COMPARISON OF NUMERICAL METHODS FOR GEOTHERMAL WELL TEST SIMULATION

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Abstract

A brief outline of numerical methods used by the author for geothermal well test simulation is given and comparison is made with other simulation methods. Results show that the method work. well except when the initial liquid saturation of the reservoir is close to 1.0. Reasons for this failure are suggested.

Introduceion

During the last four years the author, together with co-workers Zyvoloski and Krol, has developed a simulation procedure for modelling geothermal reservoirs. The simulator has been applied to the modelling of particular geothermal reservoirs and to the general analysis of geothermal flows. The details of the procedure are described in other publications (Zyvoloski et al (1979, Zyvoloski and O'Sullivan (1979)).

The purpose of this paper is to give a very brief outline of the numerical methods involved and to discuss the application of the simulation to well test analysis. Much of the work on the application of the simulation to well test analysis was carried out in collaboration with Dr HL Sorey and is described by him elsewhere (see Sorey et al 1979).

Basic equations

Conservation of mass and energy for geothermal flows can be conveniently expressed by the equations (see nomenclature list for symbol definitions):

$$\frac{\partial A_m}{\partial t} - \nabla\cdot(D_m\nabla p) + q_m = 0 \qquad (1)$$

and

$$\frac{\partial A_e}{\partial t} - \nabla\cdot(D_e\nabla p) + q_e = 0 . \qquad (2)$$

The well discharge energy q_e is related to the mass discharge q_m by the equation:

$$q_e = h_f q_m \qquad (3)$$

where h_f is the flowing enthalpy of the fluid. Similarly the energy "transmissibility" coefficient D_e is related to the mass transmissibility coefficient D_m by

$$D_e = h_f D_m . \qquad (4)$$

The quantities A_m, A_e, D_m, D and q depend on the pressure p and the fluid (in-place) enthalpy h in a highly nonlinear manner (see Coats (1977), Zyvoloski, O'Sullivan and Krol (1979) or Faust and Mercer (1979a) for example). The nonlinearities are particularly important for two-phase flows or for single phase flows where large pressure or enthalpy changes occur.

The energy per unit volume A_e can be related to the mass per unit value A_m as follows:

$$A_e = (1-\phi)\rho_r E_r + hA_m - \phi p . \qquad (5)$$

Here ϕ is porosity of the rock matrix, ρ_r is the rock density, E_r is

the internal energy in the rock and ρ_r is the fluid density. Using (3), (4) and (5) the energy equation (6) can be rearranged in the form:

$$A_m \frac{\partial h}{\partial t} - D_m \nabla p \cdot \nabla h_f + \frac{\partial}{\partial t}[(1-\phi)\rho_r E_r - \phi p] + (h_f - h)[q_m - \nabla \cdot (D_m \nabla p)] = 0 . \quad (6)$$

In this version the first two terms which correspond to the transport of energy by fluid movement are dominant, This fact is significant in the understanding of some of the difficulties encountered with the numerical procedure8 discussed below.

Numerical procedurer

The key feature of the finite difference representations of equations (1) and (2) ured by several different authors (see Coats (1977), Faust and Mercer (1979b) and Zyvoloski et a1 (1979) for 'example) is the implicit representation of the nonlinear terms. For well tert analysis radial symmetry is assumed and then the appropriate finite difference approximations of (1) and (2) is

$$\frac{1}{\Delta t}(A^{n+1}_{\alpha,i} - A^{n}_{\alpha,i}) - \left\{2r_{i+\frac{1}{2}} D^{n+1}_{\alpha,i+\frac{1}{2}} \left(\frac{p^{n+1}_{i+1} - p^{n+1}_{i}}{r_{i+1} - r_i}\right) - 2r_{i-\frac{1}{2}} D^{n+1}_{\alpha,i-\frac{1}{2}} \left(\frac{p^{n+1}_{i} - p^{n+1}_{i-1}}{r_i - r_{i-1}}\right)\right\} / (r^2_{i+\frac{1}{2}} - r^2_{i-\frac{1}{2}}) + q^{n+1}_{\alpha,i} = 0 \quad (7)$$

for a = m and e. The choice of grid layout, that is, the relationship between r_i, r_{i+1} and $r_{i+\frac{1}{2}}$ is discussed by Aziz 8ad Settati (1979) for example, Upstream weighting of transmissibility terms is reconmended in order to achieve stability for large block throughput to value ratios (see Coats (1977) or Peaceman (1977)). This leads to the

replacement of $D_{\alpha,i+\frac{1}{2}}^{n+1}$ and $D_{\alpha,i-\frac{1}{2}}^{n+1}$ in (7) by $D_{\alpha,i+1}^{n+1}$ and $D_{\alpha,i}^{n+1}$ respectively.

Equation (7) can be more simply written as

$$F\alpha(p_{i-1}^{n+1},h_{i-1}^{n+1},p_i^{n+1},h_i^{n+1},p_{i+1}^{n+1},h_{i+1}^{n+1}) = 0 , \qquad (8)$$

for $i = 1,2,\ldots,M$, where M is the total number of grid blocks. Conditions of no flow at the well $r = r_{\frac{1}{2}}$ and the outer boundary $r = r_{M+\frac{1}{2}}$ simplify the firrt and last equations of (8). (The discharge of the well is removed by means of a block discharge q_{m_s} at the firrt block centra.) The nonlinear equationr (8) are then rolved using the Newton-Raphson procedure with p_i^n, h_i^n as initial co-ordinates for p_i^{n+1}, and h_i^{n+1}, . The Newton-Rapheon procedure requires the solution of tha vector equation

$$-\underset{\sim}{F}_\alpha^k = \left(\frac{\partial \underset{\sim}{F}_\alpha}{\partial \underset{\sim}{X}}\right)^k \left(\underset{\sim}{X}^{k+1} - \underset{\sim}{X}^k\right) .$$

where $\underset{\sim}{X}^k = (p_{\cdot}^{n+1,k}, h^{n+1,k}, \ldots, p_{\cdot}^{n+1,k}, h^{n+1,k})$.

The iteration is repeated until $|\underset{\sim}{F}_\alpha^k|$ is sufficiently small. The author and co-workers (ree Zyvoloski at al 1979) introduced an important simplification of (9), alro ured by Faust and Mercer (1979b), by neglecting derivations of the transmissibility coefficient8 D_{α} and D_{α}. This approximation yields a much simpler structure of the matrix to be inverted in (9) and is particularly useful for two-dimensional reservoir problems. It should be emphasized that although the solution procedure for solving (8) is approximate once the rolution is obtained it is a valid "solution" within the tolerance set on $|\underset{\sim}{F}_\alpha^k|$. Another important aspect of the method used by the author is the use of under-relaxation in applying (9) during phase transition. Otherwise tbe

iteration diverges.

Results

Although the procedure described above was developed by the author and co-workers for reservoir modelling it seems to work well for well transient problems. Some problems are experienced with instability for high throughput to block value ratios which could probably be partially cured by following a more accurate procedure for solving (9) (see Coats (1977) or Faust and Mercer (1979b). Results obtained with the procedure compare well with earlier work by Garg (1978) and quasi-analytic colutions (see Sorey et a1 (1979).

The major failure of the method occurs in two-phase well transient problems when the initial state of the field corresponds to a liquid saturation close to 1. Data for a typical problem of this type is given in table 1. The results exhibiting s spurious oscillation in saturation are shown in figure 1. The results are certainly incorrect because the oscillation can be made smaller by mesh refinement. Host other authors (Carg (1978), Moench and Atkinson (1977)) have not identified this problem because they considered tuo-phase well-transients for initial liquid saturations well below 1.0. Coats (1977) also observed an oscillation in the solution he obtained for a well test starting at a liquid saturation of 1.0 but he did not express concern about the oscillation as the long-time accuracy of his results were good.

Results for a lower initial liquid saturation are shown in figure 2 and exhibit no large spurious oscillation.

The author is able to offer some analysis of the causes of the oscillation but so far no remedy. At high saturations when the liquid

phase is immobile and therefore relative permeability of the liquid phase is not changing the enthalpy of the fluid near the well must change rapidly as only steam can be extracted.. When the enthalpy is changing rapidly equation (6) is dominated by the first two terms and can be written as

$$\frac{\partial h}{\partial t} - V_m \cdot \nabla h_f + Q_m = 0 , \qquad (10)$$

where $V_m = D_m \nabla p / A_m$ is a hybrid velocity derived from a mass flux divided by an in-place density. Finite difference solution of advection equations of this type is known to be difficult, often (sea Richtmyer and Morton (1967) for example) leading to spurious oscillatory solutions. Therefore to improve the well test results shown in figure 1 improved finite difference techniques for handling the advection processes, involved must be introduced. Meanwhile well test simulation results for reseryoire with high initial liquid saturations should be treated with caution.

Reference8

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Nomenclature

A_m	mass of fluid per unit volume of saturated medium
A_e	energy perunit volume of saturated medium
D_m	mass transmissibility coefficient
D_e	energy transmissibility coefficients
E_r	specific energy of tho rock matrix
b	mixture enthalpy
h_f	flowing enthalpy
P	pressure
q_m	production rata
q_e	energy production
$r_i, r_{i+\frac{1}{2}}$	radii of block centres and bfock boundarier
t	time
Δt	time step
ϕ	porosity
ρ_r	density of rock

Table 1 - We11 test data

Permeability	k =	$6(10^{-14})m^2$
Porosity	ϕ =	0.15
Rock density	ρ_r =	2500kg/m
Rock specific heat	c_r =	1000 J/kg.k
Initial pressure	p =	3MPa
Initial liquid saturation	s_l =	0.99
Production rate	q_m =	16.7kg/sec
Aquifer thickness		100m
Well.radium	$r_{\frac{1}{2}}$ =	0.5m
Mesh spacing.(uniform)	Δr =	0.2, 0.3 or 0.9.
Time step	Δt =	86.4 or 172.8s

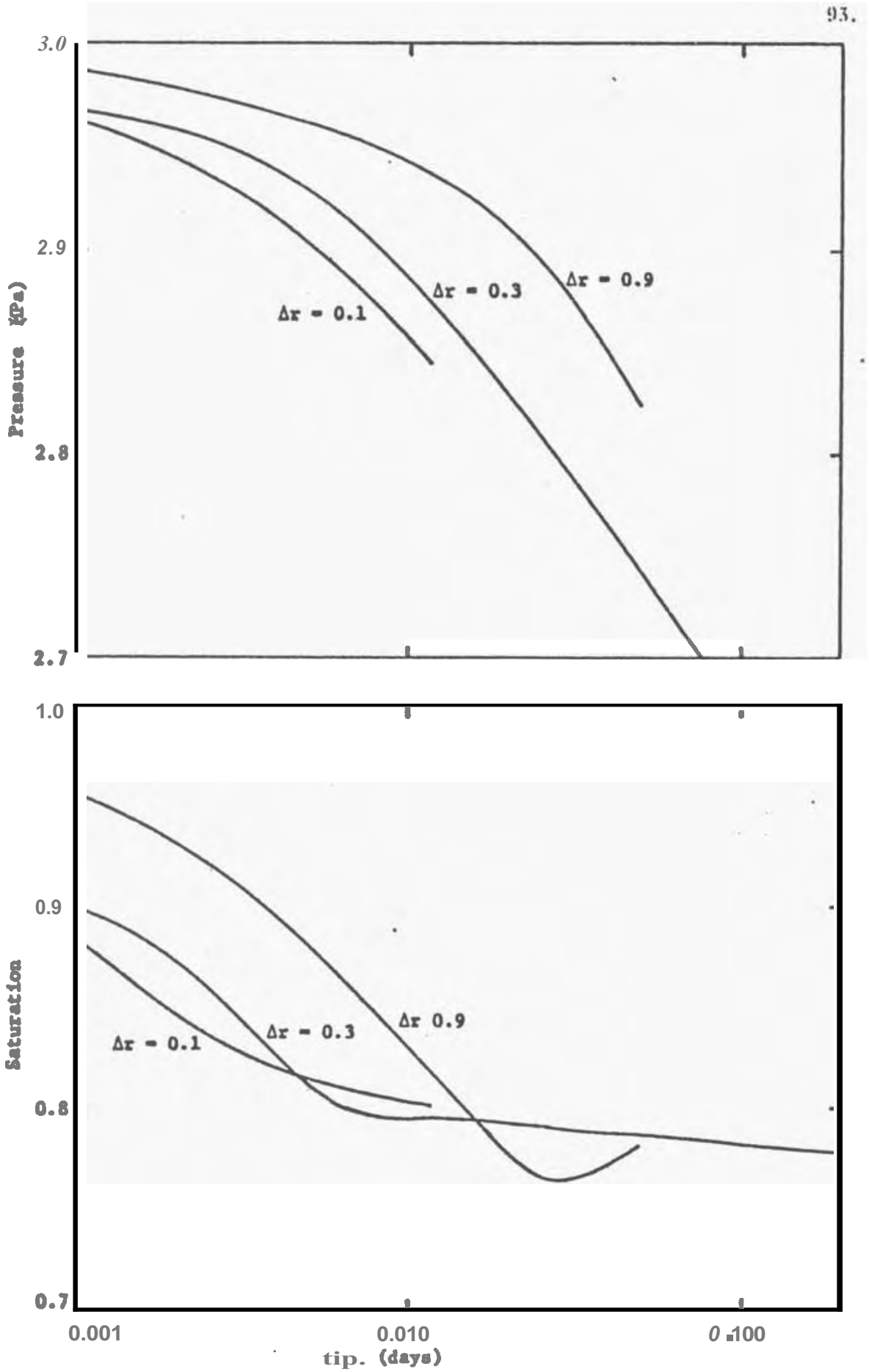

Figure 1. Pressure urd saturation at tha **well. Initial pressure** *3.0* **MPa,** initial **saturation** *0.99..*

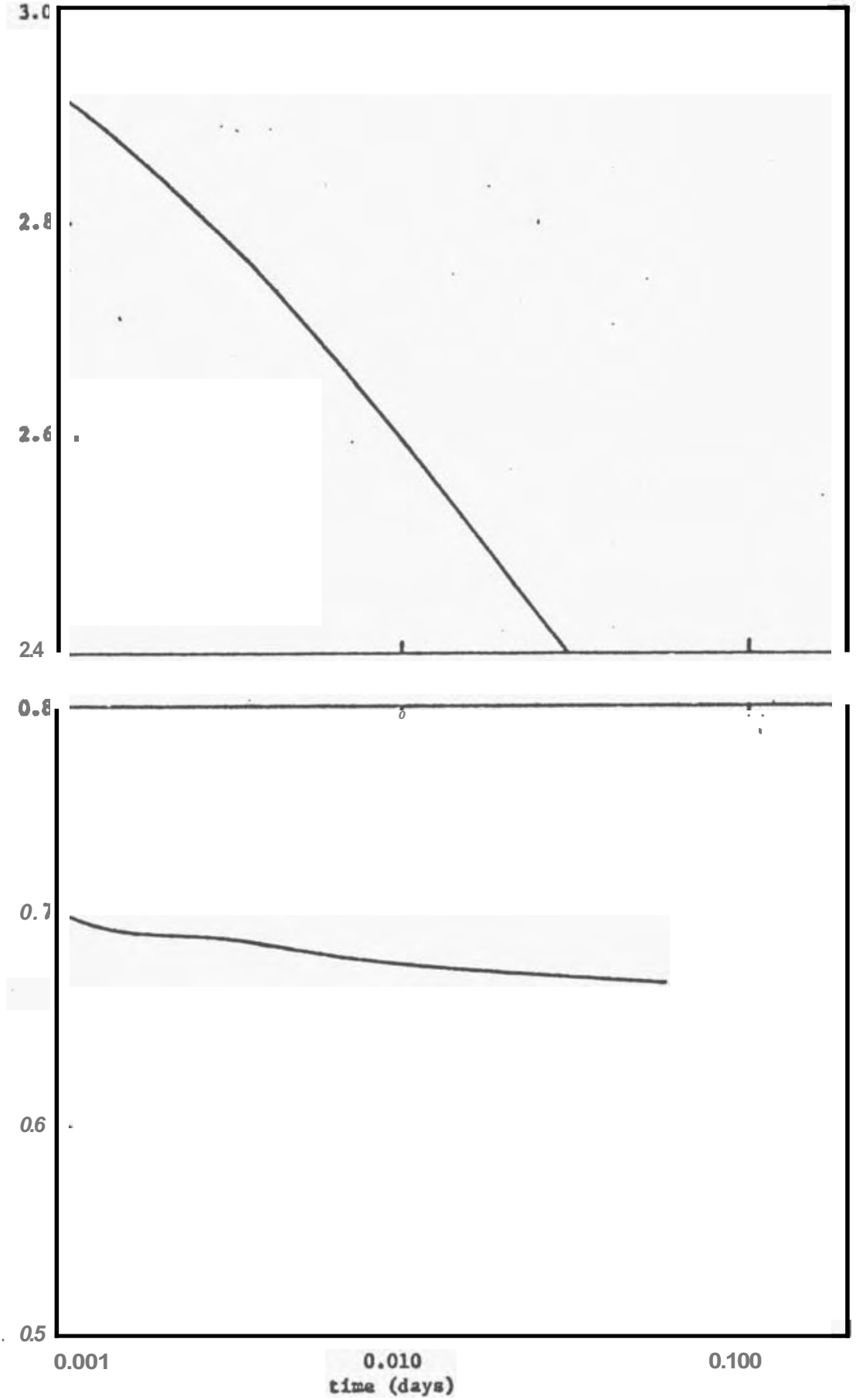

Figure 2 Pressure and saturation at tha well. Initial pressure 30 MPa, initial saturation 0.80.