TRANSIENT HEAT TRANSFER FROM A CIRCULAR CYLINDER WITH CONSTANT SURFACE HEAT FLUX IN A SATURATED POROUS LAYER; APPLICATION TO UNDERGROUND WATER VELOCIMETRY

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Transient heat transfer from a cylindrical object placed in a saturated porous layer, which is simulating a heat-generating probe vertically placed in an aquifer, has been analized. Constant heat flux is specified on the cylinder surface and cross flows are assumed. If the conditions are satisfied, it is shown that the measurements of the surface temperature rise with time and of the elapsed time to reach steady state can determine the thermal conductivity of the aquifer and the underground water velocity running through it; the physical quantities of great interest to diversifying fields of underground use.

1. INTRODUCTION Water movement in an aquifer and the associated energy and mass transfer process are the contemporary research subjects with broad interest ranging from industrial applications to environmental problems. Large scale power generation by exploitting the energy from deep hydrothermal systems is a typical example of industrial use of geothermal energy. information on the thermal properties of reservoir formation and on the hot water flows through it is crucial to modeling the hydrothermal systems. A shallow aquifer of temperature 15 °C is also a possible heat source for small scale direct use, such as house and road heating in cold regions during winter time. The prediction of extractable amount of heat from the aquifer is important in optimizing the design parameters of the heating system. The underground water velocity and the thermal conductivity of water-saturated soil will give significant effects on the heat extraction rate from the ground. Conversely, the frozen earth created in the aquifer by removing heat with large heat pipes during winter time will serve as an ideal space for storing agricultural products. Maintaining such frozen earth permanently requires good knowledge on the thermal conductivity and the flow conditions at the site. The migration of shallow underground water is also important in connection with the spread of hazardous chemical waste. Mixing species will be transported by a long distance with moving underground water. Non-mixing liquid, such as gasoline and heavy oil, will form a layer by displacing the water and by spreading over the aquifer, giving significant impact on the quality of underground water. The prediction on the size of contaminated area and its migration speed over the years requires studies on flows through saturated and unsaturated porous zones and on two phase flows of water and non-mixing liquid. Design of various underground structures, such as natural gas storage system, needs careful considerations regarding the heat transfer by under ground water.

Aforementioned problems vividly exemplify the importance of probing the thermal properties of the ground and rock formations, as well as the underground water flows through them, in developping various technologies that deal with underground space. In the present study we investigate the transient heat transfer from a circular cylinder with specified constant heat flux on the surface and placed in a saturated porous layer as shown schematically in figure 1. The water flow is assumed to be perpendicular to the cylinder axis. The problem has been investigated both analytically and numerically. Based on the study, a new method to measure the thermal properties of water saturated porous formation and the permeating water velocity simultaneously with using a single bore hole is proposed.

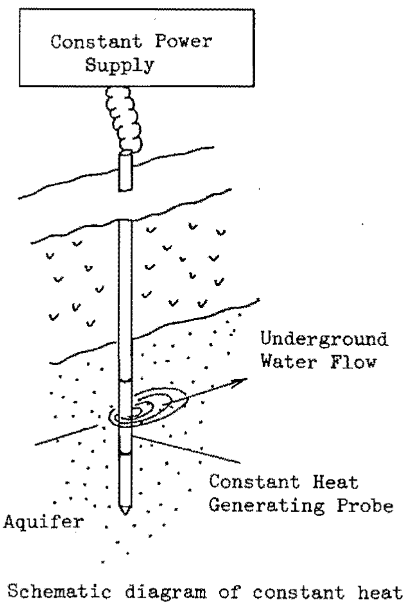

Figure 1. Schematic diagram of constant heat generating cylindrical probe placed in an aquifer.

2. MATHEMATICAL FORMULATION Prior to the analysis, we refer to the idealized physical model and the coordinates shown in figure 2. The figure shows a horizontal cross section of the cylinder surrounded by a saturated homogeneous porous medium. The cylinder is subjected to a uniform cross flow with velocity U_∞ and temperature T_∞ as indicated by a horizontal arrow in the figure.

Assuming the validity of Darcy's law, the governing equations for mass, momentum and energy [1] are expressed by

$$\nabla \cdot V = 0 \tag{1}$$

$$V = -\frac{K}{\mu}\nabla \cdot P \tag{2}$$

$$\frac{(\rho_f c_f \phi + \rho_s c_s (1-\phi))}{\rho_f c_f} \frac{\partial T}{\partial t} + V \cdot \nabla T = \frac{k_e}{\rho_f c_f} \nabla^2 T \tag{3}$$

Uniform Permeable Medium

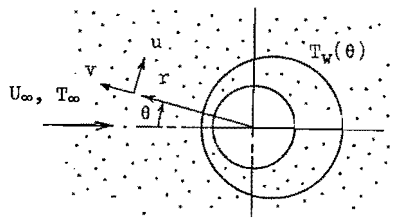

Figure 2. Idealized physical model and coordinates system

where V, K, P, T, t, c, k_e, μ, ρ, ϕ, the subscripts f and s mean two dimensional velocity vector, the permeability of the surrounding material, pressure, temperature, time, thermal capacity, the effective thermal conductivity of the water-saturated porous material, the viscosity of water, density, the porosity of the surrounding material, referring to water and the porous material respectively. The boundary conditions are given by

$v = 0$	on the cylinder surface (r=a)	(4)
V = uniform	at infinity ($r \to \infty$)	(5)
$\partial T/\partial r = q''/k_e$	on the cylinder surface (r=a)	(6)
$T = T_\infty$	at infinity ($r \to \infty$)	(7)

It should be noted that the last temperature boundary condition at infinity must be replaced by an alternative one, which expresses convecting energy in the down stream direction. The complete two dimensional solutions for flow and temperature fields may be sought by solving a set of governing equations (equations 1 through 3) subjected to the boundary conditions, at least numerically. It would be, however, useful to seek closed form solutions, which often exhibit parametric relations explicitly.

3. SOLUTIONS Recognizing the fact that at small times after the constant heat flux is imposed, the thermal boundary layer is extremely thin and convection is unimportant as a transfer mechanism. This suggests that the governing equation can be reduced to a simple unsteady heat conduction equation as long as the small time condition is satisfied. During the small time period, heat diffuses uniformly from the cylinder surface in the radial direction. Therefore, the solution is independent of the azimuthal direction θ. The following equation results [2].

$$T_w(t) - T_\infty = \frac{2q''}{k_e}\sqrt{\alpha t}\{\text{ierfc}(0) - \frac{\sqrt{\alpha t}}{a}\, i^2\text{erfc}(0) + \cdots\cdots\} \tag{8}$$

where q'', α and ierfc are the specified heat flux on the cylinder surface, the thermal diffusivity defined by $k_e/(\rho c)_f$ and the integral of complementary error function respectively. The subscript w indicates the condition at the cylinder surface. The solution implies that the surface temperature rises with 0.5 power of time at the very beginning of transient process. As time elapses, however, the thermal boundary layer grows steadily and the convection gradually comes into play. The eventual steady state is achieved by the balance between the heat removal from the outer-boundary of the thermal layer by convecting water and the heat supply from the cylinder surface into the thermal layer by conduction. If the flow velocity is relatively high, the thickness of the thermal boundary layer remains small in comparison with the cylinder diameter as the steady state is reached. The boundary layer approximation [3] may be applicable under the condition. The boundary layer simplification states that the heat diffusion in the azimuthal direction is negligibly small compared with the radial diffusion in equation (3). Such a simplification enables us to construct a steady-state solution of closed form. We use the integral technique in order to develop the solution. The integral technique assumes the temperature profile within the thermal boundary layer. It also assumes that the profile does not alter its shape as one moves in the down stream direction; the normalized profile is similar and, therefore, independent of θ. Based upon these conditions, the integral technique [4] yields the following approximate solution.

$$T_w(\theta) - T_\infty = \frac{q''a}{k_e\sqrt{Pe}} f(\theta)$$

$$\cong \frac{0.87q''a}{k_e\sqrt{Pe}}\sqrt{\theta}\ \sqrt[4]{1 + \cot^2\theta} \qquad (9)$$

where Pe is the Peclet number defined by $U_\infty a/\alpha$. Equation (9) displays the fact that if the temperature on the cylinder surface is properly normalized, there exits a universal temperature profile; a function of θ and independent of flow velocity. The finding is quite useful considering that a single curve is sufficient to describe a whole set of solutions. We have demonstrated in this paragraph that under certain approximations it is possible to develop closed form solutions, which often shed light into the physical processes that we are concerning about. However, it is also true that those solutions have their natural limitations arising from the various assumptions and simplifications made prior to the analysis. In the next paragraph we aim at giving the numerical solutions to the complete two dimensional system, which are valid over the entire transient process extending from early conductive state to final convective steady state.

Based on finite differences numerical experiments are carried out. Since the flow field can be solved exactly, only the energy equation (equation (3)) is integrated numerically. The nondimensional form of equation (3) is written in the following way.

$$\frac{\partial T}{\partial t} + Pe V\cdot\nabla T = \nabla^2 T \qquad (10)$$

where temperature, time and length are nondimensionalized by k_e/aq'', $\sigma a^2/\alpha$ and the cylinder radius respectively. σ is the thermal capacity ratio defined by $(\rho_f c_f\phi+\rho_s c_s(1-\phi))/(\rho c)_f$. The velocity components u and v are given by the followings.

$$u = (1 + \frac{1}{r^2})\sin\theta \qquad (11)$$

$$v = -(1 - \frac{1}{r^2})\cos\theta \qquad (12)$$

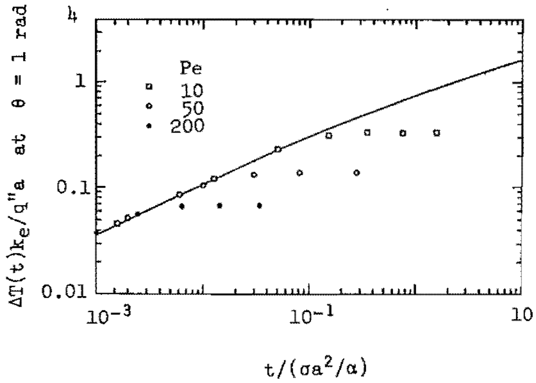

Figure 3. Surface temperature rise at θ=1 rad with time under various flow velocities.

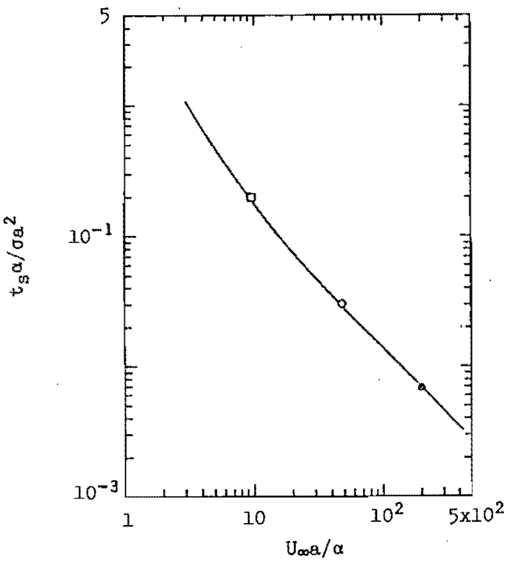

Figure 4. Time to achieve steady state at three different values of water velocity. Symbols carry the same meanings as defined in figure 3.

In the course of numerical experiments we assign three different values on the Peclet number. The numerical results are shown by three different symbols in the three consecutive figures, figure 3 through figure 5. Figure 3 shows the rise of cylinder surface temperature at θ = 1 radian as a function of nondimensional time. At small times the numerical results agree well with the analytical solution expressed by equation (8). As time passes, however, the numerical results gradually deviate from the analytical prediction (solid line) and approach to respective asymptotic values unique to the given Peclet numbers. It is also seen that the time to achieve steady state is inversely proportional to the Peclet number. This point is shown in figure 4. In the figure We plot the time to reach steady state against the Peclet number. It is clearly seen that the slope is -1 when the Peclet number is greater than 50. The surface temperature at steady state is shown in figure 5. A solid line indicates the approximate solution by equation (9) and the symbols carry the same meaning as defined in previous figures. Since heat transfer is efficient in the area facing the flow, the temperature there is kept lower than that of behind the flow. A low Peclet number case exhibits a slight deviation from the integral result. This implies that the boundary layer thickness at Pe=10 is comparable with the cylinder radius and the simplification based on the boundary layer approximations is no longer valid.

4. APPLICATION TO VELOCIMETRY Through the analysis, we have the following three major findings; (1)Surface temperature rises according to unsteady conduction at small times, (2)The time to achieve steady state is inversely proportional to flow velocity, (3)Normalized surface temperature at steady state is in dependent of flow velocity. The three statements are equivalent to saying that the right hand sides of the follwing equations possess unique values to respective systems.

$$\sqrt{k_e\sigma} = \frac{2}{\sqrt{\pi}} \cdot \frac{q''\sqrt{t}}{\Delta T(t)\sqrt{\rho_f c_f}} \qquad (13)$$

$$\frac{k_e}{\sigma} = \frac{a^2\rho_f c_f g(Pe)}{t_s} \qquad (14)$$

$$\sqrt{U_\infty k_e} = \frac{a q'' f(\theta)}{\sqrt{\rho_f c_f}\, T(\theta)} \qquad (15)$$

where g(Pe) gives a nondimensional time to reach steady state as a function of the Peclet number. The equations (13) through (15) are sufficient to determine the effective thermal conductivity of water-saturated porous layer, underground water velocity with its direction and the thermal capacity ratio. Since the right hand side of equation (14) contains implicitly the unknown ratio between the water velocity and the effective thermal conductivity, an iterative procedure is needed to solve them. Figure 6 shows a possible structure of the probe. The condition of uniform heat flux may be achieved by an electrically heated thin metallic plate tightly wound around a cylindrical insulating material.

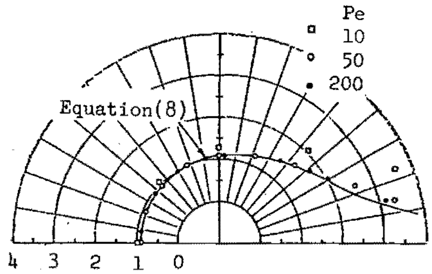

Figure 5. Surface temperature variation at steady state under various flow conditions. Solid line indicates universal temperature profile.

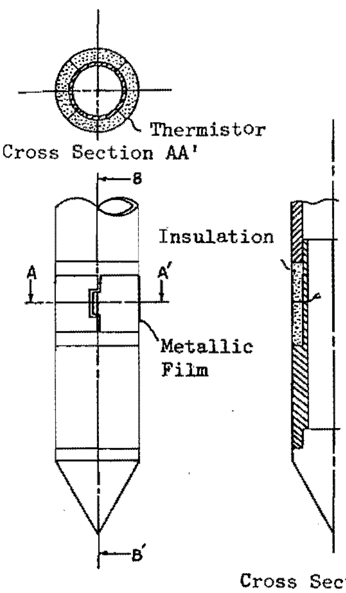

Figure 6. Diagram showing a possible structure for the probe of thermal properties and water velocity.

5. CONCLUSION In concluding the present study, transient heat transfer process from a cylinder with constant heat flux, placed vertically in a porous layer and subjected to cross flows, has been analyzed both analytically and numerically. The study reveals the nature of the process that consists of two distinctive states; one is dominated by conduction and the other by convection. The presence of the two states enables us to develop closed-form solutions valid for the respective regimes. The results also suggest a potential application to measurement that probes the thermal properties of shallow aquifer and deep permeable formation, as well as the water velocity running through them, with using a single bore hole.

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