The Boundary Element Method and the natural state of geothermal submarine systems

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Abstract
In this work is presented a new method to estimate the initial state of geothermal submarine reservoirs and an evaluation of the amount of energy contained in these important natural systems. To evaluate the natural state of these reservoirs we use the classical Boundary Element Method (BEM) and suggest a simple way to couple this technique to the simulator TOUGH2 of the Lawrence Berkeley National Laboratory through the INPUT file. Submarine geothermal reservoirs contain essentially an infinite amount of energy. The deep submarine heat is related to the existence of hydrothermal vents emerging in many places along the oceanic spreading centers between tectonic plates. These systems have a total length of about 65,000 km in the Earth′s oceanic crust. The deep resources are located at certain places along the rifts between tectonic plates of the oceanic crust at more than 2000 m below sea level. Shallow resources are found near to continental platforms between 1 m and 50 m depth and are related to faults and fractures close to the coasts. Both types of resources exist in the Gulf of California, Mexico. To model these systems the initial mathematical problem is expressed in terms of boundary integral equations, fundamental solutions and boundary conditions of mixed type. The main field functions are pressure and temperature. The versatility and power of the BEM allows the efficient treatment of very complex or unknown reservoir geometry, without requiring discretization of the whole domain occupied by the system. This capability allows efficient testing of different boundary conditions to estimate several thermodynamic initial states at any desired interior point of the domain occupied by the reservoir under specific conditions. Unfortunately, the classical BEM is limited to single-phase flow in homogeneous media and cannot be fully applied to flow problems in heterogeneous systems. In this last case there is no fundamental solution. To overcome this difficulty after an initial state is estimated, TOUGH2 can be used to improve the initial simulation. The few available data on hydrothermal vents are very useful to estimate the amount of energy flowing from the ocean floor. In this way, it is possible to estimate initial conditions knowing only heat fluxes and temperatures at fissures and chimneys using this hybrid technique.

Keywords: Boundary Element Method, submarine systems, mathematical simulation.

El Método de Elementos de Frontera y el estado natural de sistemas geotérmicos submarinos

Resumen
En este trabajo se presenta una forma nueva de evaluar numéricamente el estado inicial de reservorios geotérmicos submarinos y un cálculo aproximado de la energía contenida en esos importantes sistemas naturales. Para estimar su estado natural se usa el método clásico de los Elementos de Frontera (*Boundary Element Method* o BEM) y se sugiere una manera simple de acoplar esta técnica al simulador TOUGH2 desarrollado en el *Lawrence Berkeley National Laboratory*, a través del archivo INPUT. Los reservorios geotérmicos submarinos contienen esencialmente una cantidad infinita de

energía. El calor submarino profundo está relacionado con la existencia de respiraderos hidrotermales que emergen en muchos sitios a lo largo de los centros de dispersión oceánica entre las placas tectónicas. Esos sistemas tienen una longitud total de unos 65,000 km en la corteza oceánica. Los recursos profundos están localizados en ciertos lugares a lo largo de las cordilleras entre placas tectónicas a más de 2000 m bajo el nivel del mar. Los recursos superficiales se hallan cerca de las plataformas continentales entre 1 m y 50 m de profundidad y están relacionados con fallas cercanas a las costas. Ambas clases de recursos existen en el Golfo de California, México. Para modelar numéricamente estos sistemas el problema matemático inicial se expresa en términos de ecuaciones integrales de contorno, soluciones fundamentales y condiciones de frontera de tipo mixto. Las funciones principales de campo son la presión y la temperatura. La versatilidad y potencia del BEM permiten el tratamiento eficiente de geometrías de yacimientos muy complejas, o bien parcial o totalmente desconocidas, sin que se requiera la discretización del volumen completo ocupado por el sistema. Esta capacidad permite probar, en forma eficiente y rápida, diferentes condiciones de frontera para estimar varios estados termodinámicos iniciales en cualquier punto interior del dominio ocupado por el reservorio bajo condiciones específicas. Desafortunadamente, el BEM clásico está limitado al flujo monofásico de fluidos en medios homogéneos y no puede aplicarse completamente a problemas de flujo bifásico en sistemas heterogéneos. La razón es que para esos casos no existe solución fundamental. Para superar esta dificultad, después que se ha estimado un estado inicial, el simulador TOUGH2 puede usarse para mejorar la simulación inicial. Los pocos datos disponibles sobre respiraderos hidrotermales son muy útiles para estimar la cantidad de energía que fluye del piso oceánico. De esta forma, es posible estimar condiciones iniciales conociendo solo flujos de calor y temperaturas en fisuras y chimeneas usando esta técnica híbrida.

Palabras clave: Método de Elementos de Frontera, geotermia submarina, simulación matemática.

1. Introduction

Hydrothermal circulation at deep oceanic ridges is a fundamental complex process controlling mass and energy transfer from the interior of the Earth through the oceanic lithosphere, to the hydrosphere and to the atmosphere. Submarine hydrothermal interactions influence the composition of the oceanic crust and the oceans' chemistry.

The fluid circulating in seafloor hydrothermal systems is chemically altered due to processes occurring during its passage through the oceanic crust at elevated temperatures and pressures. This mechanism produces hydrothermal vent fields that support diverse biological communities starting from microbial populations that link the transfer of the chemical energy of dissolved chemical species to the production of organic carbon (Humphris *et al.*, 1995). The eventual transfer of some gases from the ocean to the atmosphere extends the influence of hydrothermal activity far beyond the oceans themselves. The understanding of these mass and energy flows among the complex geological, chemical, geophysical and biological subsystems requires the development of integrated models that include the interactions between them. Because of their complexity and of the scarcity of real data, the modeling and simulation of submarine reservoirs is cumbersome and uncertain.

The BEM is a numerical technique for solving elliptic and convection-diffusion partial differential equations (PDE). The BEM relates boundary data and boundary integral equations to the internal points of the solution domain in a very effective and accurate way. This is a suitable method to quickly estimate several possible initial states of reservoirs when only a few data are available. In this paper we show the potential advantages of the BEM over other numerical methods and the way it can be coupled to TOUGH2. We also outline the fundamental characteristics of submarine hydrothermal systems and present a preliminary evaluation of their energy content.

2. TOUGH2 and submarine reservoirs

TOUGH2 is a powerful numerical code for solving PDE's and for simulating the coupled transport of water, energy, air, CO_2 and other components in porous/fractured media (Pruess *et al*., 1999). It solves systems of non-linear PDE's of parabolic type. The general integral form of these equations is:

$$\frac{\partial}{\partial t}\int_{V_n} \rho_k \, dV + \int_{V_n} \vec{\nabla}\cdot\vec{F}_k \, dV = \int_{V_n} q_k \, dV \qquad (1)$$

Where V_n represents any porous medium flow domain, ρ_k is the density of some physical property (mass, energy), $\boldsymbol{F}_k$ is the flux of mass/energy and q_k is an injection/production term in V_n. The flux vector derives from the gradient of a field variable (pressure or temperature). The sub-index k means that Eq. (1) holds for a multi-phase treatment of different components in the mass/energy balance equations, including convection and heat conduction in rock, water, air, gases, etc.

Equation (1) is numerically solved using the Integral Finite Difference Method (IFDM). This technique contains aspects of both major numerical methods, Finite Differences (FD) and Finite Elements (FE). These three methods require discretization of the whole solution domain Ω in the form: $\Omega = \sum_{n=1}^{N} V_n$. The need to discretize the whole domain is the main reason for computation cost and of the total CPU time needed to solve a particular problem. In the potential application of TOUGH2 to submarine geothermal reservoirs, the first practical problems are the insufficiency of both available field data to simulate these systems and the total absence of production history.

3. The BEM for elliptic problems

During the numerical estimation of the initial state of a reservoir it is clear that, after a great number of time steps, the transient term in Eq. (1) becomes practically zero. Thus for this problem, Equation (1) becomes a PDE of elliptic type. The BEM is specifically indicated for linear elliptic PDE in homogeneous media. In this type of physical problems the BEM is clearly superior to FD, IFD and FE methods in both accuracy and efficiency. Mainly, because all these methods demand the discretization of the whole solution domain Ω.

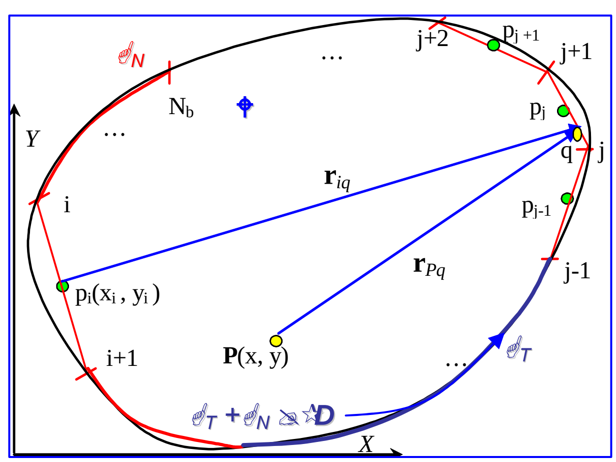

Fig. 1. Discretization of the boundary of Ω in the D solution domain of the PDE (2).

The key feature of the BEM is that only the surface of the porous medium needs to be discretized. The field variable can be calculated with high precision at any point in the interior of the domain using only the known values of the function at the boundary of

Ω. The BEM provides an effective reduction of the dimension of the PDE solution space. As a result, improved numerical accuracy and lesser use of computational resources are obtained. Differential problems that can be solved on a Notebook using the BEM, could require a cluster or a workstation, or even a supercomputer using any of the other methods for the same level of accuracy and for the same degree of geometric complexity of the reservoir boundary (Cruse and Rizzo, 1975; Ameen, 2001; Pozrikidis, 2002). To illustrate the method, we solve an elliptic problem representing a stationary temperature (or pressure) distribution, described by the Poisson's equation with mixed boundary conditions (Figure 1)

$$\begin{aligned} &\Delta T = f(\vec{P}),\ \forall\ \vec{P} = (x,y) \in \Omega \subset \mathbb{R}^2, \\ &T(\vec{P}) = u_T(\vec{P}),\ \forall\ \vec{P} \in \Gamma_T, \\ &\frac{\partial T}{\partial n}(\vec{P}) = u_n(\vec{P}),\ \forall\ \vec{P} \in \Gamma_N \\ &\partial\Omega = \Gamma = \Gamma_T \cup \Gamma_N \end{aligned} \tag{2}$$

Let's assume first that *f* = 0 (Laplace PDE). Applying the Green's theorem and the fundamental solution to the integral form of Eq. (2) we obtain:

$$\begin{aligned} &T(\vec{P}) = -\frac{1}{2\pi}\int_\Gamma \left[Ln\left\|\vec{P}-\vec{q}\right\| \frac{\partial T(\vec{q})}{\partial n} - \frac{T(\vec{q})}{\left\|\vec{P}-\vec{q}\right\|} Cos\,\theta \right] ds \\ &\forall\ \vec{P} \in \Omega,\ \vec{q} \in \partial\Omega,\ \theta = \angle(\vec{r}_{Pq}, \vec{n}),\ \vec{n} = normal\ to\ \Gamma\ at\ \vec{q} \end{aligned} \tag{3}$$

The boundary $\partial\Omega$ is discretized into Γ_j (j =1, N_b) boundary elements which can be linear, parabolic, cubic splines in 2D. In the 3D case, the boundary elements can be triangles, rectangles, arcs, etc.

BEM solution of the Poisson Equation

Let us assume now that $f \neq 0$ (Poisson PDE), applying the same methodology proposed by Katsikadelis (2002) we obtain:

$$\begin{aligned} &\forall\ \vec{P}, \vec{Q} \in \Omega:\ T(\vec{P}) = \int_\Omega v(\vec{P},\vec{Q})\, f(\vec{P})\, d\Omega \\ &-\int_\Gamma \left[v(\vec{P},\vec{q}) \frac{\partial T(\vec{q})}{\partial n} - T(\vec{q}) \frac{\partial v}{\partial n}(\vec{P},\vec{q}) \right] ds \\ &Where\ \ \vec{q} \in \Gamma\ and\ v(\vec{P},\vec{q}) = \frac{Ln\left(\left\|\vec{P}-\vec{q}\right\|\right)}{2\pi} \end{aligned} \tag{4}$$

The auxiliary function *v* is the Fundamental Solution of the singular form of Laplace Equation and plays a crucial role in the classical BEM. For time-dependent problems of parabolic type the BEM can also be applied using two subsidiary techniques:

a) Solving first the PDE in time using FD, then applying the BEM to the time-discretized equations.

b) Removing the time dependence of the PDE using the Laplace Transform.

The BEM numerical implementation: an example

Let us assume that each Γ_j is a constant linear segment. The discretization of the boundary Γ (Fig. 1) in Equation (3) implies that:

$$\Gamma_T \cup \Gamma_N = \Gamma \approx \bigcup_{j=1}^{N_b} \Gamma_j \qquad (5)$$

Consequently Eq. (3) can be discretized as:

$$\frac{T^i}{2} = -\sum_{j=1}^{N_b} \int_{\Gamma_j} v(\vec{p}_i, \vec{q}) \frac{\partial T(\vec{q})}{\partial n_j} ds \; + \; + \sum_{j=1}^{N_b} \int_{\Gamma_j} T(\vec{q}) \frac{\partial v(\vec{p}_i, \vec{q})}{\partial n_j} \qquad (6)$$

or equivalently as:

$$\sum_{j=1}^{N_b} H_{ij} T^j = \sum_{j=1}^{N_b} G_{ij} \frac{\partial T^j}{\partial n} \qquad (7)$$

The influence coefficients H_{ij} and G_{ij} are integral forms equal to:

$$H_{ij} = \int_{\Gamma_j} \left(\frac{\partial v}{\partial n_j} ds \right) - \frac{\delta_{ij}}{2}; \; G_{ij} = \int_{\Gamma_j} v(\vec{p}_i, \vec{q}) ds \qquad (8)$$

From Eq. (7) we finally obtain the linear system:

$$\mathbf{H} \cdot \vec{T} = \mathbf{G} \cdot \vec{T}_n$$
$$T^j = (u^j); \; T_n{}^j = \frac{\partial T^j}{\partial n} = (u_n{}^j) \qquad (9)$$

Because of the assumed mixed boundary conditions, u_T in Γ_T and u_n in Γ_N, there are unknown quantities in both sides of Eq. (9). Consequently we need to separate the identified *u*'s from the not known *u*'s in order to obtain a consistent system of linear equations. As an example, the system for N_b = 4 is as shown in Equation (10).

Let us suppose that the u^j are the known quantities and the T^j are the unknown variables. Moving all the unknowns to the left hand side of equation (10) we obtain the final linear system, as shown in Equation (11).

$$\begin{pmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ H_{21} & H_{22} & H_{23} & H_{24} \\ H_{31} & H_{32} & H_{33} & H_{34} \\ H_{41} & H_{42} & H_{43} & H_{44} \end{pmatrix} \cdot \begin{pmatrix} u^1 \\ u^2 \\ T^3 \\ T^4 \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{42} & G_{43} & G_{44} \end{pmatrix} \cdot \begin{pmatrix} T_n^1 \\ T_n^2 \\ u_n^3 \\ u_n^4 \end{pmatrix} \tag{10}$$

$$\mathbf{A} \cdot \vec{\mathbf{T}} = \mathbf{B} \quad \Leftrightarrow$$

$$\begin{pmatrix} -H_{13} & -H_{14} & G_{11} & G_{12} \\ -H_{23} & -H_{24} & G_{21} & G_{22} \\ -H_{33} & -H_{34} & G_{31} & G_{32} \\ -H_{43} & -H_{44} & G_{41} & G_{42} \end{pmatrix} \cdot \begin{pmatrix} T^3 \\ T^4 \\ T_n^1 \\ T_n^2 \end{pmatrix} = \cdot \begin{pmatrix} B^1 \\ B^2 \\ B^3 \\ B^4 \end{pmatrix} \tag{11}$$

$$B^i = H_{i1}u^1 + H_{i2}u^2 - G_{i3}u_n^{\ 3} - G_{i4}u_n^{\ 4}$$

The matrix in this system is full and non symmetric, but is at least four times smaller than the equivalent matrix obtained from FD, FE or IFD. This result can be easily generalized for any $N_b > 4$. We are ready to apply the BEM to submarine reservoirs.

4. Submarine geothermal systems

Most of the known vents are at the mid-ocean ridge systems (MORS) in the deep sea (Damm, 1995). Magmatic processes provide the energy to drive hydrothermal circulation of seawater through the oceanic crust causing rock-seawater interaction at temperatures between 200° C and 400° C (Suárez, 2004; Suárez and Samaniego, 2005). The resulting mechanism gives rise to venting at seafloor depth, ranging between 840 and 3600 meters depth and contributing considerably to the global balance of the total Earth's heat (Fornari and Embley, 1995). This venting is associated with fissures located directly above magma injection zones.

The submarine heat flow measured in some places of the Gulf of California was of the order of 0.34 W_T/m^2 at an average temperature of 330° C (Mercado, 1990). Using two models it can be predicted an average hydrothermal heat loss by conduction for the oceanic crust of about 1.5 W_T/m^2. The same parameter predicted for the ridges is between 2 and 100 MW_T/Km (per unit ridge length). The first value is for a slow ridge and the last value corresponds to a plume with a heat content of 1000 MW_T. Thus, the plumes remove more heat than the steady-state surface flux for the cooling lithosphere. Alt (1995) estimated that submarine hydrothermal discharges remove about 30% of the heat lost from oceanic crust.

Application of the BEM to Submarine Reservoirs, Temperature Distribution and Heat Flow

The PDE describing the natural state of a geothermal reservoir is approximately elliptic, because the transient changes are very slow. Eqs. (6-11) were programmed in a Fortran/90 code called BEMSub. We solved the line integrals of Eq. (8) by the Gauss-Legendre quadrature. We assumed a squared submarine reservoir of 1000 m × 1000 m in 2D.

Forty boundary elements were sufficient to estimate the temperature distribution in this reservoir using the BEM. The boundary conditions were zero heat flow at the lateral boundaries and constant but different temperatures at the bottom and top of the reservoir. To calculate the temperature we chosen 81 internal points uniformly distributed in the square. Although this number could be larger, it is enough to draw the surfaces and illustrate the results. We considered the numerical values shown in Table 1. The average thermal conductivity is 3.0 W/°C/m.

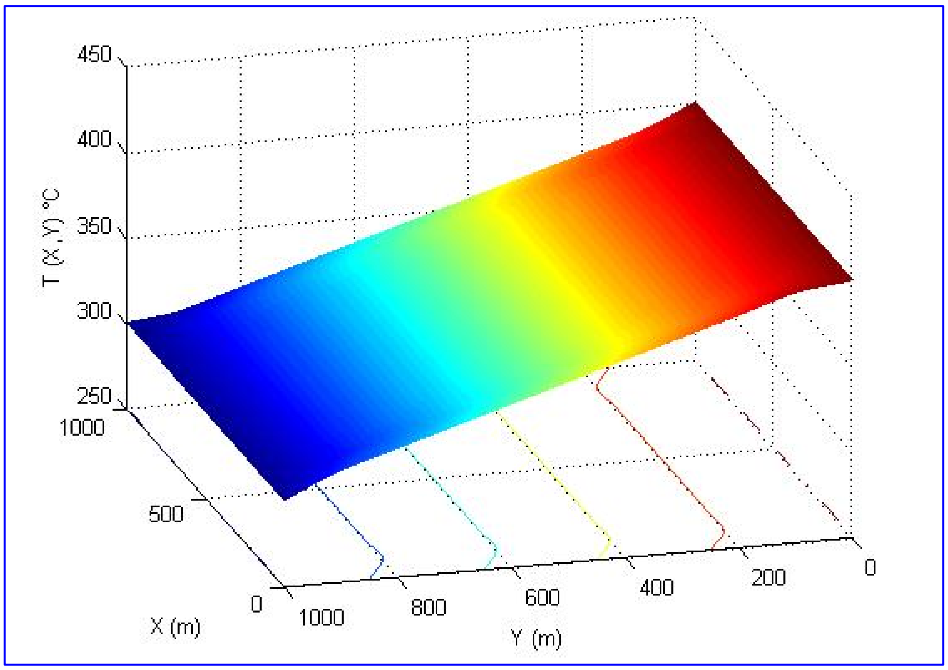

Fig. 2. Temperatures fitting the flow of heat measured in the Gulf of California.
BEM Solution for 300, 400° C, Qn = 0.3 Wm^2

Field function	Minimum	Maximum
Pressure	190 bar	300 bar
Temperature	200 °C	700 °C
Fuid flow rate	70 cm/s	250 cm/s
Heat Flux	0.34 W_T/m^2	1.50 W_T/m^2

Table 1. Some parameters of submarine reservoirs

The idea of this application is simple: knowing the range of possible temperatures, what should be the fixed temperatures at the top and bottom of the reservoir able to reproduce the measured conductive heat flow? We considered two simulations, one for the heat flow measured in the Gulf of California (0.3 W_T/m^2) and a second one for the estimated average heat flux for the oceanic crust (1.5 W_T/m^2). The results are shown in Figures 2 and 3.

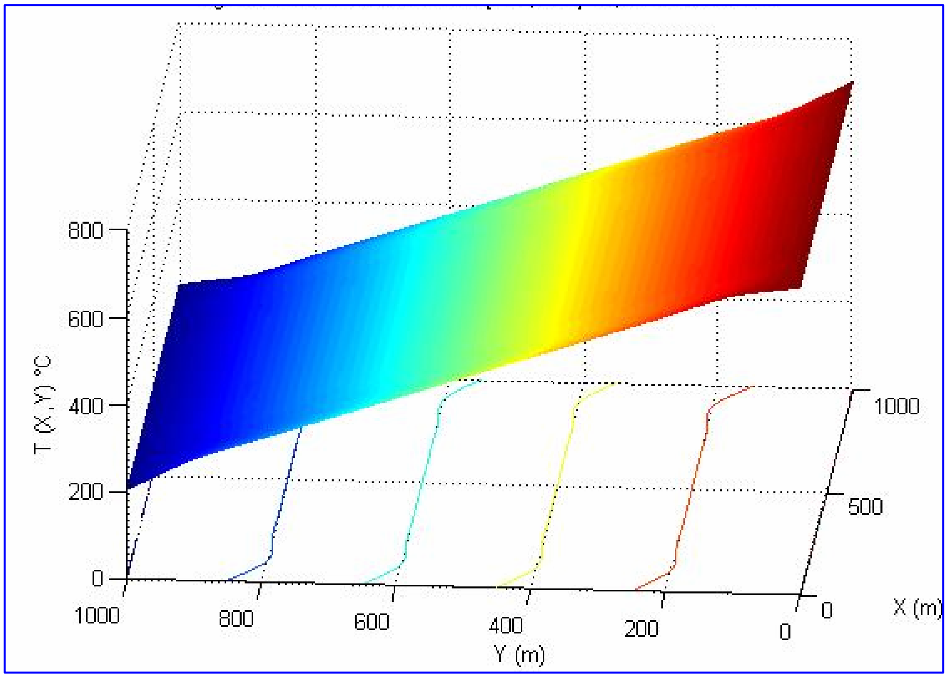

Fig. 3. Temperatures fitting the average heat flow estimated for the entire oceanic crust. BEM solution for 200, 700° C. Qn = 1.5 W/m²

Coupling the results of the BEM to TOUGH

Using the same data we prepared the INPUT deck of TOUGH for a regular mesh of 400 volumes of sizes 50 × 50 m each, for an EOS of pure water. The boundary conditions were the same, considering only the temperature initial state given by Figure 2. For the pressure we assumed a vertical linear distribution with the initial values at the top of the hypothetical submarine reservoir given in Table 2.

Field function	Top	Bottom
Pressure	190 bar	260 bar
Temperature	300° C	400° C
Fuid Density *	733.2 kg/m^3	189.4 kg/m^3
Heat Flow *	-0.29934 W_T/m^2	0.29934 W_T/m^2

Table 2. Initial and final conditions at the reservoir.

The top of the reservoir is located 2600 m below sea level and the bottom is 1000 m deeper. After around 20 time steps, the temperature obtained with TOUGH is exactly the same as the distribution shown in Figure 2. The heat fluxes calculated with TOUGH at the top and bottom of the reservoir are of equal value but of opposite sign, indicating that a steady state was reached. This value is practically equal to that obtained with the BEM (0.3 W_T/m^2). The main difference is the calculation time. The BEM takes about 0.2 seconds of CPU time for a whole single calculation using a 3 GHz PC. TOUGH needs around 10^2 more CPU time to achieve the same result. But if the starting thermodynamic point is far from these initial conditions for pressure and temperature, the total simulation time could be 10^5 times larger in the same computer, because of the need for small time steps. More advanced applications of the BEM can be found in Archer (2000).

5. Conclusions

- Deep submarine geothermal resources contain practically an infinite energy potential. Volcanic and tectonic processes control hydrothermal activity at mid-ocean ridge spreading centers, influencing all aspects of oceanography. The understanding of the mass and energy flows in these complex systems requires the development of integrated models that include the interactions among different subsystems.

- Using available data from different sources, we reported a preliminary estimation of the amount of convective and conductive energy contained in submarine systems escaping through fissures in the oceanic floor. Hydrothermal fluids at temperatures between 350° C and 400° C exit the chimneys on the seafloor at velocities of about 70 cm/s to 250 cm/s and mixes with deep seawater at 2° C. Measured thermal fluxes have an average value for a single orifice of 8 MW_T. Some mega-plumes of 750 m height correspond to heat fluxes of about 1000 MW_T.

- The main purpose of the simulation problem we have presented was to illustrate the easy and efficient use of the BEM in the estimation of the natural state of submarine reservoirs, knowing few parameters. The potential use of the BEM coupled to TOUGH could be enormously helpful in the computation of the initial state of any reservoir. This coupling could be also achieved in domain decompositions, using the extended BEM in sub-domains that require detailed calculations and mesh refinement.

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