PRESSURE TRANSIENT ANALYSIS OF GEOTHERMAL WELLS WITH PHASE BOUNDARIES

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ABSTRACT

The analysis of pressure tests in geothermal reservoirs is often complicated by two-phase effects. This paper investigates the effect of a phase boundary at a constant radial distance from the well, produced, for example, by the flashing of a water reservoir during production or by the injection of water into a steam or two-phase reservoir. This configuration may be recognized from a time shift in the buildup (or falloff) semilog straight line, and, in some cases, the distance to the phase boundary can be estimated by plotting the transition pressure response against time on Cartesian coordinates. Field data from the Broadlands geothermal field in New Zealand confirms the applicability of the technique and demonstrates that the injected volume (which is known) may be used with the buildup data to calculate porosity and "swept" volume.

The analysis also indicates the possibility of determining compressibility and permeability contrasts across the phase boundary. This enables estimation of the reservoir porosity and relative permeabilities in the two-phase region. In a few cases, wellbore storage effects can disguise the pressure response and make parameter determination more difficult. These cases are discussed.

INTRODUCTION

Radial discontinuities around a well can occur in a wide variety of reservoir situations. In hydrocarbon reservoirs the radial inflow or outflow of a fluid bank will result in a compressibility discontinuity, or an in-situ combustion or stimulation program may result in a permeability discontinuity. In a gas condensate or geothermal reservoir, reduction of reservoir pressure in the vicinity of the well will give rise to changes in relative permeabilities as the fluid changes phase, and, in the case of water, enormous changes in compressibility. The same would be true in the case of a steam flood. Recent work by Grant and Sorey (1979) indicates that the compressibility of a two-phase steam/water mixture at the boiling point has an effective compressibility four orders of magnitude larger than that of water and two orders of magnitude larger than that of steam. Clearly, the appearance of a flash front in a water region or the start of condensation in a steam region would result in a sharp discontinuity of considerable order.

Thus this study reexamined the "sands in series" problem first considered by Hurst (1960). Greater generality is added to the equations governing the transient flow of a slightly compressible fluid in a porous medium during buildup or drawdown of a single well confined in concentric regions of differing permeabilities and/or hydraulic diffusivities. The problem may be formulated in Laplace transformed space, and the use of an efficient numerical transform inverter permits solution of a wider range of boundary configurations than earlier studies using analytical inversions (Hurst, 1960; and Carter, 1966). The use of the Laplace transform also gives greater insight into the pressure response as a function of its governing parameters than is possible with directly numerical results (for example, Bixel and van Poollen, 1967; Kazemi, Merrill, and Jargon, 1972; and Merrill, Kazemi, and Gogarty, 1974). In transformed space, limiting cases and parameter groups become more apparent and provide a greater understanding of the behavior of the solutions.

FORMULATION OF THE PROBLEM

The system considered is that of a single well at the axis of a radial system of two concentric regions of differing properties (porosity, ϕ, permeability, k, fluid viscosity, μ, reservoir volume factor, B, and compressibility, c). This configuration is illustrated in Fig. 1. The outer boundary, $r = b$, can be either finite or infinite, and a number of different conditions at this boundary will be considered. The infinite case has been considered by Hurst (1960), and Carter (1966) has presented the solution to the finite case with a no-flow outer boundary. Both studies used the Laplace transform; however, the necessity of inverting the transforms analytically required some restrictions on the parameters (for example, both Hurst and Carter assume that the ϕc product was the same for both regions).

This configuration can represent several phenomena of practical interest; namely, a change in phase, the presence of differing reservoir fluids (as, for example, in a waterflood), or the change in properties caused by a previous in-situ combustion recovery. In terms of geothermal reservoirs, it is likely that the large changes in pressure between the well and the formation can cause a flash front which, together with any consequent relative permeability effects, may cause the deposition of dissolved solids. The concentric ring configuration may also be used to represent the changes in permeability due to mud invasion of the producing territory or possible stimulation close to the wellbore.

Defining nondimensional time, radius, and pressures:

$$t_{DA} = \frac{k_1 t}{\phi_1 \mu_1 c_1 a^2} \qquad (1)$$

$$r_D = r/a \qquad (2)$$

$$p_{D1} = \frac{2\pi k_1 h}{q\mu_1 B_1} (p_1 - p_i);\ 0 < r_D < 1 \qquad (3)$$

$$p_{D2} = \frac{2\pi k_1 h}{q\mu_1 B_1} (p_2 - p_i);\ 1 < r_D < b/a \qquad (4)$$

Then the flow equations in the two regions are:

$$\frac{\partial^2 p_{D1}}{\partial r_D^{\ 2}} + \frac{1}{r_D}\frac{\partial p_{D1}}{\partial r_D} = \frac{\partial p_{D1}}{\partial t_{DA}}$$

$$0 < r_D < 1;\ 0 < t_{DA} \qquad (5)$$

and:

$$\frac{\partial^2 p_{D2}}{\partial r_{D2}^{\ 2}} + \frac{1}{r_D}\frac{\partial p_{D2}}{\partial r_D} = \frac{1}{\gamma}\frac{\partial p_{D2}}{\partial t_{DA}}$$

$$1 < r_D < b/a;\ 0 < t_{DA} \qquad (6)$$

where $\gamma = \dfrac{(k/\phi\mu c)_2}{(k/\phi\mu c)_1}$.

Initial conditions are that p_1 and p_2 are p_i everywhere, so:

$$p_{D1}(r_D,0) = p_{D2}(r_D,0) = 0 \qquad (7)$$

There are three boundary conditions to consider: the inner boundary, the formation discontinuity, and the outer boundary. At the well (the inner boundary), the condition is:

$$\lim_{r_D \to 0} r_D \frac{\partial p_{D1}}{\partial r_D} = 1 \qquad (8)$$

At the formation discontinuity, the flux and pressure must be continuous, so:

$$\frac{\partial p_{D1}}{\partial r_D} = \delta \frac{\partial p_{D2}}{\partial r_D} \qquad (9)$$

$$p_{D1} = p_{D2} \qquad (10)$$

as a consequence of Darcy's law, where:

$$\delta = \frac{(k/\mu)_2}{(k/\mu)_1}$$

At the outer boundary, three cases may be considered, depending on whether the boundary is at infinity, a no-flow boundary, or a constant pressure boundary:

Infinite Boundary:

$$\lim_{r_D \to \infty} p_{D2}(r_D,t_D) = 0;\ t_{DA} > 0 \qquad (11)$$

Finite System, No-Flow Boundary:

$$\frac{\partial p_{D2}}{\partial r_D} = 0 \text{ at } r_D = b/a;\ t_{DA} > 0 \qquad (12)$$

Finite System, Constant Pressure Boundary:

$$p_{D2} = 0 \text{ at } r_D = b/a;\ t_{DA} > 0 \qquad (13)$$

As mentioned previously, the infinite boundary case has been solved by Hurst (1960) for permeability discontinuity only, and the constant pressure boundary case by Carter (1966) with less generality.

It is seen that the pressure is a function of the two independent variables r_D and t_D, plus the four parameters, γ, 6, a/r_w, and b/r_w. The simpler study by Carter (1966) assumed $(\phi c)_1 = (\phi c)_2$, in which case γ = 6.

RESULTS

Since the pressure response of the radially discontinuous system depends on so many parameters, it is most instructive to consider overall properties of its behavior. In general, the response of a radial system in an infinite system can be represented for long times as:

$$p_D = 1.15 \log t_D + \xi \qquad (14)$$

Thus the slope of p_D vs log t_D is 1.15. If we refer to p_{DII} as the dimensionless pressure in terms of region 2 properties and p_{DI} as that in terms of region 1 properties, and t_{DII} and t_{DI} accordingly, then:

$$p_{DII} = p_{DI} \frac{(k/\mu)_{II}}{(k/\mu)_I} - \delta p_{DI} \qquad (15)$$

and:

$$t_{DII} = p_{DI} \frac{(\phi\mu c/k)_{II}}{(\phi\mu c/k)_I} t_{DI} = \gamma t_{DI} \qquad (16)$$

Thus:

$$dp_{DII} = \delta dp_{DI} \qquad (17)$$

and:

$$d(\log t_{DII}) = d(\log t_{DI}) \qquad (18)$$

Thus the slope of the response line (on log-normal paper) will depend on 6 but not on y. Since in this study p_D and t_D are defined in terms of region 1, the slope of the p_D - log t_D line will be 1.15 if the region 1 response is dominant, and 1.15/δ if the region 2 response is dominant. This behavior is illustrated in Fig. 2.

Even though the slope of the semilog straight line depends only on 6, the time required to reach that straight line will depend on γ. Thus it might be expected that the response would show two semilog straight lines (one for the region 1 response and one for the region 2 response), with one log cycle between them for every power of 10 in y. This behavior is illustrated in Fig. 3 (for $\delta = 1$).

The effect of the position of the discontinuity may be examined by considering the behavior of the solution in Laplace transform space. The solutions at long times correspond to those at small values of the Laplace variable, s. Thus, for example, in the case of the infinite outer region,

$$p_{1L} = \frac{1}{s}\left(K_0(r_D\sqrt{s}) - K_0(\sqrt{s}) + \frac{\delta}{\sqrt{\gamma}} \frac{K_1(\ s)\ K_0(\ s/\ )}{K_1(\sqrt{s/\gamma}}\right) \qquad (19)$$

since $I_0 \rightarrow 1$ and $I_1 \rightarrow 0$. *Also,* since $K_1(z) + 1/z$ as $z \rightarrow 0$:

$$p_{1L} = \frac{1}{s}\left(K_0(r_D\sqrt{s}) - K_0(\sqrt{s}) + \delta\ K_0(\sqrt{s/\gamma})\right) \qquad (20)$$

The inversion of this transform yields:

$$p_{D1} = \frac{1}{2}\left[-\ Ei\left(-\frac{r_D^2}{4t_{DA}}\right) + Ei\left(-\frac{1}{4t_{DA}}\right) - 6\ Ei\left(-\frac{t_{DA}\gamma}{4t_{DA}\gamma}\right)\right] \qquad (21)$$

For large times, this may be approximated as:

$$p_{D1} = \frac{1}{2}\left(\ln r_D^2 + \delta\ \ln \frac{}{4t_{DA}\gamma} + 0.5772\right) \qquad (22)$$

From this equation it may be seen that the influence of the parameter r_D (and hence the position of the discontinuity since $r_D = r/a$) displays itself as the intercept of a line p_D vs log t_D. Unfortunately, this intercept is also dependent on 6 and y; however, it may be seen that in a buildup test the r_D dependence will disappear. Thus for a production time of t and a shut-in time of At, the pressure will be:

$$p_{D1} = -\frac{\delta}{2} \ln \frac{t+\Delta t}{\Delta t} \qquad (23)$$

yielding a straight line on a Horner plot, independent of the radius of the discontinuity. The pressure response on semilog coordinates is shown in Fig. 4 for various values of r_w/a. These lines are geometrically similar for $t_D > 0.01$, and therefore plot to a single line on a Horner plot.

The effect of the outer boundary, where one exists, may disguise the effect of the permeability or diffusivity discontinuity as the boundary effects are felt before the region 2 response becomes dominant. Although, as shown in Fig. 5, the drawdown curve passes through the straight line region 2 response, it is unlikely that this part of the curve would be long enough to determine 6 unless b/a is large.

WELL TEST ANALYSIS

In the case of a two-phase reservoir pressure transient, it is probable that the position of the compressibility/permeability discontinuity (i.e., the flash front) will depend on the length of time the well has been produced. However, on shutting in, the extent of the two-phase region may be expected to change. The effects of these changes may be circumvented by graphing the pressure 'buildup on a Homer plot. In this case the mobility/discontinuity ratio may be determined from the final slope to which the curves tend as At becomes large. When the buildup is graphed in terms of pressure increase, the final slope will be δ times the initial slope. Since 6 yields only the mobility ratio, determination of the two permeabilities/viscosities will require that a standard well test analysis be done on the first semilog straight line portion of the curve. The diffusivity ratio is more difficult to determine since a non-unity value of δ will disguise the "shift" to some extent. It should, however, be feasible to estimate γ at least to an order of magnitude by constructing a 1.15 slope line through the point at which the region 2 straight line begins. This γ estimation should be made based on the p_D vs log time plot, as opposed to the δ estimation, which can use either p_D vs log time or the Homer plot. This is because the Horner plot gives insufficient resolution for longer shut-in times, and for long producing times, the characteristic "s" in the curve may disappear altogether. In some cases, however, the Horner plut could be used to estimate γ, where multiple buildups for different shut-in periods have been determined. In such cases, the parameter γ will determine the spacing between the Horner buildup lines.

In an earlier paper, Satman, Eggenschwiler, and Ramey (1980) demonstrated that there exists a pseudosteady-state pressure response in the transition region between the inner region and outer region semilog straight lines. This pseudosteady response is indicated by a straight line on a cartesian p vs t plot, the slope of which will be dp/dt. The bulk volume of the inner region can then be estimated from:

$$V = \frac{qB_1}{c_1 \frac{dp}{dt}} \qquad (24)$$

The calculation of this volume may be of great importance in the case of injection tests or evaluation of reinjection potential, where the calculated volume may be compared to the volume of fluid known to have been injected.

FIELD EXAMPLE

Broadlands Well BR26 is a geothermal well in the North Island of New Zealand. It produced in two-phase conditions from a depth of 950 m below the casing head flange, in a formation identified as Rautawiri Breccia. The two-phase conditions are the result of the high CO_2 concentrations typical of the Broadlands field (Grant, 1978). Ambient conditions at depth 950 m are 250°C, 8400 kPa.

BR26 was pump-tested on 1 July 1978. Water was pumped into the well at a rate of 26 ℓ/sec for 7.5 hours, followed by a reduction to 7 ℓ/sec for a further 2 hours. The well was then shut in for 2 hours, injected at 7.9 ℓ/sec for 21 minutes, then shut in and the pressure falloff measured with a Kuster gauge suspended at 950 m. The pressure decline is shown in Fig. 6. What first appears to be two semilog straight lines (as the analysis predicts) is in fact a single semilog straight line at later times and a wellbore storage effect at earlier times (see Fig. 7). However, graphing the response on Cartesian coordinates (Fig. 8) shows a pseudosteady-state behavior at the end of the early time response, indicating that the apparent early semilog straight line does in fact mark the end of the inner region response. Thus the "shift" of $\log_{10} 7$ can be validly interpreted as the hydraulic diffusivity contrast between the outer and inner regions in the reservoir (the "unswept" and "swept" regions, respectively). Since the early semilog straight line is masked by storage effects, it is not possible to determine the permeability contrast. The permeability contrast is not expected to be large since, although the well produces a two-phase mixture of steam and water, it is mainly a liquid flow.

Thus:

$$c_2 = 7c_1$$

or:

$$(c_m+\phi c_T) = 7(c_m+\phi c_w)$$

The two-phase compressibility, c_T, is calculated by Grant (1978) to be 6×10^{-5} kPa^{-1}, the formation compressibility of the geologically similar Waiora formation at Wairakei was estimated by Wooding and Grant (1978) to be 2×10^{-6} kPa^{-1}, and the compressibility of water, c_w, at 250°C is 1.3×10^{-6} kPa^{-1}. Solving for the porosity, ϕ, a value ϕ = 23% is obtained. This is in the range of 20–25% anticipated for this formation.

Further information may be found using the Pseudosteady-state straight line in Fig. 8, which has a slope of 17.65 kPa/min. The volume of the inner region may thus be calculated to be 11650 m^3, or a cylinder of height 100 m and radius 6 m. Taking this to be the diffusion length (or drainage radius) at the start of the pseudosteady-state:

$$6 = \sqrt{4 \frac{k}{\phi\mu c} t}$$

where t = 60 seconds. Thus:

$$\frac{k}{\phi\mu c} = 0.15 \text{ m2/sec}$$

which, with a viscosity, μ, of 1.1×10^{-7} kPa·s and the previously determined values ϕ = 0.23 and c = 2.3×10^{-6} kPa^{-1}, yields a permeability of k = 8.7 md.

Thus it has been possible to estimate the inner region permeability despite the fact that there is no semilog straight line. By comparison, the outer region, for which there is a semilog straight line, is found to have a kh of 0.7 d-m or a permeability of 7 md, assuming the same 100 m thickness.

In this case it is not possible to estimate the porosity of the injected volume since the 2000 kPa pressure rise due to the pumping will extend the compressed liquid region beyond the liquid actually injected. Notice that 8.7×10^5 liters were injected, which would occupy a reservoir volume of 3780 m^3 at a porosity of 23%, whereas the analysis shows the "swept" volume to be 11650 m^3.

CONCLUSIONS

1) In a reservoir with radial discontinuity, the mobility ratio may be determined from the relative slopes of the early- and long-time semilog straight lines.

2) In cases where the hydraulic diffusivity is also discontinuous, the early- and long-time semilog straight lines are removed from one another by a distance dependent on the diffusivity ratio. This distance may be indeterminable if a mobility ratio also exists.

3) In the case of an infinite reservoir, the position of the discontinuity does not affect the determination of the mobility or diffusivity ratios since the pressure response is geometrically similar for physically realistic times.

4) In the case of a finite system, the interception of a boundary may occur before the appearance of pseudosteady outer region response, in which case the analysis will not be possible.

NOMENCLATURE

a = radius of discontinuity
b = radius of outer boundary
c = compressibility
h = formation thickness
k = permeability
p = pressure
q = flowrate

r = radius
s = Laplace transform variable
t = time
B = reservoir volume factor
R = b/a
γ = diffusivity ratio (outer/inner)
δ = mobility ratio (outer/inner)
μ = viscosity

Subscripts

D = dimensionless
L = Laplace transformed variable (dimensionless)
1 = inner region
2 = outer region
w = wellbore

Functions

I_0, K_0, I_1, K_1 = modified Bessel functions, order 0 and 1.

ACKNOWLEDGEMENTS

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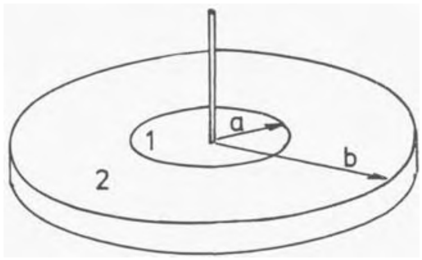


Fig. 1: Problem Configuration

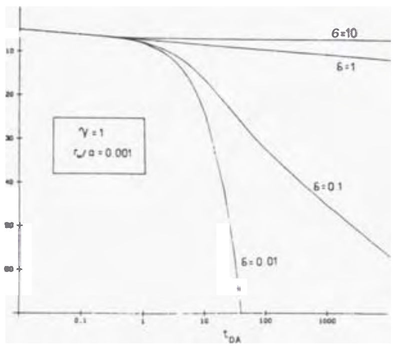
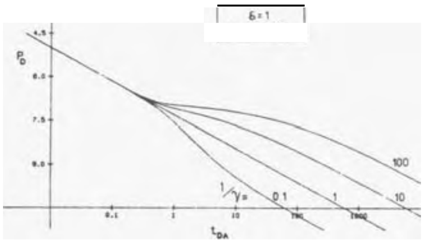


Fig. 2: Pressure Drawdown Response as a Function of Mobility Ratio. Diffusivity Ratio 1.



Eig. 3: Pressure Drawdown Response as a Function of Diffusivity Ratio. Mobility Ration 1.

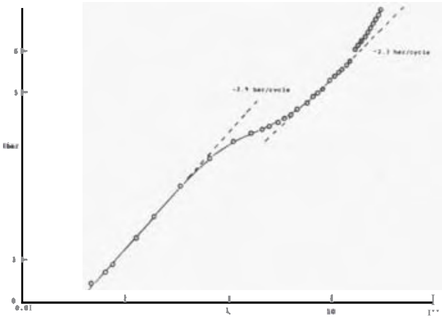


Fig. 6: Projection Fall-Off Test on Well BR26. Semilog Coordinates.

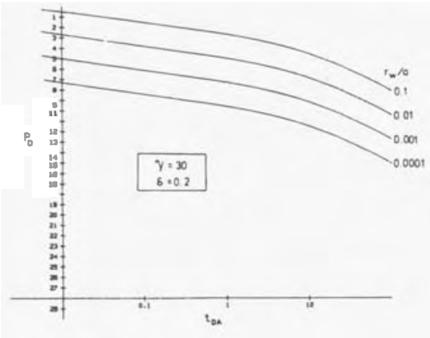


Fig. 4: Pressure Drawdown Response as a Function of Distance to Radial Discontinuity.

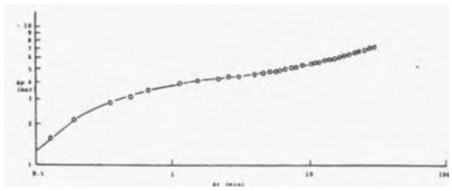


Fig. 7: Injection Fall-Off Test on Well BR26. Log-Log Coordinates.

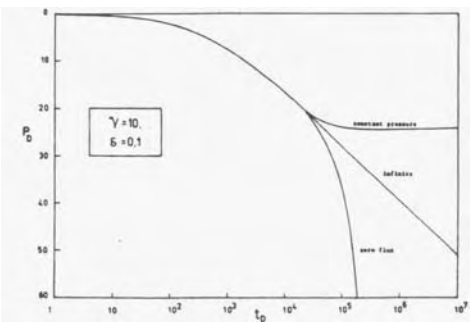


Fig. 5: Pressure Drawdown Response in Cases of Impermeable or Constant Pressure Outer Boundary.

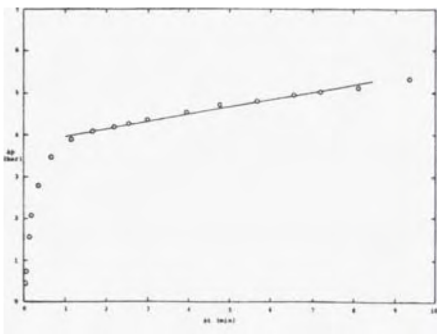


Fig. 8: Injection Fall-Off Test on Well BR26. Cartesian Coordinates.