1}^{n} \left\{ \left[\Delta p_{wf}\right]_{t=t_i} - \Delta p_{wf_i} \right\}^2 \qquad (1)$$

$\vec{\theta} = (\theta_1, \theta_2, \ldots, \theta_{np})^t$ is a vector of the np unknown reservoir parameters.

SSR is called the objective function and we seek the vector $\vec{\theta}^*$ at which it attains its minimum. This process of minimization is an optimization and if we constrain the parameters we will refer to the process as nonlinear programming.

The system of equations of condition is given by:

$$F_1 = \left[\Delta p_{wf}\right]_{t=t_1} - \Delta p_{wf_1} = 0 \qquad (2)$$

$$\vdots$$

$$F_n = \left[\Delta p_{wf}\right]_{t=t_n} - \Delta p_{wf_n} = 0$$

The equations in this system are nonlinear with respect to the unknown reservoir and well parameters. The basis of the Gauss method is linearization of the equations by expanding each function F_i in a Taylor series up to the first derivative term. We note that the derivatives cannot always be easily obtained in real space because the functions F_i are known only in the Laplace domain. However, we employ one of the theorems of the Laplace transform theory:

$$\frac{\partial F}{\partial \theta_p}(\vec{\theta}, t) = L^{-1}\left\{ \frac{\partial f}{\partial \theta_p}(\vec{\theta}, z) \right\} \qquad (3)$$

where:

p = parameter number

θ_p = reservoir parameter

$\vec{\theta}$ = vector having the reservoir parameters as components

f = function F in the Laplace domain, i.e., $\overline{\Delta p}_{wf}(\vec{\theta}, z)$.

After linearizing the system of equations (2) the normal equations are constructed by the usual procedure. Then, the normal equations are solved, the corrections added to the solution vector to form a new estimate of the unknowns. We repeat the process until convergence within a prescribed accuracy.

The Gauss method is known to be very sensitive to the initial guess and may converge slowly or even diverge if it is too far from the true solution. It may be improved greatly using the Marquardt (1983) modification and further improved by the addition of penalty functions (Rosa and Horne, 1983).

The inversion of the equation From Laplace space to real space was done numerically using the Stehfest (1970) algorithm. The accuracy variable N in that algorithm was chosen to be 8. With this value results from the evaluation of the pressure at the wellbore agree closely with those presented by Agarwal (Al-Hussainy and Ramey, 1970), within four or five significant figures.

Table 1 presents data obtained from a pressure drawdown test on a new oil well, which is strongly influenced by wellbore storage.

TULE 1

DATA FOR EXAMPLE TEST

After Earlougher and Kersch (1974)

q_{sc} = 179 STB/d $\qquad c_t$ = 8.2 x 10^{-6} psi^{-1}

B_o = 1.2 bbl/STB $\qquad \phi$ = 0.18

h = 35 ft $\qquad \mu$ = 1.0 cp

r_w = 0.276 ft

Drawdown Pressure Data

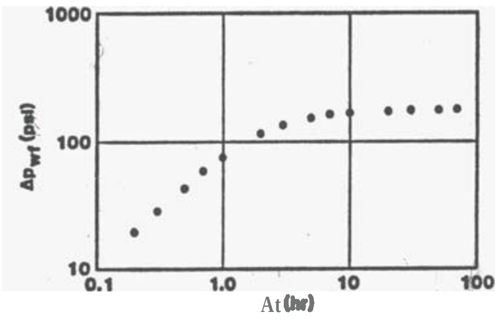
Δt(hr)	Δp_{wf}(psi)
0.2	19.7
0.3	28.1
0.5	43.1
0.1	58.3
1.0	75.1
2.0	114.5
3.0	135.5
5.0	152.2
7.0	163.2
10.0	166.7
20.0	171.2
30.0	173.9
$0.0	175.2
70.0	111.1

Figure 1 shows the log-log plot. Matching with the Gringarten et al (1979) type curves we estimated

k = 104 md = 0.104 darcy
s = 12.3
C = 0.08659 bbl/psi = 202,388. cm^3/atm

Earlougher and Kersch (1974) obtained the following results using type curve matching:

k = 132 md = 0.132 darcy
s = 18
C = 0.0942 bbl/psi = 220,172. cm^3/atm



Pig 1. Log-log plot of example data (after Earlougher and Kersch, 1974).

We performed the semi-log analysis, as presented in Figure 2, and calculated:

k = 102.8 md = 0.103 darcy
s = 121

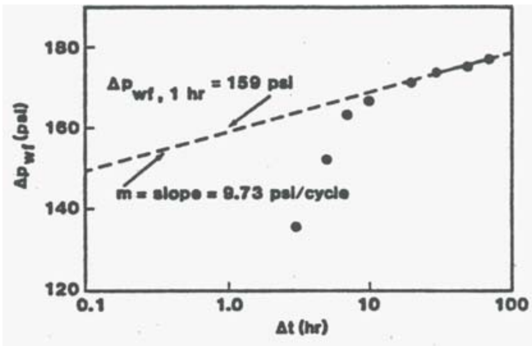


Fig 2. Semi-log plot of example data.

According to the approximate start of semi-log straight line indicated by the matching with the Gringarten et al (1979) type curves. only two or at, most three points fall on that line. This makes the conventional analysis rather difficult and subjective. In order to investigate this point, we employed the Gauss method using all the data points but also without some of them. The results are summarized in Table 2.

The matching using all the data points gave a sum of squares of residuals of *0.0478* atm^2 and the largest individual residual,' not shown, was 2.3 psi. The estimated permeability from the conventional remi-log analysis is 13.6% above the result from the Gauss method and the skin factor is 23.6% above.

The conventional semi-log analysis is difficult with so few points on the straight line, so it is interesting to observe what happens if the last two data points are not present. In this case the conventional analysis would be impossible but the putomated matching still produces reasonable figures for permeability and skin, within 18%, and practically the same wellbore storage coefficient.

Excluding the first three points it would be impossible to estimate the wellbore storage by conventional analysis using the unit slope line. However, the Gauss method yields almost the same results as those obtained when using all the data points.

These examples point out the efficacy of the automated procedure. It is clear that it is feasible to estimate the three reservoir parameters using data that fall only in the transition region between.the unit slope line (dominated by storage effects) and the semi-log straight line (no storage effects anymore). The only way of performing this analysis is automatically, using a regression technique as proposed, because there is no simple analytical expression for the pressure drop in the transition region and the visual matching becomes completely subjective.

TABLE 2 - RESULTS OF THE GAUSS METHOD

Data from Table 1

Example	Number of Data Points	Initial Guess			Solution			Iteration	SSR $(atm)^2$
		Darcy System of Units							
		k	s	C	k	s	C		
1	14	0.100	18.	220,000.					0.0478
2	12 (excluding the last two points)	0.104	12.3	202,386.	0.0806	8.07	200,360.	4	0.03857
3	11 (excluding the first three points)	0.100	18.	220,000.	0.0899	9.79	201,490.	6	0.04524
4	10 (excluding the first two & last three points)	0.104	12.3	202,366.	0.0810	8.14	200,560.	1	0.03811

Quite apart from this, the automatic approach also permits confidence intervals to be placed on all the parameter estimates. as shown in Rosa and Horne (1083). This quantification of the "fuzziness" of the estimates is not possible in the conventional analysis.

Variable Flow Rate

Given that the model pressure drop $\left[\Delta p_{wf}\right]_{t=t_i}$ and its derivatives can be obtained by numerical inversion of their Laplace transforms. it is no longer necessary to require that the flow rate be constant during the test · provided it has been measured. For the simple case without storage and skin. the model function can be written down in terms of the exponential integral function E_1, without recourse to the Laplace transform:

$$\left[\Delta p_{wf}\right]_{t=t_i} = \frac{\mu}{4\pi kh}\sum_{j=0}^{n-1} \frac{q(\tau_{j+1})+q(\tau_j)}{2}$$

$$\left[E_1\left(\frac{\varphi\mu c r_w^2}{4k(t_i-\tau_j)}\right) - E_1\left(\frac{\varphi\mu c r_w^2}{4k(t_i-\tau_{j+1})}\right)\right] \quad (4)$$

Here $q(\tau_j)$ are ?he set of n flow rates measured throughout the test · each treated as constant from time τ_j to time τ_{j+1}. Guillot (1983) also examined the case where the variable flow rate is fltted using a spline, however the step function approach was found to give better results in much less computer time. Figure 3 shows an example of a flow rate variation and its approximation in terms of (a) step functions and (b) a cubic spline. The resulting pressure histories are shown in Figure 4. The non-linear regresrion program can flt both pressure curves with good accuracy, however only the step function approach arrives at estimates of the reservoir parameters which coincide with the values used to generate the data. The spline approximation approach gives answers which deviate as much as 10% from the actual values.

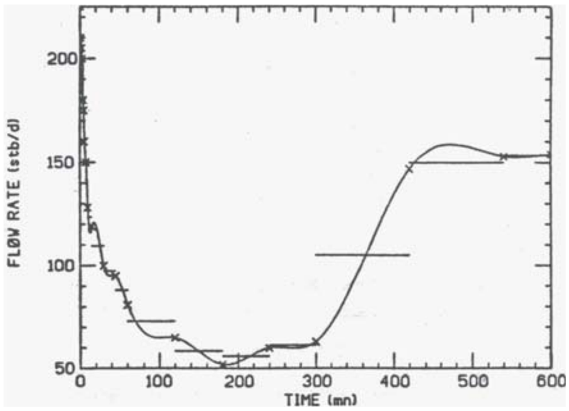


Fig 3. Flow rate vs. time: x = data; broken curve - step functions; solid curve - spline approximation.

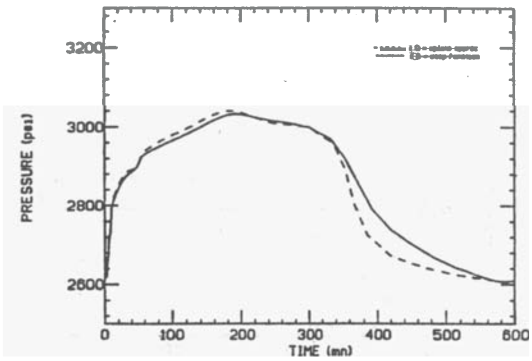


Fig 4. Pressure vs. time resulting from the flow rate approximations of Fig 2.

This difficulty in smoothing the data manifests itself in an important way in real data. There is commonly a "step" in flow rate measurements when the spinner stalls as the flow becomes small. This step is not a true representation of the actual calculated pressure response in the same manner as the "wiggles" produced by the spline. Examples considered by Cuillot (1983) showed two difficulties in fitting such real data - firstly. the algorithm has difficulty converging on the observed data (since the model forces the non-existent step), and secondly the fit eventually 'obtained does not give reasonable values of the reservoir parameters. Using only that part of the data which precedes the stalling of the spinner results in a much better flt. This again emphasizes the efficacy of the computerized technique, since the reservoir estimates are obtained purely from what would normally be the afterflow period - before the downhole flow rate reaches zero in a buildup test. The additional hours of data collected in pursuit of the correct straight line is not required, and in fact make the fit more difficult to obtain.

Summary

(1) The use of the Laplace transform of a reservoir m'odel pressure transient (and its derivatives with respect to unknown reservoir parameters) makes possible computerized fitting of many common type curves. The transforms need to be inverted numerically in most cases.

(2) The computerized method does not analyze a sub-region of the data (such as the semilog straight line). but flts all the data at once. It is therefore possible to correctly At many data sets that do not demonstrate easily recognizable graphical features.

(3) By fitting the whole data set at one time the reservoir parameter estimates obtained are at least consistent. The reduction in degrees of freedom is also likely to make the estimates more correct.

(4) The flow rate need not be constant, provided it has been measured. Analysis of variable flow rate tests gives better answers than constant rate tests since wellbore storage effects do not effect the results, and the number of degrees of freedom are substantially reduced.

(5) The computerized well test analysis procedure is capable of fitting data from much shorter tests than are necessary for conventional analysis. This represents a significant saving in time for wire-line crews, and a shortening of exposure of the tools to the downhole environment.

(6) The methods are sensitive to errors in the measurements, but fortuitously less sensitive to flow rate errors than to pressure errors. An exception is the flow rates indicated when the spinner "stalls", and this data requires careful handling.

ACKNOWLEDGEMENT

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