AN ANALYTICAL MODEL FOR ESTIMATING TERRAIN INFLUENCE ON RESERVOIR PRESSURES - IMPLICATIONS FOR HOHI THERMAL AREA

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Introduction

When geothermal fields are found in mountainous areas, it is usually observed that natural discharge points (hot springs) are located in regions of relatively low ground surface elevation. In the natural state, it is sometimes possible to correlate stable underground reservoir pressures with the local ground surface elevation (in addition to correlation with absolute elevation with respect to sea level). That is, underground pressure at a fixed elevation will be higher under mountains than under valleys.

A simplified linearized analytical model for these phenomena is presented. Observations of hot spring discharge rates and underground pressures may be used in connection with this model to obtain approximate regionally averaged horizontal and vertical rock permeabilities. The method is illustrated by application to the Hohi thermal region located on Kyushu Island, Japan.

Background

Consider a mountainous region characterized by alternating mountain peaks and valleys. The variation in the elevation of the ground surface around the mean value is given by:

$$E_s = \bar{E} \cos\left(\frac{2\pi x}{X}\right) \cos\left(\frac{2\pi y}{Y}\right) \tag{1}$$

where x and y are the horizontal coordinate directions and $\bar{E}$, X and Y are constants with dimensions of length. We will assume that $\bar{E}$ << smaller of (X,Y). The region of interest is infinite in lateral extent, so $-\infty \le x \le \infty$, $-\infty \le y \le \infty$.

The z-coordinate measures distance downward into the earth, and is equal to zero at the mean ground surface elevation. We assume that the subsurface rock is porous and permeable, and saturated with liquid water. A distinction is made between horizontal (x,y) rock permeability (k_H) and vertical (z) rock permeability (k_V). The vertical coordinate is unbounded; $0 \le z \le \infty$.

We assume that k_H and k_V are functions of depth (z) only, as is the fluid temperature (T). We further impose the following two constraints upon the permeability distribution:

$$\frac{\left(k_H k_V\right)^{1/2}}{\nu} = \frac{\bar{k}}{\nu} = \text{constant} = \tau \tag{2}$$

$$\left(k_V/k_H\right)^{1/2} = \text{constant} = \alpha \tag{3}$$

which serves to define the transmissivity τ and the anisotropy α. The quantity ν is the fluid kinematic viscosity, taken to depend on temperature (and thus depth) only. We also assume that no regional horizontal average pressure gradient is present.

The Underground Pressure Distribution

In the absence of surface elevation variations (that is, in the special case $\bar{E} = 0$), the underground distribution of fluid pressure would depend only upon z and would be hydrostatic. No fluid flow would take place. In this special case, the pressure would be simply:

$$P_h(z) = P_{air} + g \int_0^z \rho(z)\,dz \tag{4}$$

where P_{air} (constant) is air pressure, g (also constant) is the acceleration due to gravity, and the fluid density (ρ) is assumed to depend on temperature (and hence depth) only.

In the more general case, the pressure distribution must be found from the steady-state fluid conservation principle:

$$\frac{\partial}{\partial x}\left[\frac{k_H}{\nu}\frac{\partial P}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{k_H}{\nu}\frac{\partial P}{\partial y}\right] + \frac{\partial}{\partial z}\left[\frac{k_V}{\nu}\left(\frac{\partial P}{\partial z} - \rho g\right)\right] = 0 \tag{5}$$

This may be written more compactly as:

$$\left(\frac{\tau}{\alpha}\right)\frac{\partial^2 P}{\partial x^2} + \left(\frac{\tau}{\alpha}\right)\frac{\partial^2 P}{\partial y^2} + (\alpha\tau)\frac{\partial}{\partial z}\left(\frac{\partial P}{\partial z} - \rho g\right) = 0 \tag{6}$$

If we define:

$$p(x,y,z) = P(x,y,z) - P_h(z) \tag{7}$$

(see Equation 4, above), Equation 6 becomes simply:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \alpha^2 \frac{\partial^2 p}{\partial z^2} = 0 \tag{8}$$

Note that the transmissivity (τ) does not appear in Equation 8.

The proper boundary conditions are: (1) the deviation from hydrostatic pressure (p) should approach zero as $z \to \infty$, and (2) the pressure at the local ground surface elevation must equal P_{air}. Since $\bar{E} \ll X$ or Y, this latter condition may be adequately represented by imposing, at $z = 0$:

$$p(x,y,z=0) = \rho_0 g\, E_s(x,y)$$

$$= \left[\rho_0 g \bar{E}\right] \cos\left[\frac{2\pi x}{X}\right] \cos\left[\frac{2\pi y}{Y}\right] \tag{9}$$

where $\rho_0 = \rho(z = 0)$. Note that it is not essential to the argument that P_{air} be applied exactly at the ground surface; the only essential assumption is that the air-pressure isobar lies within a fixed distance of the local ground surface everywhere.

As may be verified by substitution, the solution for the pressure deviation (p) is given by:

$$p(x,y,z) = \left[\rho_0 g \bar{E}\right] \cos\left[\frac{2\pi x}{X}\right] \cos\left[\frac{2\pi y}{Y}\right] \exp\left[\frac{-2\pi z}{D}\right] \tag{10}$$

where the constant "D" (the "characteristic depth") is given by:

$$D = \alpha\left[\frac{1}{X^2} + \frac{1}{Y^2}\right]^{-1/2} \tag{11}$$

The Natural Discharge Rate

Only in the special case $\bar{E} = 0$ is the solution static (no fluid motion). If $\bar{E} > 0$, fluid will flow downward into the ground near mountains (where $E_s > 0$) and will be discharged (in equal amounts) from valleys (where $E_s < 0$). Darcy's law provides the upward mass flow rate per unit area for any point on the surface:

$$\dot{m}(x,y) = \left[\frac{k_V}{\nu} \frac{\partial p}{\partial z}\right]_{z=0} \tag{12}$$

Differentiating Equation 10 with respect to z and incorporating Equations 2 and 3 yields:

$$\dot{m}(x,y) = -\frac{2\pi\alpha\tau}{D}\, p(x,y,z=0) \tag{13}$$

This expression may be integrated in the x-y plane (for regions where $E_s < 0$) to find the total discharge rate from the system:

$$\dot{M} = \frac{2A\tau}{\pi} \left[\rho_0 g \bar{E}\right] \left[\frac{1}{X^2} + \frac{1}{Y^2}\right]^{1/2} \tag{14}$$

where A is the total surface area considered (containing many mountains and valleys). Note that the anisotropy coefficient (α) does not appear in Equation 14.

Application to Hohi Thermal Area

The Hohi thermal area (about 100 km^2; $10^8 m^2$) contains several mountains and mountain ridges, notably Mt. Waita to the west, the Mt. Goto-Mt. Sensui crest centrally, and Mt.

Iwoyama to the east. Previous studies of the area are summarized by Pritchett, *et al.* (1985). We take the terrain description in the area to be: X = 5000 m (east-west); Y = 15,000 m (north-south), and $\bar{E}$ = 400 m (800 meters peak-to-trough).

For water, the kinematic viscosity as a function of temperature for the range of interest may be taken to be:

$$\nu(m^2/s) = 3.2 \times 10^{-5}/(T + 9) \tag{15}$$

where T is temperature in degrees Celsius.

Solving Equation 14 for the transmissivity τ yields:

$$\tau = \frac{\pi \dot{M}}{2A\rho_0 g\bar{E}} \left[\frac{1}{X^2} + \frac{1}{Y^2}\right]^{-1/2} \tag{16}$$

and recall that $\bar{k}$ $(=[k_H k_V]^{1/2})$ is given by $(\tau\nu)$; to estimate $\bar{k}$ in the Hohi study area, we use the following approximate values:

$A = 10^8 m^2$
$\rho_0 = 10^3\ kg/m^3$
$g = 9.8\ m/s^2$
$\bar{E} = 400\ m$
$X = 5 \times 10^3 m$
$Y = 15 \times 10^3 m$
$\dot{M} = 150\ kg/s$

The resulting expression for $\bar{k}$ (expressed in millidarcies) is:

$$\bar{k}(md) = 91.2/T + 9) \tag{17}$$

which takes on the following values for the temperature range of practical interest:

T(°C)	$\bar{k}$(md)	T(°C)	$\bar{k}$(md)
100	0.84	200	0.44
125	0.68	225	0.39
150	0.57	250	0.35
175	0.50	275	0.32

It is important to note that these values are quite low (less than one millidarcy).

Next, we note that feedpoint pressures in shutin Hohi wells show a strong correlation with overlying surface elevation (see Pritchett, *et al.*, 1985). For these wells (with feedpoint depths from ~ 1000 m to ~ 2500 m, with average value ~ 1500 m), the pressures can be represented by an expression of the form:

$$P(z) = a + b\left[z + \frac{3}{4}E_s\right] \tag{18}$$

The coefficient for surface elevation (3/4) suggests that, at these depths (z ≈ 1500 m), the quantity:

$$\exp(-2\pi z/D)$$

(see Equation 10) is about equal to 3/4. Therefore, we obtain for the characteristic depth D:

$$D \approx 33{,}000 \text{ meters}.$$

Noting that X ≈ 5,000 m and Y ≈ 15,000 m, and using Equation 11, we may obtain the anisotropy coefficient:

$$\alpha = 6.96$$

so that

$$k_V/k_H = 48$$

which suggests that vertical permeabilities at Hohi are generally much greater than horizontal permeabilities. If, for example, the system were instead isotropic ($k_V = k_H$; $\alpha = 1$), this model predicts that the correlation between feedpoint pressure and overlying surface elevation (observed to be ~ 3/4) would be much smaller (~ 1/7). These results for the anisotropy coefficient α permit the estimation of separate horizontal and vertical permeabilities at Hohi, depending somewhat on the temperature considered representative:

T(°C)	k_H(md)	k_V(md)
100	0.12	5.8
150	0.08	4.0
200	0.06	3.0
250	0.05	2.5
300	0.04	2.1

The clear implication is that horizontal permeabilities in the Hohi area are extremely low, at least in a regional sense. This suggests, in turn, that water motions are predominantly vertical, at least to the depths so far reached by drilling (~ 3000 m). Of course, these conclusions are only considered valid in a regional sense and do not preclude the existence of local relatively high-permeability subregions embedded in a larger area characterized by low average horizontal permeability.

References

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