SOME RESULTS BEING USE TO STABLE CONTROL OF AN ARTIFICIAL GEOTHERMAL CRACK IN A HOT DRY ROCK

SHIBUYA, Y., Dept. of Mechanical Eng. for Production, Akita Univ., Akita 010, Japan
SEKINE, H., Dept. of Eng. Science, Tohoku Univ., Aoba, Sendai 980, Japan
TAKAHASHI, Y., Dept. of Mechanical Eng. for Production, Akita Univ., Akita 010, Japan

1. Introduction

An abundant amount of geothermal energy could be recovered from hot dry rocks by circulating fluid through geothermal cracks which created by a hydraulic fracturing technique. In the development of geothermal extraction system of this type, the theoretical and experimental studies of problem for fluid-filled geothermal cracks have been made on the basis of the fracture mechanics approach. A comprehensive survey can be found in literature.[1]

During the extraction of heat, the surfaces of the geothermal cracks are cooled by circulating fluid and the thermal contraction of the rocks occurs. Therefore, it is necessary for controlling the geothermal cracks to make the fracture mechanics study of them including thermoelastic effects.[2][3]

In this paper, discussion is focused on the behavior of an artificial geothermal crack during extraction of heat, which is use to stable control of geothermal cracks. The analysis is based on the theory of quasi-static thermoelasticity. Firstly, we concerned with the effect of the initial rock temperature gradient on the fluid temperature at the outlet and the behavior of the geothermal crack. Next, the effect of the change of supply flow rate is considered. Finally, the time limit for the stable geothermal crack is discussed.

2. Artificial Geothermal Crack Model

Consider a two-dimensional crack as a model of a vertical artificial geothermal crack in the earth's crust, as shown in Fig. 1. In the analysis, we employ a Cartesian coordinate system (x,y) whose origin O is located at the position of equal distance ℓ from the inlet and outlet on the crack line. The upper and lower tips of the geothermal crack are $y=L_1$ and $y=-L_2$, respectively.

The energy equation for the fluid flowing through the geothermal crack is written as

$$c_W \rho_W Q \frac{dT_W}{dy} = -q(y,t) \qquad (|y|<\ell, t>0) \tag{1}$$

where c_W is the specific heat of the fluid, ρ_W the fluid density, Q the flow rate, T_W the fluid temperature, q(y,t) the heat flux across the crack surface and t time. When the temperature of the rock varies linearly with respect to the depth in the earth's crust, the initial condition of the rock temperature T_r is given by

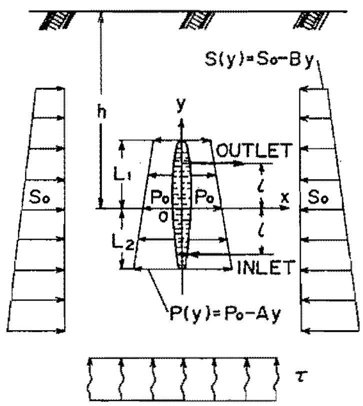


Fig.1 Artificial geothermal crack and coordinate system

$$T_r(x,y,0) = T_\infty + \tau(\ell-y) \tag{2}$$

where T_∞ is the rock temperature at $y=\ell$ and τ is the temperature gradient. Moreover, the rock temperature on the crack surface is given by

$$T_r(0,y,t) = T_W(y,t), \quad T_r(0,-\ell,t) = T_W(-\ell,t) = T_i \tag{3}$$

where T_i is the fluid temperature at the inlet which is assumed to be constant.

The rock is assumed to be obey the Duhamel-Neuman relation. When the fluid pressure P(y) acts on the surface of the geothermal crack, the boundary condition of the disturbed stress field σ^*_{xx}, σ^*_{yy} and σ^*_{xy} by existence of the fluid-filled geothermal crack subject to compressive tectonic stress S(y) are given following.

(a) On the surface of the geothermal crack: $\sigma^*_{xx} = -\{P(y)-S(y)\}, \quad \sigma^*_{xy} = 0$ (4)

(b) At infinity: $\sigma^*_{xx} = \sigma^*_{yy} = \sigma^*_{xy} = 0$ (5)

The fluid pressure P(y) and tectonic stress S(y) can be written as

$$P(y) = P_0 - Ay, \qquad S(y) = S_0 - By \tag{6}$$

where P_0 and S_0 are the fluid pressure and tectonic stress at y=0, respectively, and A and B are the linear gradient of the fluid pressure and tectonic stress, respectively.

3. Temperature Field and Stress Intensity Factors

By integration of Eq.(1), the fluid temperature is given as

$$T_w(y,t) = T_i - \frac{1}{c_w \rho_w Q}\int_{-\ell}^{y} q(\xi,t)\,d\xi \tag{7}$$

The rock temperature is obtained after determining q(y,t) by

$$T_r(x,y,t) = T_\infty + \tau(\ell-y) + \frac{1}{4\pi\lambda_r}\int_0^t\int_{-\ell}^{\ell}\frac{q(\xi,s)}{t-s}\exp\left[\frac{-\{x^2+(y-\xi)^2\}}{4\kappa_r(t-s)}\right]d\xi ds \tag{8}$$

where λ_r and κ_r are the thermal conductivity and the thermal diffusivity of the rock, respectively. The heat flux q(y,t) can be obtained by solving the singular integral equation which is derived by substituting Eqs.(7) and (8) into Eq.(3).

For the analysis of stress intensity factors, the singular integral equation is derived by replacing the geothermal crack with a continuous distribution of edge dislocations. The detail of the analysis is seen in Ref.(2). The stress intensity factors k_1 at the upper and lower tips of the geothermal crack are obtained from the dislocation density function in the forms.

$$\left.\begin{aligned} k_1(L_1) &= \{(P_0-S_0) - (A-B)\frac{(L_1-L_2)}{2}\}(\frac{L_1+L_2}{2})^{\frac{1}{2}} - \frac{1}{2}(A-B)(\frac{L_1+L_2}{2})^{3/2} \\ &\quad -\frac{1}{\pi}\int_{-L_2}^{L_1} F(\zeta,t)(\frac{L_2+\zeta}{L_1-\zeta})^{\frac{1}{2}}d\zeta \\ k_1(-L_2) &= \{(P_0-S_0) - (A-B)\frac{(L_1-L_2)}{2}\}(\frac{L_1+L_2}{2})^{\frac{1}{2}} + \frac{1}{2}(A-B)(\frac{L_1+L_2}{2})^{3/2} \\ &\quad -\frac{1}{\pi}\int_{-L_2}^{L_1} F(\zeta,t)(\frac{L_1-\zeta}{L_2+\zeta})^{\frac{1}{2}}d\zeta \end{aligned}\right\} \tag{9}$$

where

$$\left.\begin{aligned} F(y,t) &= \frac{k\kappa_r}{2\pi\lambda_r}\int_0^t\int_{-\ell}^{\ell} q(\xi,s)[\{\frac{1}{(y-\xi)^2} + \frac{1}{2\kappa_r(t-s)}\}\exp\{\frac{-(y-\xi)^2}{4\kappa_r(t-s)}\} - \frac{1}{(y-\xi)^2}]\,d\xi ds \\ k &= E\gamma/(1-\nu) \end{aligned}\right\} \tag{10}$$

Here, E is the Young's modulus, γ the thermal expansion, ν the Poisson's ratio of the rock.

When the stress intensity factor attains the fracture toughness of the rock, k_c, the geothermal crack begins to propagate. During the extraction of heat energy, the expanding geothermal crack must satisfy the following equilibrium condition:

$$k_1(L_1^*) = k_c \quad , \qquad k_1(-L_2^*) = k_c \tag{11}$$

where L_1^* and L_2^* are the positions of the upper and lower tips of the crack in the equilibrium state, respectively. Moreover, the equilibrium state is stable if and only if

$$\left.\frac{\partial k_1(L_1)}{\partial L_1}\right|_{L_1=L_1^*} < 0, \qquad \left.\frac{\partial k_1(-L_2)}{\partial L_2}\right|_{L_2=L_2^*} < 0 \tag{12}$$

4. Numerical Results and Discussion

On the basis of the aforementioned method, numerical calculations are performed. Then the following nondimensional quantities are used; $\overline{T}_0=\{T_w(\ell,t)-T_i\}/(T_\infty-T_i)$ for the fluid temperature at the outlet and $\alpha=c_w\rho_w Q/\lambda_r$ for the fluid flow rate. Furthermore, the following values are used; E=35GPa, ν=0.12, $\gamma=1.2\times10^{-5}\,K^{-1}$ and $k_c=1.6MPa\cdot m^{\frac{1}{2}}$ for granite, $B-A=6051Pa\cdot m^{-1}$, ℓ=50m and $T_\infty-T_i$=180K.

4.1 Effect of Rock Temperature Gradient

The initial rock temperature which increases linearly with the depth is considered. The variation of the fluid temperature at the outlet against time is shown in Fig.2 for various values of the linear temperature gradient τ. Here, the nondimensional flow rate are α=100 and 200. The nondimensional flow rate α=200 corresponds to about 0.45t/h per 1m in the direction perpendicular to x-y plane. On the abscissa, the nondimensional time $\kappa_r t/\ell^2$=0.01 corresponds to about 309 days. It is seen from the figure that the

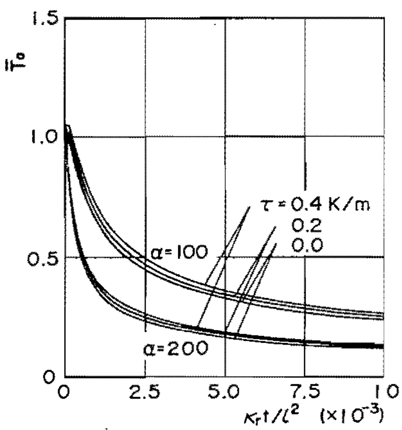


Fig.2 Fluid temperature at the outlet against time

outlet fluid temperature increases slightly at the earlier state of the extraction of heat energy. However, the outlet fluid temperature decreases markedly with time after that and the tendency is marked with increasing α. The effect of the temperature gradient on the outlet fluid temperature is, generally speaking, small as shown in the figure.

The cross-sectional shape of the geothermal crack is shown in Fig. 3. The solid, broken and one-dot chain lines indicate in case of the temperature gradient τ =0, 0.2 and 0.4K/m, respectively. Here, the positions of the crack tips is determined to satisfy Eqs.(11) and (12), and the opening displacement W(y) can be calculated by

$$W(y) = -\int_{-L_2^*}^{y} b(\zeta)\, d\zeta \qquad (13)$$

where $b(\zeta)$ is the density of edge dislocations. The length and opening displacement of the geothermal crack become large with time and the opening displacement is almost constant from the outlet to the inlet. As the rock temperature gradient becomes large, the opening displacement becomes large near the inlet. However, the effect of the temperature gradient on the propagation of the geothermal crack is small.

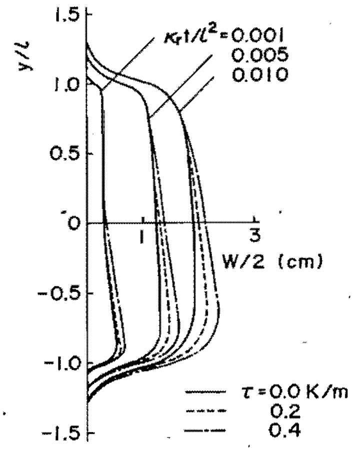
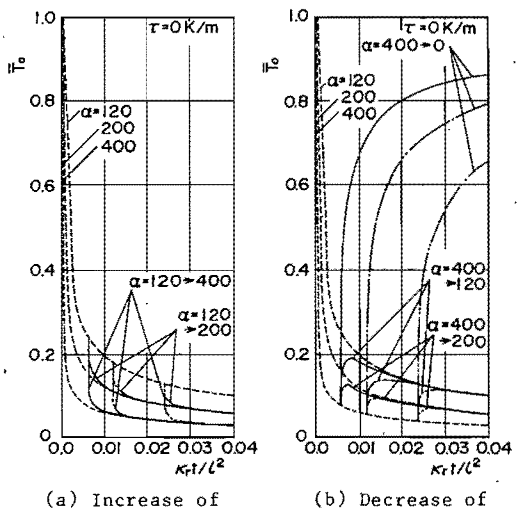


Fig.3 Cross-sectional shape of the Geothermal crack

4.2 Effect of Change of Supply Flow Rate

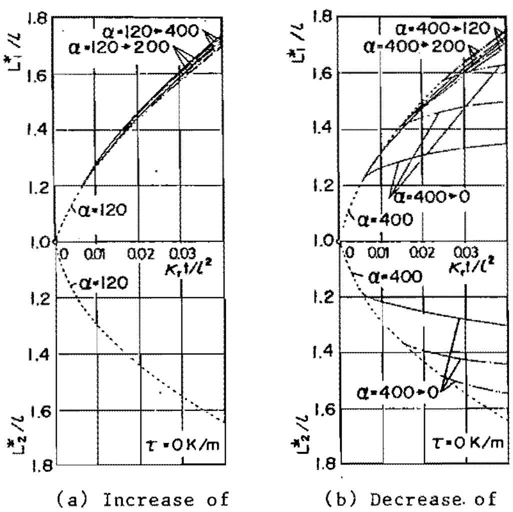
It is necessary to change the supply flow rate for controlling the geothermal crack. Here, the effect of the change of the supply flow rate is discussed. The variation of the outlet fluid temperature is shown in Fig. 4. Fig. 4(a) is in case of increasing flow rate from α=120 to α=200 and 400, and Fig. 4(b) is in case of decreasing flow rate from α=400 to α=200, 120 and 0. When the flow rate is changed, the outlet fluid temperature varies abruptly.

The stable positions of crack tips are shown in Fig. 5. The change of flow rate has a little influence on the propagation of geothermal crack. The figure also shows that the geothermal crack propagates slightly even if α=0, i.e., the supply of the fluid into the geothermal crack is stopped.



(a) Increase of flow rate (b) Decrease of flow rate

Fig. 4 Fluid temperature at the outlet in case of the change of supply flow rate



(a) Increase of flow rate (b) Decrease of flow rate

Fig. 5 Positions of crack tips in case of the change of supply flow rate

4.3 Time Limit for Stable Geothermal Crack

From Eq.(12), the critical condition of the stable crack propagation is

$$\left.\frac{\partial k_1(L_1)}{\partial L_1}\right|_{L_1=L_1^*} = 0, \qquad \left.\frac{\partial k_1(-L_2)}{\partial L_2}\right|_{L_2=L_2^*} = 0 \qquad (14)$$

Generally speaking, since the tectonic stress becomes large with the depth, the crack propagation is more stable at the lower tips. Therefore, we note the crack propagation at the upper tip on the basis of the stress intensity factor. The stress intensity factor at the upper

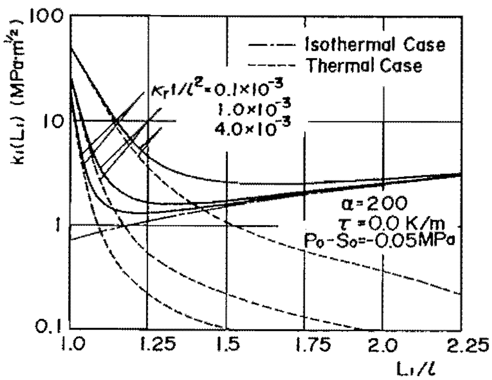


Fig. 6 Stress intensity factor at the upper crack tip

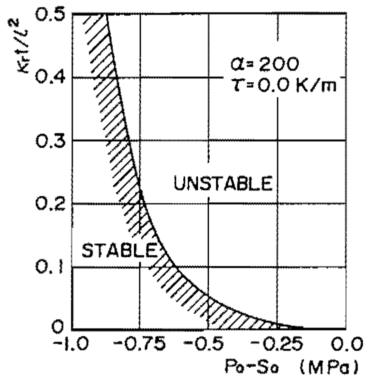


Fig. 7 Time at the critical state against the pressure difference P_0-S_0

crack tip is shown in Fig.6 for $L_1=L_2$. The broken line indicates the thermal stress intensity factor, and the one-dot chain line the stress intensity factor due to fluid pressure and tectonic stress. The actual stress intensity factor at the upper crack tip is shown by the solid line as the sum of them. The stress intensity factor has a minimum value. If the fracture toughness kc of the rock is between the minimum value and the value at $L_1/\ell=1$, the two positions of crack tips satisfying Eq.(11) exist. In these cases, the smaller tip is stable. When the minimum value of stress intensity factor is equal to the fracture toughness, the condition is critical state as indicated in Eq.(14). The time at the critical state is shown against the pressure difference P_0-S_0 between the fluid pressure and tectonic stress at y=0 in Fig. 7. In the figure, the stable region is hatched and the other region is unstable. The critical time becomes shorter with increasing of fluid pressure. Fig. 8 shows the relation between the positions of the crack tips and the pressure difference P_0-S_0 at the critical state. As the fluid pressure decreases, we can keep the larger geothermal crack for a long time in a hot dry rock.

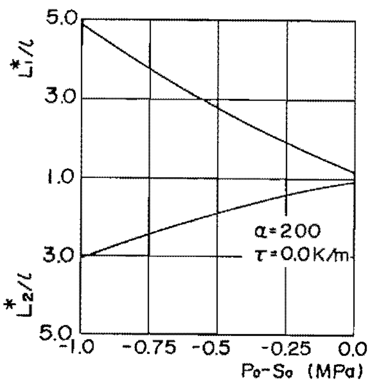


Fig. 8 Relation between the positions of the crack tips and the pressure difference P_0-S_0 at the critical state

5. Conclusion

In this paper, the effect of the initial rock temperature gradient, the change of supply flow rate and the time limit for the stable geothermal crack are discussed. The main results are summarized as follows:

(1) The effect of the rock temperature gradient on the outlet fluid temperature and propagation of the geothermal crack is small.

(2) The change of flow rate has a little influence on the crack propagation. The geothermal crack propagates slightly even if the supply of fluid into the geothermal crack is stopped.

(3) When the minimum value of stress intensity factor is equal to the fracture toughness, the condition is critical state as indicated in Eq.(14). The critical time becomes shorter with increasing of fluid pressure.

The authors wish to their gratitude to Messrs. K. Kitagawa, T. Saito and K. Nakayama of Tohoku University, Department of Engineering Science, for their assistance.

Reference

[1] Nemat-Nasser, S., Abé, H. and Hirakawa, S., eds., Hydraulic Fracturing and Geothermal Energy, Martinus Nijhoff, Hague, 1983.

[2] Abé, H., Sekine, H. and Shibuya, Y. (1983) Thermoelastic Analysis of a Cracklike Reservoir in a Hot Dry Rock During Extraction of Geothermal Energy, Trans. ASME, J. Energy Resources Technology, Vol.105, pp.503-508.

[3] Shibuya, Y., Sekine, H., Takahashi, Y. and Abé, H. (1984) Analysis of a Crack-Like Reservoir for Extraction of Geothermal Energy from HDR (Thermal Stresses around the Crack-Like Reservoir), Journal of the Geothermal Research Society of Japan, Vol.6, pp.1-11 (in Japanese).