

PRESSURE LOSSES IN FRACTURED DOMINATED RESERVOIRS: THE WELLBORE CONSTRICTION EFFECT

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ABSTRACT

Improved energy production from many types of energy reservoirs such as hot dry rock geothermal as well as hydraulically fractured oil, gas, and other geothermal reservoirs requires a better understanding of the fluid mechanics in the vicinity of the fracture-wellbore intersection. Typically, the aperture (smallest dimension) of a hydraulic fracture is only of the order of 1 mm (0.04 in.) so that reasonable energy production rates from geothermal systems require fairly large flow velocities within the fractures, particularly so as the wellbore-fracture intersection is approached. The high velocities and accelerations result in non-Darcian, often turbulent, flow and increased pressure losses. These flow phenomena were investigated experimentally for the simple case where the fracture plane and the wellbore drilling axis are orthogonal and the implication of these experimental results are examined by investigating the pressure losses in a hot dry rock reservoir.

INTRODUCTION

Efficient operation of many types of energy reservoirs such as hot dry rock geothermal as well as hydraulically fractured oil, gas, and other geothermal reservoirs requires a better understanding of the fluid mechanics in the vicinity of fracture-wellbore intersections. This is particularly important for high flow rate situations that are almost always present in geothermal systems, particularly hot dry rock geothermal reservoirs. Hot dry rock reservoirs are formed by drilling into low permeability basement rock to a depth where the temperature is high enough to be useful; creating a large hydraulic fracture by pressurizing the well above the in situ compressive tectonic stresses; and then completing the circulation loop by drilling a second hole to intercept the hydraulic fracture (Smith, et al., 1975). Energy is produced from this system by pumping cold water down the first well and into the fracture where the water is heated as it contacts the freshly exposed hot rock surfaces. The heated water is conveyed to the surface via the second well, where the thermal energy is converted to electricity or used for other beneficial purposes. Preliminary results of field tests of this concept are reported by Tester and Albright (1979) and Murphy (1979a). The

mechanics and modeling of the heat extraction process are described by Harlow and Pracht (1972) and McFarland and Murphy (1976).

Typically, the aperture (smallest dimension) of a hydraulic fracture, regardless of the type of reservoir, is only of the order of 1 mm, (0.04 in.) (Perkins and Kern, 1961) so that significant rates of energy production require fairly large flow velocities within the fracture which often result in non-Darcian flow. Several distinct flow regimes can be identified. Because these regimes are conceptually the same for both the inlet and the outlet of the fracture, we start with the flow deep within the fracture and consider the changes that occur as the reservoir fluid approaches the outlet, turns into the production wellbore, and then continues up this well. For simplicity we consider the case where the fracture plane and production well are nearly orthogonal. A sketch of the geometry is shown in fig. 1, which illustrates a vertical fracture intersecting a wellbore that has been deviated from the vertical, using directional drilling, to the extent that its axis at the intersection is horizontal. This of course is an extreme case; more realistic configurations are discussed in the discussion, but it has the important advantage of representing a simpler two-dimensional geometry, as well as providing upper bounds to actual pressure losses. As indicated in the discussion, the plane of the hydraulic fracture is normally orthogonal to the minimum (least compressive) component of the tectonic earth stress. In tectonically relaxed geological settings, this stress is expected to be a horizontal one at depths greater than about 1 km (3000 ft). The attainment of usefully high temperatures for hot dry rock geothermal reservoirs usually requires depths of 3 to 5 km, so that fractures are expected to be vertical. However, some other types of reservoirs may be situated at shallow depths, possibly resulting in horizontal fractures and thus orthogonal intersections with vertical wells. The flow sketched in fig. 1, after a 90° rotation of the figure, then also represents a close approximation to conditions for these reservoirs.

In the first flow regime, deep within the fracture, i.e., far from either the inlet or outlet, the flow velocities and local Reynolds numbers are relatively small compared to their values near the inlet and outlet, so that if the walls of the

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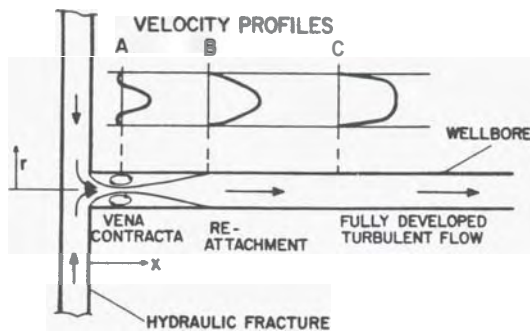


Fig. 1 Flow geometry and velocity profiles. Fracture aperture is shown in exaggerated scale for clarity.

confining rock can be considered hydraulically smooth, then the local velocities are related linearly to the local pressure gradients by means of the fluid viscosity and the fracture permeability. This permeability is one-twelfth the square of the fracture aperture. In actuality the fracture walls are rough so a linear pressure gradient-velocity relation cannot always be expected even at Reynolds numbers below the usual transition value of about 2000. Thus at high flow rates the flow may be turbulent and hydrodynamically rough so that pressure gradients are proportional to roughly the square of the velocity.

As the outlet is approached, but far enough away so that the effects associated with the turning of the flow into the production well can still be ignored, an additional complexity is introduced. Now the flow passage area, given by the product of aperture and 2π times the radial distance measured from the axis of the outlet well, continuously decreases as the outlet is approached. Thus the local velocities increase and acceleration increases in importance. The required pressure loss must increase not only to account for this acceleration but also because of the increased friction caused by the "blunting" of the velocity profile and the increased velocity gradients at the fracture walls. In this regime can be proportional to the square of the velocity, even for laminar flow with hydrodynamically smooth walls (Murphy et al., 1978). This flow situation is referred to as flow regime two.

In regime three, where the turn into the outlet well is approached, a significant component of momentum is directed in the longitudinal, or well-drilling direction; however, because radial momentum still increases with decreasing radius, the flow at the turn resembles a vena contracta. In other words the flow entering the wellbore appears as if it enters from a hole with a diameter very, much smaller than the wellbore diameter, as sketched in fig. 1. The flow then expands to fill the entire well diameter. The expansion is accompanied by a reduction of longitudinal flow momentum and results in an adverse pressure gradient in which the pressure increases in the flow direction. This results in enhanced turbulence, possibly flow

separation (velocity profiles with negative values near the wall), and subsequently greater viscous losses.

With continued passage up the production well the flow reattaches to the wellbore wall, the turning and vena contracta effects of regime three are attenuated, and the flow eventually becomes a typical fully developed pipe flow in which velocity profiles and pressure gradients no longer change in the downstream direction. This is regime four.

For either laminar or turbulent flows the fully developed pipe flow of regime four is well understood and is not considered further here. Laminar accelerated radial flow (regime two) has been investigated by Murphy (1979b). The unaccelerated radial flow of regime one is reasonably well understood although there are some remaining questions currently under investigation regarding the effects of large roughness-to-aperture ratios on friction and laminar-to-turbulent transition (Pearce and Murphy, in prep.). The remaining topic, the flow turning effects of regime three, was investigated experimentally and is reported here.

Although conceptually the same, there are important differences in detail between the inlet case, flow from wellbore to fracture, and the outlet case. We have chosen to investigate the outlet case first, because under most operating conditions, as discussed in the Discussion, the pressure at the outlet is lowest. Thus the aperture and, consequent flow passage area at the outlet are expected to be smaller and the pressure losses are expected to be much larger at this location.

PROCEDURE AND APPARATUS

The apparatus consisted of two flat, smooth, parallel discs of Plexiglas each 0.597 m (2 ft) in diameter, connected to a Plexiglas tube with a 75 mm (3 in.) inner diameter and a reversed blower, as schematically shown in fig. 2. The discs simulate the fracture and the pipe simulates the exit wellbore. The flow surfaces of both the discs and the pipe were hydrodynamically smooth. As mentioned above, the simulation of possible fracture roughness is an important consideration in flow regime one and possibly even regime two, but is not important in regime three, where the pressure loss is primarily due to the dynamic acceleration followed

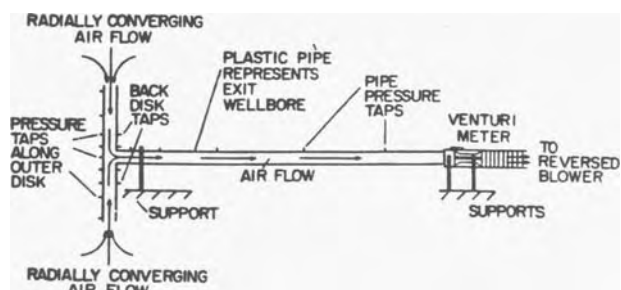


Fig. 2 Experimental apparatus.

by deceleration as the flow turns into the wellbore.

Both figs. 1 and 2 suggest that the outlet wellbore terminates at the fracture intersection. In actuality it is extremely unlikely that the fracture is located precisely at the end of the well, so that a length of open-hole wellbore may exist on the other side of the fracture. However, the fluid in this open-hole section would be relatively stagnant--nearly all the fracture flow would enter the wellbore and flow toward the surface as indicated in fig. 1, so that the failure to model the open-hole section in the experimental apparatus is expected to have little practical consequence.

Air at room temperature and pressure was drawn between the two discs and then into the pipe with the reversed blower. The choice of air as the modeling fluid simplified piping and pressurization and, when the results are properly nondimensionalized according to similarity principles, the fluid used is immaterial so long as it can be considered as a Newtonian incompressible fluid with constant properties. All experiments were conducted at room temperature and atmospheric pressure and the pressure losses in the test section were limited to 5 mm of Hg (0.1 psi) so the assumption of constant properties was easily satisfied.

Static pressures were read along two near-radial chords of each disc and at increasing longitudinal distances along the pipe. Flow rates were measured with a venturi meter. Flow rates as well as simulated fracture apertures were varied so that results could be obtained as a function of Reynolds number and the aperture-to-well diameter ratio. Further details about the apparatus and procedure were described previously by Murphy (1979b).

RESULTS

The results of interest in this study included those details of the vena contracta that could be inferred from pressure distributions, the locations of the transition from flow regime two to three and the transition from regime three to four, and the overall pressure drop associated with regime three.

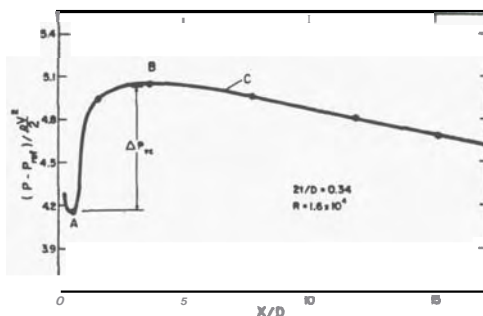


Fig. 3 Typical wellbore pressure loss. Pressure is nondimensionalized in terms of average wellbore velocity. Distance x is measured from the wellbore inlet (see fig. 1).

Vena Contracta, Reattachment, and Fully Developed Flow Positions.

A representative plot of pressure loss along the pipe or simulated wellbore is shown in fig. 3. The longitudinal distance, x , has been nondimensionalized by D , the well inner diameter. Pressure, P , has been nondimensionalized in terms of $\rho V^2 / 2$ where ρ is the air density and V is the mean velocity in the wellbore. The grouping $2t/D$ shown in fig. 3 is the ratio of fracture aperture to well diameter. To the left, the pronounced pressure reduction identified with the letter A is due to the vena contracta (V.C.) following the flow turning. The precise location of the minimum was difficult to determine because of the paucity of pressure taps in this region. In all cases the pipe pressure tap nearest the discs, located at $x/D = 0.5$, had the lowest pressure. The adjacent nearest taps were located at $x/D = 0.0$ and 1.5 respectively.

The point of reattachment, point B, is the local maximum in fig. 3 and was also difficult to determine because of the low density of pressure taps, though this difficulty was not as great as it was in identifying point A. In all cases the pressure tap at $x/D = 3.5$ indicated the maximum pressure. Point C, where fully developed flow begins, is shown in fig. 3 as the onset of a linear relationship between pressure loss and distance. The dimensionless distance required to attain fully developed flow is referred to as $(x/D)_{\text{linear}}$. Because the minimum wellbore Reynolds number, $R = VD/\nu$, during all these runs was 3000, and since no effort was made to defer the transition from laminar to turbulent flow beyond the usual Reynolds number of about 2000, by, for example, attempting to provide an extremely vibration- and disturbance-free environment, these fully developed pipe flows were turbulent. The kinematic viscosity of the air is denoted as ν .

As was the case for points A and B, the exact location of point C was often difficult to determine, but rough estimates were made using extrapolation and the results for each of the 14 experimental runs were plotted in fig. 4. The average

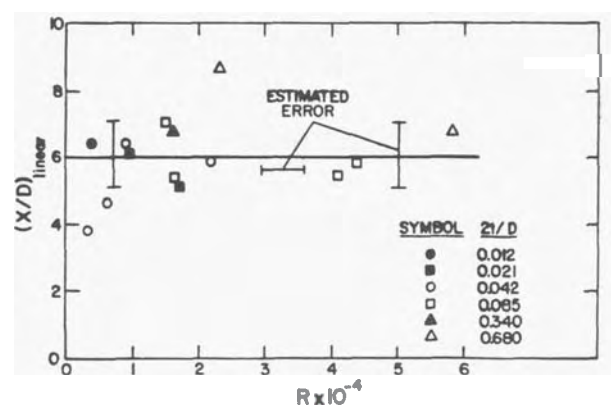


Fig. 4 Number of wellbore diameters from entrance for fully developed turbulent flow within the wellbore.

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value of (x/D) is 6.0 with a standard deviation of 1.2, and appears, to a first approximation, to be independent of Reynolds number and $2t/D$. The value of (x/D) derived here is substantially lower than the values of 25-40 reported by Nikuradse (Schlichting, 1968) and 50-100 reported by Kirsten (Schlichting, 1968). However, their work was based on air entering pipes with essentially uniform velocity profiles. The vena contracta in the present situation resulted in unstable, highly turbulent flow in the region with adverse pressure gradient, and strong mixing occurred accompanied by large radial momentum transfer. This mixing may explain the early onset of fully developed flow.

Departure From Purely Radial Flow Between the Discs. Of interest is the point where outlet flow turning effects within the discs appear, that is, the position where the purely radial flow of regime two ends and regime three begins. Two suggested methods of determining this point are: (1) compare a plot of predicted pressure distribution along the disc radius (Murphy et al., 1978), which does not account for turning effects, to the actual measured pressure distribution, and take, as suggested in fig. 5, the onset of discrepancies between the two as the beginning of flow regime three; (2) examine the measured pressure distributions for the disc closest to the pipe and the opposite disc and determine the radius where significant differences occur. Both methods suffered major drawbacks. The first method emphasized a greater than desired reliance on subjective curve fitting, whereas the second method suffered, once again, from the paucity of pressure taps in the region of interest for this study. However, in most cases the weight of the evidence suggested that the inward-most radial tap at $r = 0.06$ m (2-1/4 in.) was the point where flow turning effects were first noticeable.

Regime Three Pressure Drops. The last question to be addressed is that of the pressure loss associated with flow regime three. For the sake of

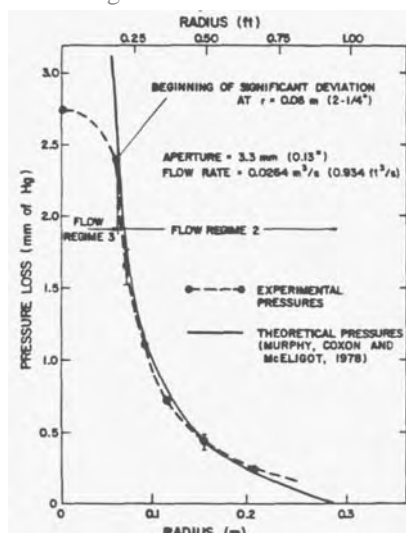


Fig. 5 Comparison of predicted and measured fracture pressure losses.

choosing an easily identifiable quantity that could be experimentally reproduced, ΔP_3 was defined as the loss occurring from the point where turning effects were noticed within the discs, i.e., at $r = 0.06$ m, to the point where fully developed turbulent flow occurred in the pipe. Using these end points in the definition of ΔP_3 is convenient for practical applications since total pressure losses can then simply be summed as ΔP_3 plus the easily calculated pressure losses for purely radial laminar fracture flow and fully developed, either laminar or turbulent, pipe flow. The results are plotted in fig. 6 as dimensionless pressure losses $(\Delta P_3 / \rho V^2)$ as a function of Reynolds number. The dimensionless pressure loss is equivalent to the often-used loss coefficient, C_f . It should be recalled that V is the average velocity in the wellbore, not the velocity in the fracture. Figure 6 shows that C_f is larger for small $2t/D$ and that for large R , C_f is nearly independent of R . This is to be expected as inertial effects outweigh viscous effects at large R .

It may be noted that for some extremes of $2t/D$, flow regime three has some similarities to flow through an abrupt contraction or expansion. For example, if $2t/D$ was to approach infinity, the flow would be perfectly analogous to that of flow from an infinite reservoir into a pipe. Under these conditions, the loss coefficient is 0.58 (Benedict and Carlucci, 1966). Referring to fig. 6, it can be seen that for the largest aperture-to-diameter ratio, $2t/D = 0.68$, the measured C_f is 1.3. The explanation for the difference lies in the fact that when $2t/D$ is finite, the radial velocities, for a given flow rate, are greater than when $2t/D \rightarrow \infty$. Larger radial velocities result in a smaller vena contracta at the beginning of the expansion, so that the pressure losses associated with the re-expansion of the flow are greater. It is these re-expansion losses that actually contribute most of the pressure loss in flow situations

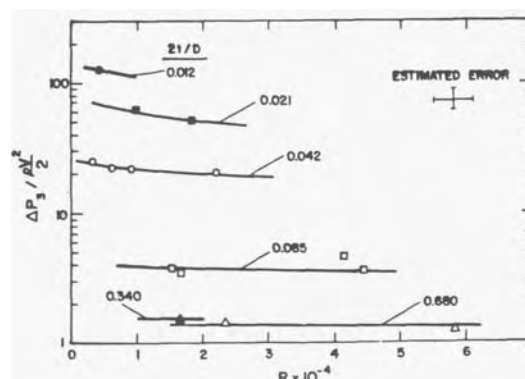


Fig. 6 Regime three pressure loss, ΔP_3 , as a function of wellbore Reynolds number for different fracture apertures. Pressure loss is nondimensionalized in terms of average wellbore velocity.

which appear, geometrically at least, to be abrupt contractions.

At the other extreme, $2t/D \rightarrow 0$, the flow appears similar to an abrupt expansion, although in this case the expansion would be from a radial to a longitudinal flow configuration rather than from the more usual case of expansion from one longitudinal flow to another, as typified by flow from a small pipe to a larger coaxial pipe. In the latter case the loss coefficient is usually based upon the average upstream pipe, i.e., the larger velocity, and when the upstream pipe diameter is very small compared to the downstream diameter the loss coefficient is one (Benedict and Carlucci, 1966). Overlooking for the moment the differences between the present flow and the simple collinear longitudinal expansion, the larger upstream velocity, corresponding to the maximum radial fracture velocity in the present flow, is $D/8t$ times the average wellbore velocity. Therefore if $C_L = 1$ when defined on the basis of the larger velocity, then $C_L = (D/8t)$ when defined in terms of the wellbore velocity. From fig. 6 it may be noted that for the smallest value of $2t/D$, 0.012, the measured C_L is about 150, whereas $(D/8t)^2 = 430$. In this case the actual loss is smaller than one would expect from estimates of collinear expansions, apparently because the cross-sectional area of separated flow is less. In the simple pipe expansion the upstream flow enters the expansion area as a jet with an area nearly equal to the upstream area. The area difference between the jet and the full expansion area is filled with a separated, recirculating and highly turbulent fluid. In the radial-to-longitudinal expansion, the initially radial jet is transformed to a longitudinal jet. This longitudinal jet has its minimum cross-sectional area at the vena contracta, and this area, while certainly smaller than the wellbore area, may be larger than the minimum fracture radial flow area. It appears that when $2t/D$ is very small, the vena contracta area is indeed larger than $2\pi Dt$, thus resulting in a smaller than expected loss coefficient.

DISCUSSION

The implications of these results are examined by investigating the pressure losses in a hot dry rock geothermal reservoir. While this application may appear at first to be of parochial interest, the geometrical conditions are likely to be typical of those encountered in other fractured reservoirs, although flow rates may vary from case to case. Specifically we examine the latest geothermal reservoir proposed for the wellbores EE-2 and EE-3 currently being drilled at the Fenton Hill site near the Valles Caldera in northern New Mexico. The goal of this reservoir is to produce 20 to 30 MW(t) for approximately 20 years. We initially assume the following parameters: fracture aperture ($2t$) = 1 mm (0.04 in.); wellbore diameter (D) = 0.2 m (8 in.); water temperature = 200°C (390°F); flow rate = 0.063 m³/s (1000 gal/min). With these values the pressure loss associated with regime three alone, ΔP_3 , would be estimated to be about 0.8 MPa (120 psi). As noted above, this pressure loss is considerably less than would be

expected for the expansion from a small collinear pipe with the same minimum fracture area to a larger pipe with the same diameter as the wellbore. Under these latter conditions the pressure loss would be six times higher.

It was mentioned in the introduction that a fracture aperture of 1 mm is typical of fractures held open against the tectonic stresses by internal pressurization. Although one might expect the fracture near the inlet well to be so "inflated" by the high-injection pressures required simply to overcome the pressure losses associated with flow through the fracture, the fracture near the outlet well would not be inflated unless a high "back pressure" was applied to the production well. Such a back pressure is operationally undesired as it increases the rate of water permeation loss to the rock surrounding the fracture and it also requires that either the geothermal energy conversion equipment at the surface be designed for high pressure or else the production well pressure be throttled to near zero before the fluid enters this equipment, and that the pumps then make up the throttling loss before reinjection. To avoid these problems, little or no back pressure is preferred, so that the fracture near the outlet would be only partially held open due to the "self propping" of the misaligned rough surfaces produced during fracturing. It is estimated (Murphy, 1979a) that the effective opening of a self-propped aperture is 0.2 mm (0.01 in.), in which case ΔP_3 would be 13 MPa (2000 psi). Because of its dependence upon $2t/D$, ΔP_3 does not quite scale with $(2t)^3$ as might be expected from purely dynamic pressure loss considerations in accordance with the inviscid Bernoulli equation. The pressure loss cited here for the self-propped case is a crude approximation because of the extrapolation required for this value of $2t/D$. The pressure drop corresponding to the collinear situation discussed above would be nine times larger.

It is emphasized that the calculations above represent a worst case, corresponding to a self-propped fracture whose plane is perfectly orthogonal to the well axis. Upon reaching depths of interest, 3.5 km (11,000 ft), the EE-2 and EE-3 wellbores are currently planned to be deviated about 30° from the vertical for the remainder of the drilling. This deviation should allow multiple fractures to be produced from the same wellbores, thus increasing the reservoir heat transfer area. (Experience to date indicates that the horizontal tectonic stresses are one-half the vertical overburden stress. Therefore, it is expected that induced fractures will be very nearly vertical.) A 30° angle of intersection will result in a fracture periphery at the wellbore about 54% greater than that afforded by an orthogonal intersection. Neglecting the fully three-dimensional nature of the flow produced by such an intersection, we can, as a first order approximation, assume that the flow is still a purely radial transition to longitudinal flow, but now with an effective wellbore diameter which is 54% greater. Under these conditions the ΔP_3 for the self-propped fracture would be reduced to approximately 50 MPa (700 psi). If n multiple,

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parallel fractures could be created, each flowing at $1/n$ of the total flow rate specified earlier, then ΔP_3 would be even further reduced by the factor n^{-2} .

These sorts of calculations illustrate the important effect that the fracture flow passage area at the wellbore has upon reservoir pressure losses. To achieve low flow impedances, one should attempt to increase one or more of the following: (1) the effective wellbore diameter at the intersection, (2) the fracture aperture, or (3) the number of flow paths that convey flow from the main hydraulic fractures to the wellbore. For example, one could locally enlarge the bore diameter with underreaming, stimulate more flow paths by multiple hydraulic fracturing or the use of explosive fracturing or "jetting," or acidize the existing flow paths so as to increase their near-wellbore apertures. In the absence of such man-initiated actions there also exists the possibility that natural events may foster such impedance reductions. For example, the high pressures associated with initially "tight" fracture-to-wellbore intersections may be high enough to open natural joints and fractures that are normally closed by tectonic stresses. The high velocities through these connections may result in chemical dissolution and erosion of either the host rock or the depositional products, e.g., calcite, that often fill these natural joints. Finally, Murphy (1978) has suggested that as the geothermal reservoir is thermally depleted the thermal contraction stresses so produced may be sufficient to open natural joints or produce new thermal fractures that result in additional flow paths to the wellbore. In fact, field evidence (Tester and Albright, 1979; Murphy, 1979a) provided by downhole temperature and flow rate well logs from the first reservoir at the Fenton Hill site show the development of major new flow paths, fostered by the natural events cited above.

CONCLUSIONS

Parameters related to flow regime three, associated with radially flowing fluid turning and entering the wellbore from an orthogonal fracture, were measured. These measurements included the associated pressure loss ΔP_3 ; the effect of the constriction of the vena contracta; and the points where attachment and then fully developed turbulent wellbore flow begins. Fully developed pipe flow was observed at 5 to 7 wellbore diameters from the fracture-well intersection for Reynolds numbers (based on the wellbore diameter and mean velocity) from 3000 to 58000 and $2t/D$ ranging from 0.01 to nearly 0.7. This entry development length is to be compared to literature values of 25 to 100. The early development noted in this study is attributed to enhanced turbulent mixing following the vena contracta at the wellbore entrance. The pressure loss ΔP_3 associated with regime three was found to be larger for smaller $2t/D$, and nearly Reynolds number-independent at large R . Results for dimensionless pressure losses, or loss coefficients, were presented for $2t/D$ ranging from 0.01 to 0.7, and for R ranging from 3000 to 58000.

These results were applied to a hot dry rock geothermal reservoir under construction near the Valles Caldera in northern New Mexico, and it was found that the pressure loss, ΔP_3 , would be 0.8 MPa (120 psi) for a single fracture with an aperture of 1 mm, but would increase to 13 MPa (2000 psi) if the aperture was as small as 0.2 mm. The first value is consistent with a fracture held open, i.e., inflated by high pressure, whereas the second value is consistent with previous estimates of self-propped fractures. It was pointed out that the fracture flow passage area at the wellbore intersection is critical in these calculations, so that if this area can be increased, either naturally or by stimulation, a significant reduction in flow impedance could be accomplished.

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