Correlation and Estimation of Holdup in Two-Phase Flow

by

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1. Introduction

Butterworth has suggested that an equation of the form

$$\frac{\bar{R}_L}{\bar{R}_G} = A\left[\frac{1-x'}{x'}\right]^p \left(\frac{\rho_G}{\rho_L}\right)^q \left[\frac{\mu_L}{\mu_G}\right]^r \qquad [1]$$

is useful in expressing the results of a large number of correlations of holdup in two-phase horizontal gas-liquid flow. Chen and Spedding[2] have shown theoretically that the use of equation [1] is justified for the cases of the ideal stratified and annular flow regimes where the flow of the phases are either both turbulent or both laminar. However, the analysis showed that the values of the indices A, p, q and r varied depending on the value of $\bar{R}_G/\bar{R}_L$ and the flow pattern. This is in agreement with the findings of Butterworth[1]. Thus there is no general correlation which can be expected to predict holdup but specific correlations can be obtained to cover a limited range of conditions.

Experimental data plus the results of other workers[3-7] have established the usefulness of two holdup relations namely the empirical Armand equation[8] and the Nguyen and Spedding model[9]. The Armand equation may be written as

$$\frac{\bar{R}_L}{\bar{R}_G} = \frac{1}{K}\left[\frac{1-x'}{x'}\right]\frac{\rho_G}{\rho_G} + \frac{1-K}{K} \qquad [2]$$

while the Nguyen and Spedding analysis gives

$$\frac{\bar{R}_L}{\bar{R}_G} = C_o\left[\frac{1-x'}{x'}\right]\frac{\rho_G}{\rho_G} + (C_o - 1) + \frac{B}{V_{SG}} \qquad [3]$$

which is essentially of the same form.

Equation [1] can be written in terms of volumetric flow rates and thus for a given system at constant temperature and pressure often reduces to

$$\frac{\bar{R}_G}{\bar{R}_G} = H \left(\frac{Q_G}{Q_L} \right)^s \qquad [4]$$

Figure 1 gives a plot of the data in the form Suggested by equations [2] and [4].. It transpires that all plug and slug flow data, i.e. with $Q_G/Q_T < 0.9$, obeyed the transformed Armand relation of equation [2], quite independently of the system or the temperature and pressure involved.

Annular flow data were observed to generally follow a relation of the form suggested by equation [1] with some additional variation due to pipe diameter. A relation was developed similar to equation [4] which would be wed with advantage for any particular system since the variations of holdup due to D, $\frac{\rho_G}{\rho_L}$ and $\frac{\mu_L}{\mu_G}$ only affected the value of the constant H. The actual relation for annular flow cast in the form of equation [4] proved to be in general agreement with the transformed Lockhart-Martinelli equation[10] albeit 20% higher in value. The theoretical predictions of Chen and Spedding[2] did not follow the annular flow data; the failure of the model being bound up with the occurance of disturbance waves on the liquid-gas interface.

2. Theoretical Development

Stratified flow data'** exhibited a large scatter when plotted in the form suggested by equation [4] because the. data seemed to be strongly dependent on the nature of the flow that is whether the phases were laminar or turbulent in nature. However, the problem has some solution by using an idealised stratified flow anlaysis.

Figure 2 outlines the idealised form of stratified flaw in which the liquid passes along the base of the pipe such that the gas-liquid interface io smooth. If the system is in equilibrium a force balance taken separately for the gar, and the liquid phases yield the following equations.

$$-A_L \left(\frac{dP}{dx} \right)_{LF} - \tau_{WL} S_G + \tau_i S_i = 0 \qquad [5]$$

$$-A_G \left(\frac{dP}{dx} \right)_{GF} - \tau_{WG} S_G - \tau_i S_i = 0 \qquad [6]$$

It is assumed that the pressure gradients in the liquid and the gas phases are equal as done by Taitel and Dukler[11]. That is,

$$\left(\frac{dP}{dx} \right)_{LF} = \left(\frac{dP}{dx} \right)_{GF} \qquad 171$$

By substitution of equation [5] and [6] into equation [7]

$$\tau_{WG} \left(\frac{S_G}{A_G} \right) - \tau_{WL} \left(\frac{S_L}{A_L} \right) + \tau_i S_i \left(\frac{1}{A_L} + \frac{1}{A_G} \right) = 0 \qquad [8]$$

The shear stresses may be evaluated as in the case of single phase flow

$$\tau_{WL} = f_L \frac{\rho_L \bar{u}_L^2}{2} \qquad [9]$$

$$\tau_{WG} = f_G \frac{\rho_G \bar{u}_G^2}{2} \qquad [10]$$

$$\tau_i = f_i \rho_G \frac{(\bar{u}_G - \bar{u}_i)^2}{2} \qquad [11]$$

where f is the friction factor which may be expressed in the form

$$f_L = C_L \left(\frac{\bar{D}_L \bar{u}_L}{\nu_L} \right)^{-m} \qquad [12]$$

$$f_G = C_G \left(\frac{\bar{D}_G \bar{u}_L}{\nu_G} \right)^{-n} \qquad [13]$$

Generally it can be assumed that $\bar{u}_G >> \bar{u}_i$ as by Bergelin and Gazley[12] and that for a smooth interface $f_i = f_G$. Then equation [11] becomes,

$$\tau_i = f_G \frac{\rho_G \bar{u}_G^2}{2} \qquad [14]$$

while

$$\bar{D}_G = \frac{4 \bar{R}_G A}{(S_i + S_G)} \qquad [15]$$

$$\bar{D}_L = \frac{4 \bar{R}_L A}{S_L} \qquad [16]$$

$$\bar{u}_G = \frac{Q_G}{\bar{R}_G A} \qquad [17]$$

$$\bar{u}_L = \frac{Q_L}{\bar{R}_L A} \qquad [18]$$

substitution of equation [9] to [18] into equation [8] yields

$$\left(\frac{\bar{R}_G}{\bar{R}_L}\right)^2 = \frac{C_G\, 4^{-n}\, \rho_G\, Q_G^{\,2-n}\, \nu_L^{\,-m}}{C_L\, 4^{-m}\, \rho_L\, Q_L^{\,2-m}\, \nu_G^{\,-n}}\; S_L^{\,-m} \left[\frac{1}{S_i + S_G}\right]^{-n} \left[\frac{S_G \bar{R}_L + S_i}{\bar{R}_G S_L}\right] \qquad [19]$$

which can be expressed in terms of the type of flow in the two phases.

Turbulent Gas and Liquid Flow

When both gas and liquid phases are flowing in the turbulent regime $C_G = C_L = 0.046$ and $n = m = 0.2$, so equation [19] reduces to

$$\frac{\bar{R}_G}{\bar{R}_L}\left[\left(\frac{S_i + S_G}{S_L}\right)^{-0.2}\left(\frac{\bar{R}_G S_L}{S_G \bar{R}_L + S_i}\right)\right]^{0.5} = \left(\frac{Q_G}{Q_L}\right)^{0.9}\left(\frac{\rho_G}{\rho_L}\right)^{0.4}\left(\frac{\mu_L}{\mu_G}\right)^{-0.1} \qquad [20]$$

$$\frac{\bar{R}_G}{\bar{R}_L}\, P^{0.5} = \left(\frac{Q_G}{Q_L}\right)^{0.9}\left(\frac{\rho_G}{\rho_L}\right)^{0.4}\left(\frac{\mu_L}{\mu_G}\right)^{0.1} \qquad [21]$$

From an inspection of Figure 2 it can be deduced that P may be expressed as a function of $(\bar{R}_G/\bar{R}_L)$. Thus P can be solved analytically or numerically in terms of $(\bar{R}_G/\bar{R}_L)$. The numerical solution is given in Figure 3 from which it can be observed that P may be divided into a number of distinct ranges each with a particular approximate relationship, details of which are presented in Table 1 Substitution of these values of P into Equation [20] leads to relationships of the type

$$\frac{\bar{R}_G}{\bar{R}_L} = B \left(\frac{Q_G}{Q_L}\right)^a \left(\frac{\rho_G}{\rho_L}\right)^b \left(\frac{\mu_L}{\mu_G}\right)^c \qquad [22]$$

or by rearranging in equation [1] and equation [4]. Tabulated values are given in Table 2.

Laminar Gas and Liquid Flow

In this case $C_G = C_L = 16.0$ and $n = m = 1.0$ thus Equation [19] reduces to

$$\frac{\bar{R}_G}{\bar{R}_L} \left[\frac{\bar{R}_G S_L^2}{(S_i + S_G)(S_G \bar{R}_L + S_i)}\right]^{0.5} = \left(\frac{Q_G}{Q_L}\right)^{0.5} \left(\frac{\mu_G}{\mu_L}\right)^{0.5} \qquad [23]$$

Using a similar technique to the case of turbulent flow

$$U = \left[\frac{\bar{R}_G S_L^2}{(S_i + S_G)(S_G \bar{R}_L + S_i)}\right] \qquad [24]$$

which is solved numerically in terms of $(\bar{R}_G/\bar{R}_L)$ and is plotted in Figure 4. There are three ranges in this case and their relationships are given in Table 3. substitution of these values of U into equation [23] yields a relationship of the type presented in equation [22] or by rearrangement in equation [1]. Tabulated values of constants are given in Table 2.

Turbulent Gas and Laminar Liquid Flow

This situation is represented by taking the following values for C_G, C_L, n and m in equation [19]

$C_G = 0.046$ $\qquad$ $n = 0.2$

$C_L = 16.0$ $\qquad$ $m = 1.0$

yielding

$$\left(\frac{\bar{R}_G}{\bar{R}_L}\right)^2 = \frac{0.046}{16}\,\frac{4^{-0.2}}{4^{-1.0}}\,\frac{\rho_G}{\rho_L}\,\frac{Q_G}{Q_L}^{1.8}\,\nu_L^{-1}\left(\frac{1}{\nu_G}\right)^{-0.2} S_L^{-1}\left(\frac{1}{S_i + S_G}\right)^{-0.2}\left(\frac{S_G\,\bar{R}_L + S_i}{\bar{R}_G\,S_L}\right) \qquad [25]$$

Simplifying and rearranging giver

$$\left(\$1' \left[\frac{S_L}{(S_i + S_G)^{0.2}}\left(\frac{\bar{R}_G\,S_L}{S_G\,\bar{R}_L + S_i}\right)\right]\right. = 0.008715\,\frac{\rho_G}{\rho_L}\,\frac{Q_G}{Q_L}^{1.8}\,\frac{\nu_G}{\nu_L}^{0.2} \qquad [26]$$

For equation [26] to be of practical use, the term in square brackets on LHS must be solved in terms of $(\bar{R}_G/\bar{R}_L)$. Let

$$M = \frac{S_L\,D^{-0.8}}{(S_i + S_G)^{0.2}}\left(\frac{\bar{R}_G\,S_L}{S_G\,\bar{R}_L + S_i}\right) \qquad [27]$$

where $D^{-0.8}$ is introduced to make M independent of pipe size. Equation [26] becomes

$$\left(\frac{\bar{R}_G}{\bar{R}_L}\right)^2 M = 0.008715\,\frac{\rho_G\,Q_G^{1.8}\,\nu_G^{0.2}}{\rho_L\,Q_L\,\nu_L\,D^{0.8}} \qquad [28]$$

Equation [27] is solved numerically in terms of $(\bar{R}_G/\bar{R}_L)$ and is plotted in Figure 5 and piecewise straight line curve fitting yields:

$$M = A_1\,(\bar{R}_G/\bar{R}_L)^{a_1} \qquad [29]$$

where the values A_1 and a_1 are given in Table 4. Substitution of equation [28] to equation [29] yields

$$\frac{\bar{R}_G}{\bar{R}_L} = \left[A_2 \frac{\rho_G}{\rho_L} \frac{Q_G^{\;1.8}}{Q_L} \frac{\nu_G^{\;0.2}}{\nu_L} \frac{1}{D^{0.8}} \right]^{\frac{1}{a_2}} \qquad [30]$$

with the values of A_2 and a_2 given in Table 5.

<u>Laminar Gas and Turbulent Liquid Flow</u>

This situation is represented by taking the following values of C_G, C_L, n and m in equation [19].

C_G =	16.0	n =	1.0
C_L =	0.046	m =	0.2

yielding

$$\left(\frac{\bar{R}_G}{\bar{R}_L}\right)^2 = \frac{16 \;\; 4^{-1}}{0.046 \;\; 4^{-0.2}} \frac{\rho_G}{\rho_L} \frac{Q_G}{Q_L^{\;1.8}} \nu_L^{\;-0.2} \left(\frac{1}{\nu_G}\right)^{-1} S_L^{\;-0.2} \left(\frac{1}{S_i + S_G}\right)^{-1} \left(\frac{S_G \bar{R}_L + S_i}{\bar{R}_G \, S_L}\right)$$

[31]

Simplifying and rearranging gives

$$\left(\frac{\bar{R}_G}{\bar{R}_L}\right)^2 \left[\frac{S_L^{\;1.2} \bar{R}_G}{(S_i + S_G) \;\; (S_G \bar{R}_L + S_i)}\right] = 114.74 \frac{\rho_G}{\rho_L} \frac{Q_G}{Q_L^{\;1.8}} \frac{\nu_G}{\nu_L^{\;0.2}} \qquad [32] .$$

The term in the square bracket on the LHS of equation [32] needs to be solved in terms of $(\bar{R}_G/\bar{R}_L)$. Let

$$N = \frac{\bar{R}_G \, S_i^{\;1.2} \, D^{0.8}}{(S_i + S_G) \;\; (S_G \bar{R}_L + S_i)} \qquad [33]$$

Equation [33] is solved numerically and is plotted in Figure 6 giving

$$N = A_3 \;\; (\bar{R}_G/\bar{R}_L)^{a_3} \qquad [34]$$

where the values of A_3 and a_3 are given in Table 6.

Substitution of equation (32) to [34] gives

$$\left(\frac{\bar{R}_G}{\bar{R}_L}\right) = \left[A_4 \quad \frac{\rho_G}{\rho_L} \quad \frac{Q_G}{Q_L^{1\cdot 8}} \quad \frac{\nu_G}{\nu_L} \quad D^{0\cdot 8}\right]^{\frac{1}{a_4}} \qquad [35]$$

with the values of A_4 and a_4 given in Table 7.

TABLE I

Relationship between P and the holdup ratio.

Range of $\bar{R}_G/\bar{R}_L$	Approximate Relation P
2.5×10^{-6} - 3×10^{-4}	$0.95\ (\bar{R}_G/\bar{R}_L)^{0\cdot 6}$
a $\times 10^{-4}$ - 2.1×10^{-2}	$0.71\ (\bar{R}_G/\bar{R}_L)^{0\cdot 57}$
2.1×10^{-2} - 2.5×10^{-1}	$0.51\ (\bar{R}_G/\bar{R}_L)^{0\cdot 49}$
2.5×10^{-1} - 1.3	$0.40\ (\bar{R}_G/\bar{R}_L)^{0\cdot 33}$
1.3 - 8.0	$0.42\ (\bar{R}_G/\bar{R}_L)^{0\cdot 18}$
8.0 - 1.4 x 10'	0.61
1.4×10^{2} - 10^{5}	$0.84\ (\bar{R}_G/\bar{R}_L)^{-0\cdot 064}$

TABLE II

Holdup Data from Various Models and Correlations

Flow Type	Flow Regime	Z_G / R_L	equation [1] A	p	q	r	eq. [4] H	s	equation [22] B	a	b	c
			Value of Constants									
Turbulent	Stratified	2.5~10' - 3×10^{-4}	0.98	0.69	0.38	0.05	0.094	0.69	1.02	0.69	0.31	0.08
"	"	3×10^{-4} - 2.1×10^{-2}	0.875	0.70	0.39	0.08	0.105	0.73	1.143	0.70	0.31	0.08
"	"	2.1×10^{-2} - 2.5×10^{-1}	0.763	0.72	0.40	0.08	0.112	0.72	1.31	0.72	0.32	0.08
"	"	2.5×10^{-1} - 1.3	0.175	0.77	0.43	0.09	0.107	0.77	1.48	0.77	0.34	0.09
"	"	1.3 - 8.3	0.671	0.83	0.46	0.09	0.083	0.83	1.49	0.83	0.37	0.09
"	"	8.0 - 1.4×10^{2}	0.78	0.90	0.5	0.1	0.059	0.90	1.28	0.9	0.4	0.1
"	"	1.4×10^{2} - 10^{5}	0.914	0.93	0.52	0.1	0.047	0.93	1.09	0.93	0.41	0.1
Laminar	"	10^{-3} - 0.2	0.68	0.45	0.45	0.45	0.239	0.45	1.46	0.45	0	0.45
"	"	0.2 - 3.0	0.51	0.50	0.50	0.50	0.262	0.50	1.95	0.30	0	0.50
"	"	3.0 - 10'	0.55	0.57	0.57	0.57	0.185	0.53	1.83	0.57	0	0.57
Turbulent	Annular	-	1.0	0.9	0.5	0.1	0.046	0.90	1.0	0.9	0.4	0.1
Laminar	"	-	1.0	0.5	0.5	0.5	0.134	0.50	1.0	0.5.	0	0.5
Model												
Homogeneous Model			1	1	1	0	1.0	1.0	1	1	0	0
Zivi Model[13]			1	1	0.67'	0	0.111	1.0	1	1	0.33	0
Turner Wallis Model[14]			1	0.72	0.4	0.08	0.086	0.72	i	0.72	0.32	0.08
Lockhart - Martinelli[18]			0.28	0.64	0.36	0.07	0.417	0.64	3.37	0.64	0.28	0.07
Thom[15]			1	1	0.89	0.18	0.233	1.0	i	1	0.11	0.18
Baroczy[16]			1	0.74	0.65	0.13	0.325	0.74	1	0.74	0.09	0.13
Harrison[7]			1	0.80	0.515	0	0.149	0.80	1	0.80	0.285	0

TABLE III

Relationship between U and $\bar{R}_G/\bar{R}_L$

Range of $\bar{R}_G/\bar{R}_L$	Approximate Relation U
10^{-3} to 0.2	$0.426\ (\bar{R}_G/\bar{R}_L)^{0.33}$
0.2 to 3.0	0.26
3.0 to 10^3	$0.35\ (\bar{R}_G/\bar{R}_L)^{-0.26}$

TABLE IV

$\bar{R}_G/\bar{R}_L$ range	λ_1	a_1
0.1 to 0.7	0.63	0.25
0.7 to 3.5	0.575	0.0
3.5 to 20.0	0.724	-0.174
20.0 to 200.0	1.055	-0.3

TABLE V

$\bar{R}_G/\bar{R}_L$ range	λ_2	a_2
0.1 to 0.7	0.01383	2.25
0.7 to 3.5	0.01516	2.0
3.5 to 20.0	0.012	1.826
20.0 to 200.0	0.00826	1.70

TABLE VI

$\bar{R}_G/\bar{R}_L$ range	λ_3	a_3
0.04 to 0.2	0.213	0.25
0.2 to 6.0	0.182	0.153
6.0 to 150.0	0.242	0.0

TABLE VII

$\bar{R}_G/\bar{R}_L$ range	A_4	a_4
0.04 to 0.2	538.69	2.25
0.2 to 6.0	630.44	2.153
6.0 to 150.0	474.13	2.0

3. Application of Stratified Flow Equations to Experimental Data

A complete set of equations covering the four possible combinations of flow regimes in stratified flow is now available for the prediction of holdup. The equations had been derived with the assumption of a smooth gas-liquid interface. It would be interesting to see how the equations apply to practical cases when surface disturbances are a common feature.

Since the formulations for the equations for stratified flow were similar to those used by Taitel and Dukler[11] in their derivation of the theoretical equations for the Lockhart-Martinelli type of relationship it is expected that the holdup predicted for stratified flow from this set of equations should agree with those of the Taitel and Dukler predictions. Based on the X parameters calculated, the predicted $\bar{R}_L$ were plotted against the $\bar{R}_L$ - X line derived by Taitel and Dukler as shown in Figure 7 and 8. Only data of gas-turbulent, liquid-laminar and gas-turbulent, liquid turbulent are shown. Agreement is excellent. In Figure 9 data of Govier and Omer covering the four regimes as predicted by the author's holdup prediction are shown against Taitel and Dukler's predictions. Again agreement between them is excellent.

It has been shown so far that the holdup predicted by the equations derived in this work agreed exactly with the predictions of the Lockhart-Martinelli type of $\bar{R}_L$ - X relationship derived by Taitel and Dukler. Next, experimental results obtained in this work will be compared with the values calculated by using the derived equations. This is done by plotting the predicted $\bar{R}_L$ against the measured $\bar{R}_L$ in Figures 10, 11, 12 and 13 depending on the flaw characteristics of the gas and liquid phases whether it is laminar or turbulent. Data of Govier and Omer plotted in Figure 9 have also been included.

Data of Govier and Omer deviate in a similar fashion to those obtained in this work. In the Turbulent-Turbulent regime plotted in Figure 10 the predicted values a m higher than the measured. This is believed to be caused by the existence of faster moving waves at the interface where a large proportion of the liquid is being transported at a higher speed as discussed by Ueda[17] and Quandt[18]. In the laminar turbulent regime shown in Figure 11 the predicted is lower than the measured and the reason for it is not clear. Them am very few data in the laminar-turbulent and laminar-laminar regimes. The Laminar-Laminar data of Govier and Omer plotted in Figure 13 showed excellent agreement with predictions indicating that it is perhaps in this case that the mal situation resembles most closely that used in the derivation of the equationr.

It therefore appears that the set of holdup equations for stratified flow presented in this work are quite valid despite the assumption of smooth interface. It agreed excellently with the Lockhart-Martinelli type of holdup prediction derived by Taitel and Dukler. Moreover, the agreement with experimental data was within reason.

The application of the holdup equations presented in this work to pressure drop prediction is not pursued here because pressure drop prediction in stratified flow ham already been covered adequately by using the Lockhart-Martinelli type of relationship[11,19].

4. Conclusions

The general holdup correlation suggested by Butterworth[1] has been shown to bo valid from theoretical and experimental evidence. The plug and slug flow data of a number of workers has been shown to follow the Armand equation[8] for a wide range of conditions. The annular flow regime can be predicted using the general relation

$$\frac{\bar{R}_G}{\bar{R}_L} = H \left(\frac{Q_G}{Q_L} \right)^s \qquad [4]$$

which is a special form of the Butterworth relation. This equation does show some variation with experimental conditions and the type of system.

Stratified flow data can be predicted using a complete set of equation derived from an idealised flow model. The resulting equations were shown to give agreement with the Lockhart-Martinelli[10] type of $\bar{R}_L$ versus X relationship for stratified flow derived by Taitel and Dukler[11].

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6. Symbols Used

A	-	factor given in equation [1]
A_G	-	flow area of gas phase L^2
A_L	-	flow area of liquid phase L^2
a	-	factor defined by equation [22]
A_1, a_1	-	factors defined by equation [29], with values given in Table 4.
A_2, a_2	-	factors defined by equation [30], with values given in Table 5.
A_3, a_3	-	factors defined by equation [34], with values given in Table 6.
B'	-	initial function given in equation [3], LT^{-1}
B	-	factor defined by equation [22].
b	-	factor defined by equation [22]
a	-	factor defined by equation [22]
C_o	-	distribution parameter given in equation [3]
C	-	constant in Blasins equation equals 16.0 for laminar flow, 0.016 for turbulent
D	-	pipe diameter L
$\bar{D}$	-	hydraulic diameter L
f	-	friction factor

H	-	factor defined by equation [4]
K	-	Armand's factor given in equation [2] equal to 0.83 for most situations
M	-	function defined by equation [27]
m,n	-	exponent in Blasins equation equals 1.0 for laminar flow, 0.2 for turbulent flaw
N	-	function defined by equation [33]
P	-	pressure $M L^{-1} T^{-2}$ also function defined by equation [21]
p	-	factor in equation [1]
Q	-	volumetric flow rate $L^3 T^{-1}$
q	-	factor in equation [1]
$\bar{R}$	-	average holdup
r	-	factor in equation [1]
S	-	length of wall or interface in contact with thr phases shown in Figure 2.
o	-	factor in equation [4]
U	-	function defined by equation [24]
$\bar{u}$	-	true average velocity
V_{SG}	-	superficial gas velocity
x	-	axial distance along pipeline L
x'	-	dryness fraction
X	-	Lockhart-Martinelli flow modulus, equals the square root of the presoure drop ratio of single phase liquid flow to single phase gam flow.

Subscripts

F	-	frictional component
G	-	gas phase
i	-	interface
L	-	liquid phase
W	-	wall

Greek Letters

μ	-	absolute viscosity $M L^{-1} T^{-1}$
ν	-	kinematic viscosity $L^2 T^{-1}$
ρ	-	density $M L^{-3}$

7. Acknowledgements

The D.S.I.R. of New Zealand is to be thanked for financial support of this work.

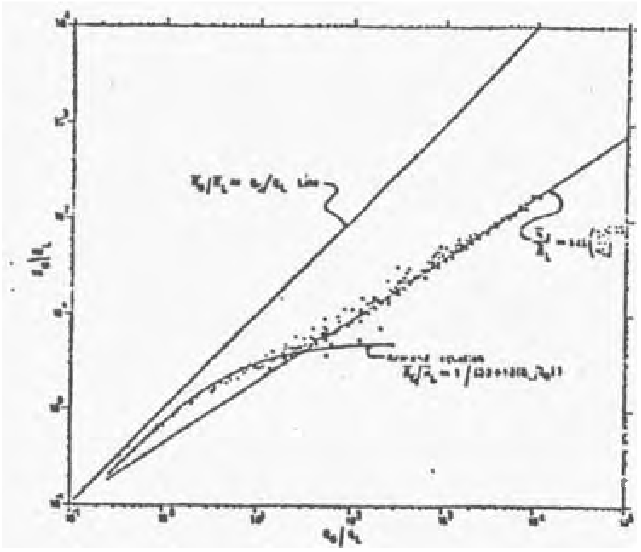


Figure 1

Hold-up relations for slug, plug and annular flow regimes. Chisholm and Laird's data.

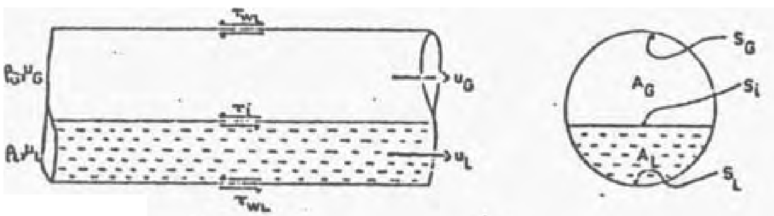


Figure 2

Schematic diagram of ideal stratified flow.

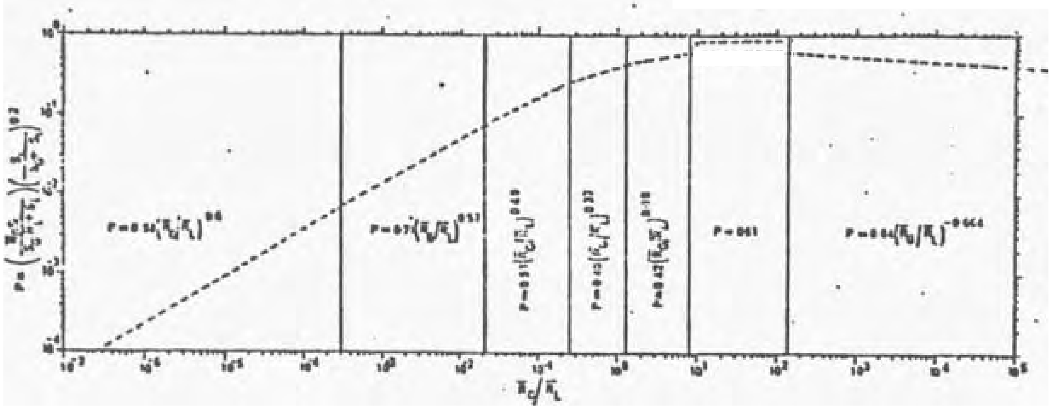


Figure 3

Relationship between P and $\bar{R}_G/\bar{R}_L$ from equations [20] and [21] which are derived for turbulent gas and liquid stratified flow.

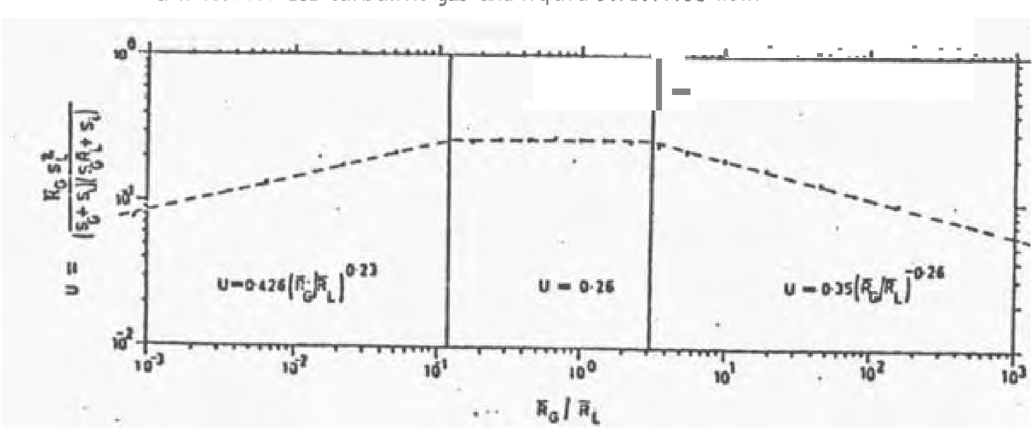


Figure 4

Relationship between U and $\bar{R}_G/\bar{R}_L$ from equations [23] and [24] which are derived for laminar gas and liquid stratified flow.

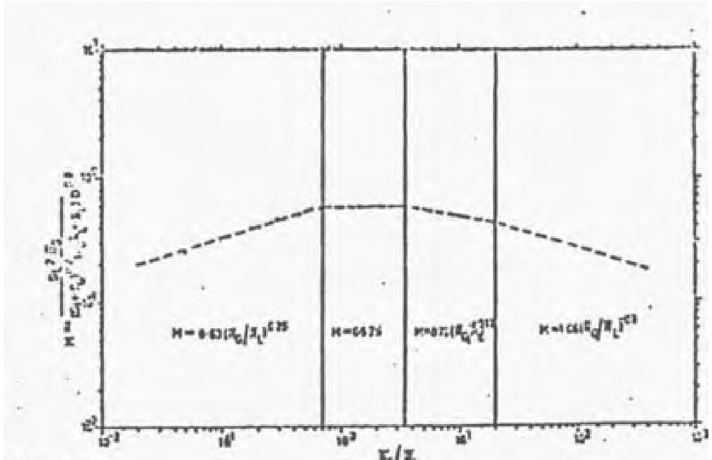


Figure 5

Relationship between M and $\bar{R}_G/\bar{R}_L$ from equations [26] and [27] which are derived for turbulent gas and laminar liquid stratified flow.

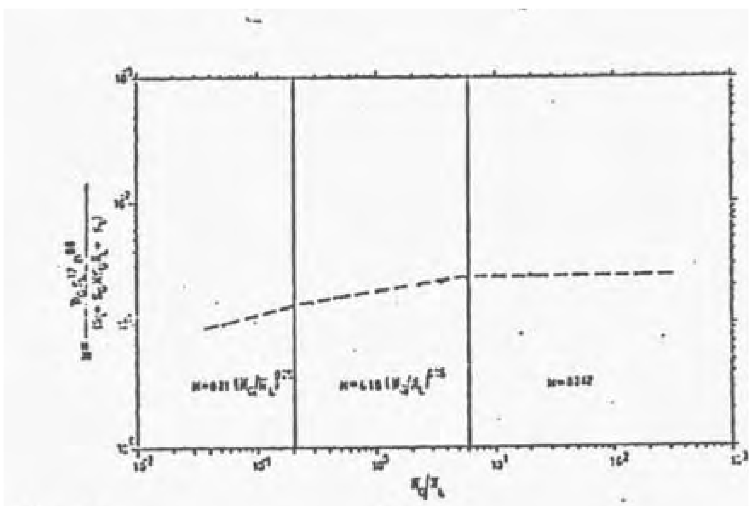


Figure 6

Relationship between N and $\bar{R}_G/\bar{R}_L$ from equations [32] and [33] which are derived for laminar gas and turbulent liquid stratified flat.

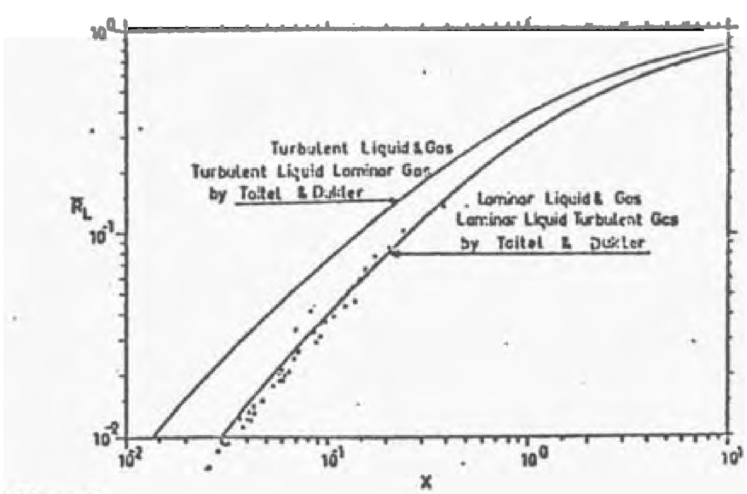


Figure 7

Comparison of $\bar{R}_L$ prediction using equation [30] and the R_L - X plots of Taitel and Dukler[11] for turbulent gas and laminar liquid stratified flow.

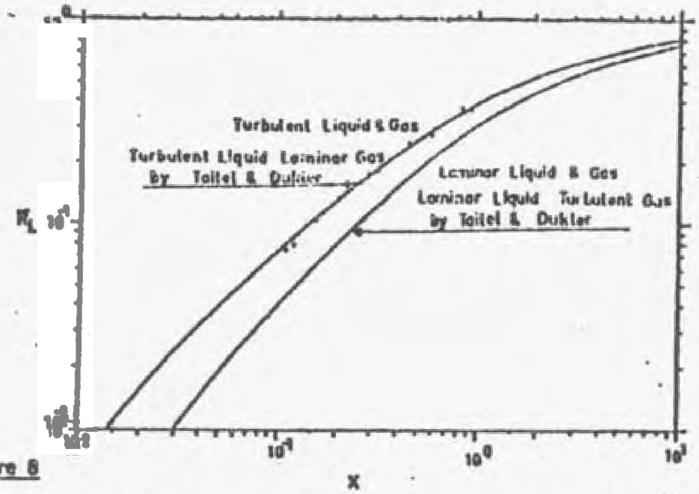


Figure 8

Comparison of R_L predictions using equation [21] and the R_L - X plots of Taitel and Dukler[11] for turbulent gas and liquid stratified flow.

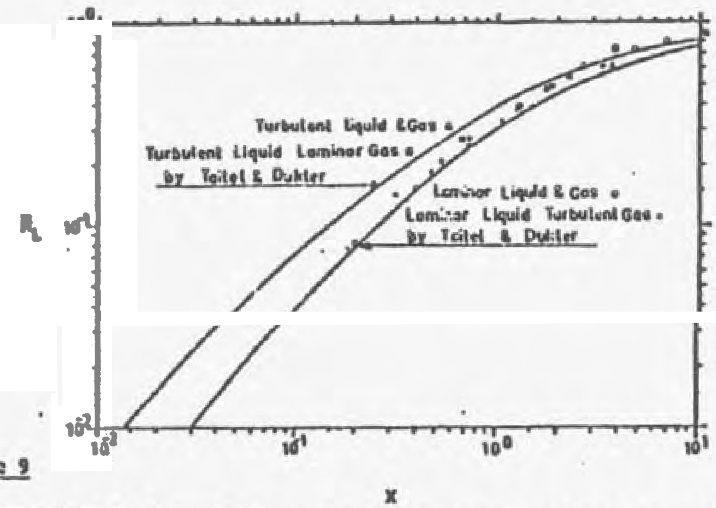


Figure 9

Comparison of R_L predictions using equations [21], [23], [30] and [31] and the R_L - X plots of Taitel and Dukler[11] for all four possible flow regimes based on Govier and Omer's[4] data.

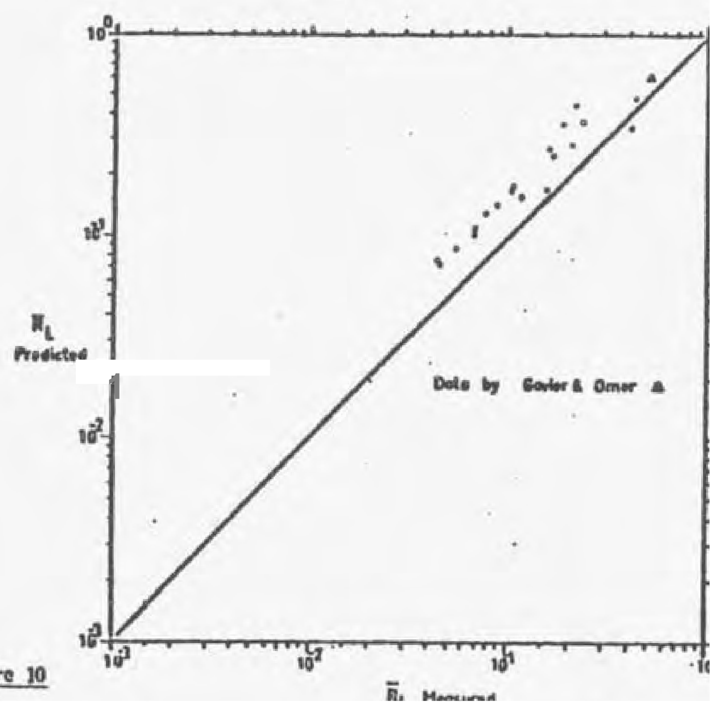
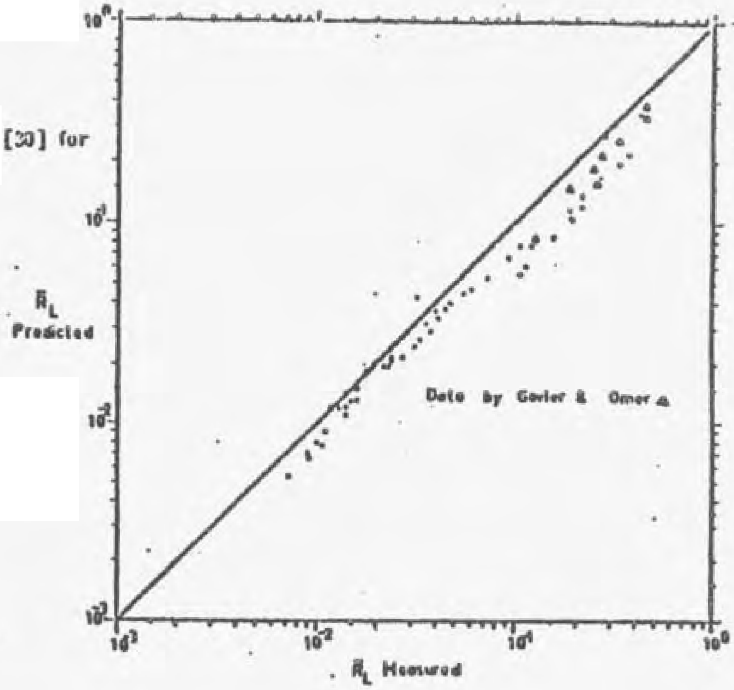


Figure 10

Measured liquid hold-up against that predicted by equation [21] for ure 11

Measured liquid hold-up against that predicted by equation [30] for case of laminar liquid and turbulent gas stratified flow.



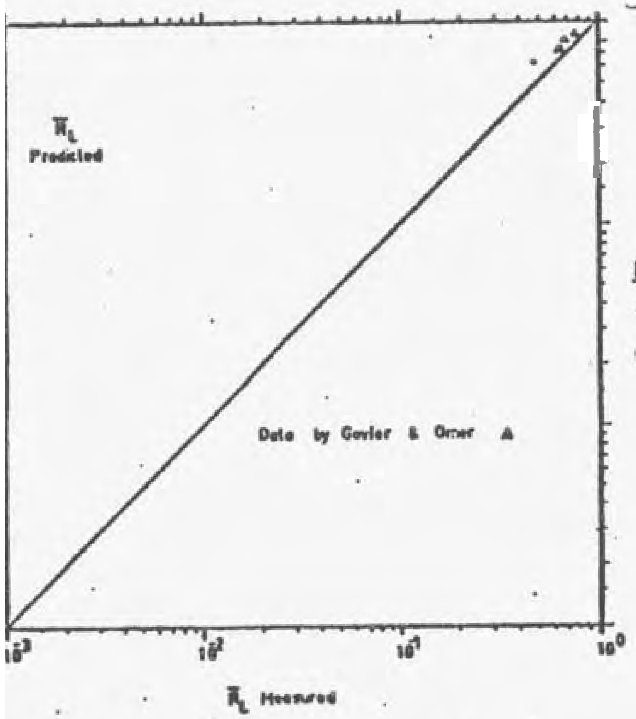
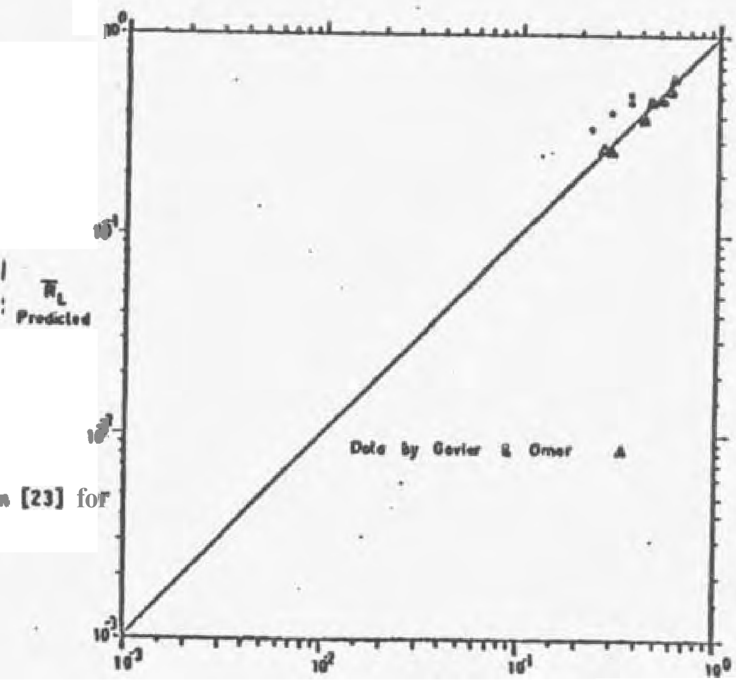


Figure 12

Measured liquid hold-up against that predicted by equation [35] for the case of turbulent liquid and laminar gas stratified flow.



ure 13

Measured liquid hold-up against that predicted by equation [23] for case of laminar liquid and gas stratified flow.