THE SCALE ANALYSIS OF CONVECTION IN POROUS MEDIA

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The objective of this lecture is to outline the basic rules of the method of scale analysis (or back-of-the-envelope order of magnitude reasoning), and to show its applicability in the field of convection through fluid-saturated porous media. The method and its convection applications are presented in greater detail in the book Convection Heat Transfer by Adrian Bejan (Wiley, New York, 1984).

Scale analysis is a highly efficient theoretical approach to obtaining useful engineering results. In many cases, its results differ from the most accurate solutions by percentage points. Furthermore, the use of scale analysis can provide a much needed bird's-eye view of the physics of complex heat, mass and fluid flow phenomena. It can identify the proper dimensionless parameters, in terms of which complicated numerical and experimental results can be reported.

The list of specific convection configurations that will be subjected to scale analysis during the lecture includes:

1. The isothermal wall adjacent to a porous medium saturated with a fluid of a different temperature
2. The wall with uniform heat flux
3. The plume rising above a horizontal line heat source
4. The plume above a concentrated point heat source
5. Enclosed porous media subjected to a temperature difference in the horizontal direction
6. Penetrative flows, i.e. flows that do not spread throughout the porous medium
7. Examples of new (scaling-correct) correlations of older experimental and numerical data
8. The combined buoyancy effects due to temperature and concentration gradients in saturated porous media
9. Horizontal porous layers heated from below (Benard convection).

As an example, the scale analysis of the last configuration (Benard convection, item 9 above) indicates that in the Darcy regime the Nusselt number must increase linearly with the Rayleigh number, more precisely,

$$Nu = \frac{1}{40} Ra \quad , \quad \text{if} \quad Ra > 40 \qquad (1)$$

where Nu is the actual heat transfer rate divided by the pure conduction heat transfer rate across the same layer. The Rayleigh numer is $Ra = Kg\beta\Delta TH/(\alpha\nu)$, where the symbols represent, in order, the Darcy permeability, gravitational acceleration, volumetric coefficient of thermal expansion, the bottom-top temperature difference, the height of the fluid saturated porous layer, the thermal diffusivity of the medium, and the kinematic viscosity of the fluid.

At sufficiently high Rayleigh numbers, where the effect of fluid inertia becomes important, the scale analysis recommends an entirely different behavior,

$$Nu \sim Ra^{1/2} Pr_p^{1/2} \quad , \quad \text{if} \quad Ra > Pr_p \qquad (2)$$

It also unveils a new dimensionless group, the now-called "porous medium Prandtl number" Pr_p,

$$Pr_p = \frac{H}{bK} Pr \qquad (3)$$

where b is Forchheimer's constant and Pr is the ratio ν/α. Equations (1) and (2) were combined recently into a successful correlation of existing Nu(Ra) measurements of Benard convection, reported by many sources in the range $40 < Ra < 4000$,

$$Nu = \left\{ \left(\frac{Ra}{40}\right)^n + \left[c\,(Ra Pr_p)^{1/2} \right]^n \right\}^{1/n} \qquad (4)$$

The values of the two empirical constants are $n = -1.65$ and $c = 1896.4$.

This example shows not only how directly scale analysis pinpoints the trends (asymptotes) of an important quantity (Nu), but also how to correlate the many empirical data into a compact and useful engineering tool [equation (4)]. This conclusion will be reinforced by the convection examples exhibited in the lecture.

The Occurence of Hot-Water at the Nibetsu Area, Akita, North Japan.

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1. Introduction

Since a hot-spring was detected at the Nibetsu area about 15 km north-east of the center of Akita city in 1987, this area has attracted much more citizens' attention as a resort area (see Fig.1). So, a study on the occurence of hot-water is required for the development, utilization and management of the hot-spring in the future.

We investigate the the occurence of hot-water and the geological structure in this area using the data obtained from the geological survey, radioisotope exploration, well logs and chemical analysis and try to estimate the hot-water velocity in the fractured rock.

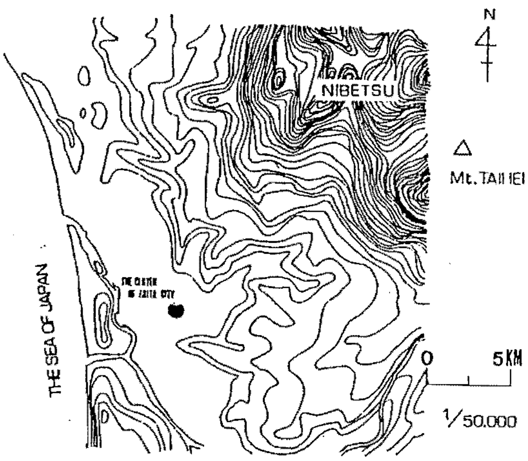

Fig.1. The location of Nibetsu Area

2. Topography and Geology

This area belongs to the Nibetsu mountenous district and shows a steep geographical features in the prime of life and its slope is covered with forests. The maximum elevation of this area is 682 m and its elongation is continuous to Mt.Taihei (1,171 m) on the east side of this area (see Fig.1). The drainage relief and the drainage density are 300 m/km^2 and 25 km^2, respectively.

Geology in this area is composed mainly of the volcanic rocks of haginari, ookuramata and sunakobuchi formations, and their associated sedimentary rocks which consists of the mudstone. The bed strikes N20°-40°W and dips west(see Fig.2). The basement composed of Granodioritic and Tertiary granitic rocks is considered as a heat source in this area.

3. Occurence of Hot-Water

A review of where and how hot-water exists, necessitates to describe the occurence of hot-water in this area.

From the results of radioisotope exloration in the Nibetsu area, there are many points of the bismuth concentration of over 20% near the estimated faults and the lineaments. This suggests the possible presence of fractures which may be opened to the surface.

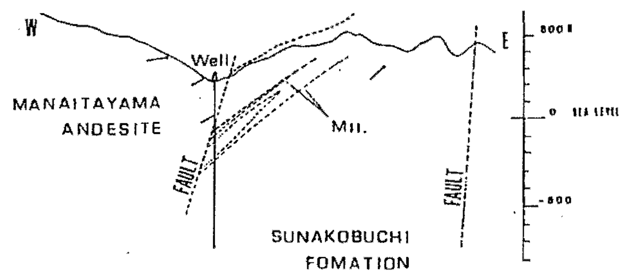

Fig.2. Geological Cross-Section

Fig.3 shows the well completion, the geological column, the location of lost circulation and the temperature profile of the well.

The results of intensity log indicate the presence of fractures at the depths corresponding to the parts of lost circulation in the borehole during the well drilling.

Many cracks and fissures are founded in the cores extracted from the formation of the lost circulation.

Therefore, these fractured zones also are considered to strike N20°-40°W, dipping west as same as the bed strike previously presented.

The amount of the residue on evaporation per kilogram of the hot-water sampled from the depths of 550 and 1000 m are 480 and 669 mg, respectivly, and the total amounts of dissolved matters per kilogram of the hot-waters in both depths under 1000 mg. The tritium concentration of the hot-water ranges from 2.6 to 2.7 TR.

From the result of the chemical analysis mentioned above, these waters can be thought of as a part of hydrologic cycle, including surface and meteoric waters.

Therefore, the surface and meteoric waters are considered to infiltrate into the ground through the fractured zones previously described.

4. Estimate of Hot-Water Velocity

Assuming the fractured zone is expressed by means of a channel model with a rectangular cross-sectional area as shown in Fig.4, a flow rate of hot-water "Q" through the channel is written as

$$Q = B*H*L/t \qquad (1)$$

where

Q=the flow rate of hot-water	(m^3/day)
B=the width of a channel	(m)
H=the height of a channel	(m)
L=the distance from the recharge area on the surface to the production well	(m)
t=the date of hot-water	(day)

On the other hand, using analysed data from pumping tests such as transmissivity "T" and wellhead shut-in pressure " P", Q also is expressed as

$$Q = B*T*P/(\gamma *L) \qquad (2)$$

where

T = transmissivity	(m^2/day)
P = wellhead shut-in pressure	(Kg/cm^2)
γ = specific weight of hot-water	(Kg/m^3)

From eqs.(1) and (2), hot-water velocity "V"

$$V = \sqrt{\frac{T* P}{\gamma *t*H}} \qquad (3)$$

can be written.

Table 1 indicates the analysed data from the pumping tests. From this Table, T = 11.7 (m^2/day), P = 2.7 (Kg/cm^2), H = 20 m and t = 12—30 (years). Thus, the range of hot-water velocity "v" through the fractured zone is estimated to become from 0.1 to 0.2 (m/day).

Refferences

1) Fujiboring Co.Ltd.(1987): A report of Nibetsu hot-spring.

2) D.K.Todd (1959): Ground water hydrology, John Wiley & Sons.

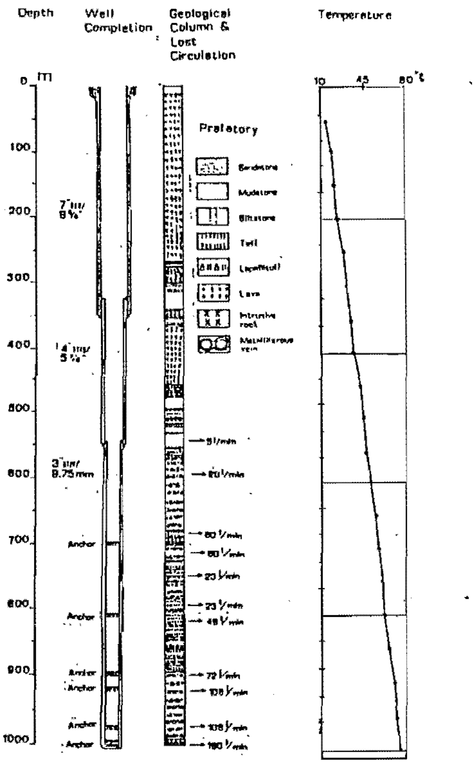

Fig.3. Well completion, Geological column Lost circulation and Temperature profile

Table.1 Analysed data from pumping test

shutin pressure	P	2.7 Kg/cm^2
thickness	H	20.0 m
transmissivity	T	11.7 m^2/min
flow rate	q	300.0 l/min
bottom-hole temperature	θ	67.7 C

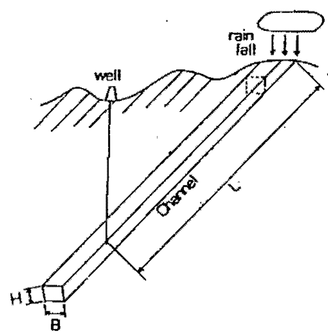

Fig.4. A channel model