EFFECTS OF HYDRAULIC GRADIENT AND DIFFUSIVITY ON ISOTHERM AND FLOW PATTERNS IN TWO-DIMENSIONAL LIQUID-DOMINATED HYDROTHERMAL SYSTEMS

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ABSTRACT

This work is intented to illustrate isotherm and flow patterns in two-dimensional liquid-dominated hydrothermal systems. They can be simulated numerically by assuming that a density-driven flow exists in the system. The equations that govern the density-driven flow are the differential equations of flow and transport. The model of isotherm and flow patterns is executed by FAST-C(2D) code. The isotherm and flow patterns in the system are influenced by hydraulic gradient at the top of boundary and thermal diffusivity. The mushroom shape of isotherm pattern is found in the free convection system where no hydraulic gradient is set at the top of boundary. In this case, an ascending flow region and convection pattern emerge in the system. When different hydraulic gradients are set at the top of boundary, different isotherm and flow patterns appear where the isotherm pattern is distorted from the initial shape. When thermal diffusivity is larger, the the size of isotherm patterns will also be larger.

1. INTRODUCTION

Flow patterns of fluid flow can be influenced by density differences in the fluid system. This fluid flow is called as density-driven flow. Therefore, the main condition for this type of flow is that density differs from one place to another. This condition holds for both steady and transient states. Those fluid density differences can result in convective patterns. Density-driven flow may occur in all types of fluids. It can be observed in systems that contain water only. It can also be observed in a porous medium that contains water, such as in a liquid-dominated hydrothermal system.

The distribution of density as a function of time and space is also related to density-driven flow. In density-driven flow systems it is often not only the density itself which is studied, but also the variables that influence on density. The density of fluids in a porous medium is mainly influenced by temperature and salinity.

The density-driven flow can be modelled numerically. This model can be carried out if the system of flows can be expressed in corresponding mathematical equations. This paper will be discussed the patterns of density-driven flow in two-dimensional liquid-dominated hydrothermal systems. It is assumed that the system consists of porous media which is saturated by water.

2. STUDY AREA AND METHOD

2.1. Hydrothermal System

The study area of this work is a numerical modelling of hydrothermal systems. Hydrothermal systems are classified into a convective geothermal system (Rybach, 1981; Hochstein, 1990). The convective geothermal system is a geothermal field whose high temperatures and expose some surface activities such as fumaroles, hotsprings and mudpools (Grant, Donaldson, and Bixley, 1981).

In convective geothermal systems the distributions of temperature and fluid are determined by fluid flow through porous media. This natural state of systems is dynamic, and a knowledge of the natural fluid flow is needed if we want to know the energy feature. The knowledge of natural fluid flow may give the information of parameter geothermal systems, such as vertical permeability which is difficult to determine by other means. If surface features, such as geysers, hot springs, and mudpools are associated with geothermal systems, these features are the end points for some part of this natural flow.

In low-temperature the fluid of hydrothermal systems can be assumed as liquid water. When we consider the higher temperature systems, steam can also be present. Therefore, hydrothermal systems are classified in two categories, liquid-dominated and vapour-dominated (Nicholson, 1993). Liquid-dominated systems have a pressure distribution that is close to hydrostatic, vapour-dominated systems have a pressure distribution that is close to steamstatic. In each case the dominant phase controls the pressure distribution, although the other phase may be present. If the fluid of hydrothermal system is all water, we call the system liquid; if steam is also present we call it two phases.

Liquid-dominated systems are found in Wairakaei (New Zealand), Salak (West Java), and Dieng (Central Java). Vapor-dominated systems are found in The Geysers (USA), Larderello (Italy), Matsukawa (Japan), Kamojang and Darajat (West Java).

Fluid flows in porous medium in a liquid-dominated hydrothemal system occur because of density differences that are caused by temperature differences. The density differences may also occur because of concentration differences.

2.2. Governing Equations

Differential equations that govern density-driven flows are flow and transport equations. This two differential equations must be solved simultaneously as a coupled equation.

For two-dimensional systems with horizontal x-axis and vertical z-axis, the differential equations which is concerned with the density-driven flow are expressed as follows (Holzbecher, 1998). The flow equation can be expressed as

$$\frac{\partial}{\partial x}\left(\frac{1}{k_z}\frac{\partial \psi}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{1}{k_x}\frac{\partial \psi}{\partial z}\right)=-\frac{g}{\mu}\frac{\partial \rho}{\partial T}\frac{\partial T}{\partial x} \qquad (1)$$

where ψ is stream function, g is gravity acceleration, ρ is density of fluid, μ is dynamic viscosity of fluid, T is temperature, k_x and k_z are permeabilities in x- and z-directions, respectively. Partial derivatives of streamfunction are related to velocity as

$$\frac{\partial \psi}{\partial x}=v_z \qquad \frac{\partial \psi}{\partial z}=-v_x \qquad (2)$$

where v_x and v_z are the components of velocity in x- and z-direnctions, respectively. The transport equation can be expressed as

$$\frac{1}{\gamma}\frac{\partial T}{\partial t}=\frac{1}{\gamma}\nabla\cdot(D\nabla T)-\mathbf{v}\cdot\nabla(T) \qquad (3)$$

where γ is ratio of heat capacities of liquid to rock, **v** is velocity vector, and D is diffusivity.

The dependence of fluid properties in density-driven flow is usually studied only for one variable, temperature or concentration. In order to come to a formulation that valid for thermal and saline transport, the transformations of equation have to be performed. If the dimensions of system are L in length and H in height, these transformations are

$$\begin{array}{l} x\rightarrow x/H \quad z\rightarrow z/H \quad t\rightarrow tD/H^2 \\ v\rightarrow v\gamma H/D \quad \psi\rightarrow \psi\gamma/D \end{array} \qquad (4)$$

so that

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial z^2}=Ra\frac{\partial\theta}{\partial t} \quad \text{with} \quad Ra=\frac{k\gamma\Delta\rho g H}{D\mu} \qquad (5)$$

for flow equation. The dimensionless constant *Ra* is so called the Rayleigh number, and Δρ is density difference. If minimum and maximum temperatures in the system are T_{min} and T_{max}, respectively, the transport equation becomes to

$$\frac{\partial\theta}{\partial t}=-\mathbf{v}\cdot\nabla\theta+\nabla^2\theta$$
with
$$\theta=(T-T_{\min})/(T_{\max}-T_{\min}) \qquad (6)$$

2.3. Numerical Model

Density-driven flow modeling is based on the differential equations of flow and transport, like equations (1) and (3) or equations (5) and (6). The analytical solution of these differential equations is difficult, so the numerical solution of them is needed. In this paper they are solved numerically by implementing FAST-C(2D) code (Holzbecher, 1998).

The first step in changing differential equations to numerical formulation is known as discretization. Discretization which is applied here is finite differences (FD). The first and second order terms of differential equation in space are approximated with standard method, i.e. BIS (backward in space) scheme.

The first and second order terms of differential equation lead to linear systems of equations which are solved by using matrices. These are sparse matrices, i.e. in each row or column there are only few nonzero elements. In rectangular grids the nonzero elements in the matrices show a diagonal stucture. For five points patterns there are five diagonals in the matrices. These equation systems can be solved directly or iteratively. In this paper Picard iterative method is used to solve the flow and transport equations.

In transient simulation temporal discretization is needed. In this paper generalized Crank-Nicholson method is used, with a time level weighting factor is chosen as ½.

Two types of boundary condition in a model area are determined, i.e. Dirichlet-type and Neumann-type conditions. For flow equation, Dirichlet-type condition is specified by streamfunction and Neumann-type condition is specified by velocity paralel to boundary. For transport equation, Dirichlet-type condition is specified by temperature and Neumann-type condition is specified by derivative normal to the edge. In transient simulations, initial condition is required to describe the initial state of model area. This state is described as a no-flow, hydrostatic, isothermal state.

Figure-1 shows the scheme of simple two-dimensional liquid-dominated hydrothermal systems which includes grids and bondary conditions. This model area has L = 5000 meter in length and H = 1000 meter in height. On the bottom center of this model there is a heat source. This area model is assumed as porous media that contain saturated water, so that water move through the cavity of pores.

The area model is divided into 200×40 block grids. This model use hyphotetical parameter: T_{min} = 20°C, T_{max} = 320°C, Δρ = 307.66 kg/m^3, γ = 1, μ = 1.46 × 10^{-3} Pa.s, and g = 9.80 m/s^2. All boundary conditions with prescribed streamfunction are specified with the same value (ψ = 0). The entire boundary line – from the top left corner of the model down, along the bottom, up top right corner – become a streamline. The normalized temperatuture of heat source is specified with 1. The transient model starts with a hyphotetical situation of constant temperature and streamfunction.

This paper discusses the isotherm and flow patterns for some hydraulic gradients at the top of system: 0%, 1% , 2%, 4%, and 7% , respectively. Thermal diffusivity (D) is the ratio of porous media thermal conductivity (λ) and specific heat capacity (ρc). Rayleigh number can be found by using equation (5). **Table-1** shows some parameters which are used to find thermal diffusivities and Rayleigh numbers (Rosca and Antics, 1999). The diffusivities and Rayleigh numbers are shown in the last two columns. The other parameters can be looked up in the Steam Tables (JSME, 1980).

3. Result and Discussion

Some patterns of isotherm and flow in two-dimensional liquid-dominated hydrothermal systems are shown in **Figures-2** to **8**. **Figures-2** to **6** show the development of temperature and flow patterns for hydraulic gradients: 0, 1, 2, 4, and 7% at the top of boundary.

When no hydraulic gradient is set on the top boundary, in the free convection case, an ascending flow region emerges as shown in **Figure-2**. The temperature distributions show the mushroom shape of isotherm pattern. Upward flow of hot water can be observed in the vicinity of the centre of system.

Different isotherm and flow patterns appear with increasing simulation time. Some convection flows can be observed in **Figures 2** to **4**. No convection flow is shown in **Figures-5** to **6**. When hydraulic gradients at the top of boundary are increased, the potential flow regime dominates. The strong potential flow that is mainly horizontal resists the up flow of hot water plumes. Hot water isotherm patterns distort to the right. When hydraulic gradient at the top of boundary is stronger, the distortion of isotherm pattern will be larger. These phenomena can be observed in **Figure-7**.

Figures-8 to **9** show isotherm and flow patterns for different thermal diffusivities. The shape of hot water isotherm for some thermal diffusivities are similar each other. However, the size of hot water isotherm is different. The size of isotherm pattern will be wider when thermal diffusivities is larger.

4. Conclusion

Isotherm and flow patterns in two-dimensional liquid-dominated hydrothermal system can be simulated numerically by assuming that a density-driven flow occurs in porous medium. The density differences in this system occurs because of different temperature. The isotherm and flow patterns in the system are influenced by hydraulic gradient at the top of boundary. Thermal diffusivity influences the size of isotherm pattern.

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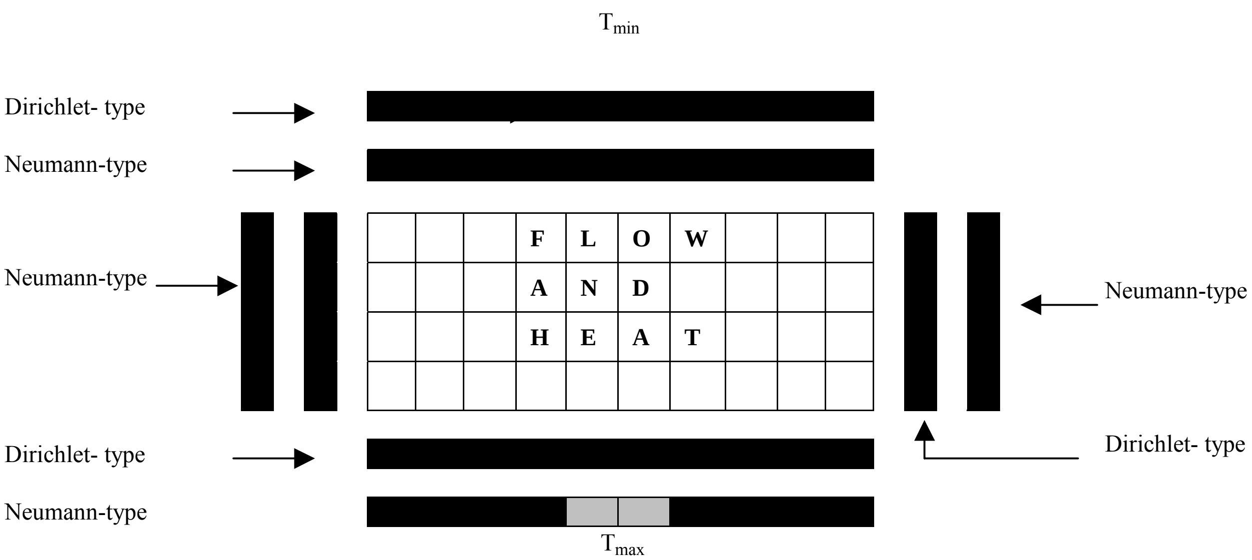

Figure-1
Model area, grids, and boundary conditions (inner: flow, outer: transport).

Table-1
Some parameters, diffusivities, and Rayleigh numbers.

	ρ kg/m^3	ϕ	K m^2	λ W/m.K	c J/kg.K	D m^2/s	Ra
Quarternary	1,900	0.30	1.0×10^{-12}	2.989	885.6	1.8×10^{-6}	1147
Upper Pliocene	1,950	0.25	5.0×10^{-13}	2.821	892.7	1.6×10^{-6}	645
Miocene	2,100	0.10	1.0×10^{-13}	2.761	914.2	1.4×10^{-6}	147
Lower Cretaceous	2,300	0.10	2.0×10^{-13}	2.789	919.7	1.3×10^{-6}	318

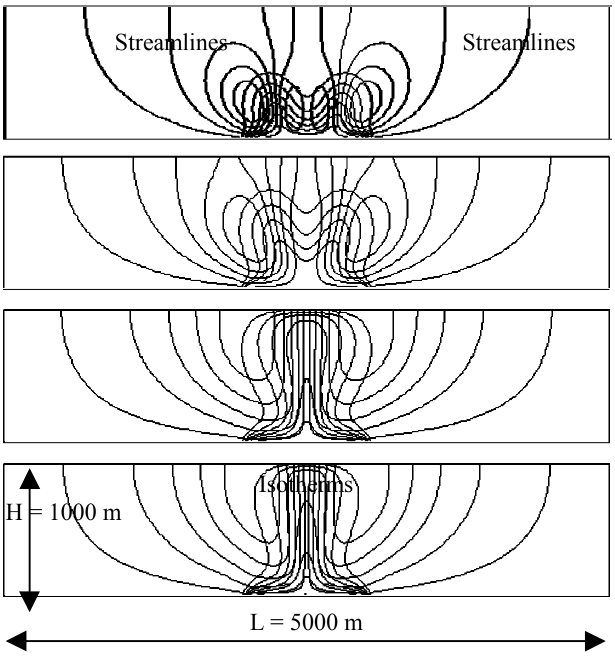

Figure 2. The development of temperature and flow patterns for D = 1.4×10^{-6} m^2/s and $\partial h/\partial x$ = 0%.

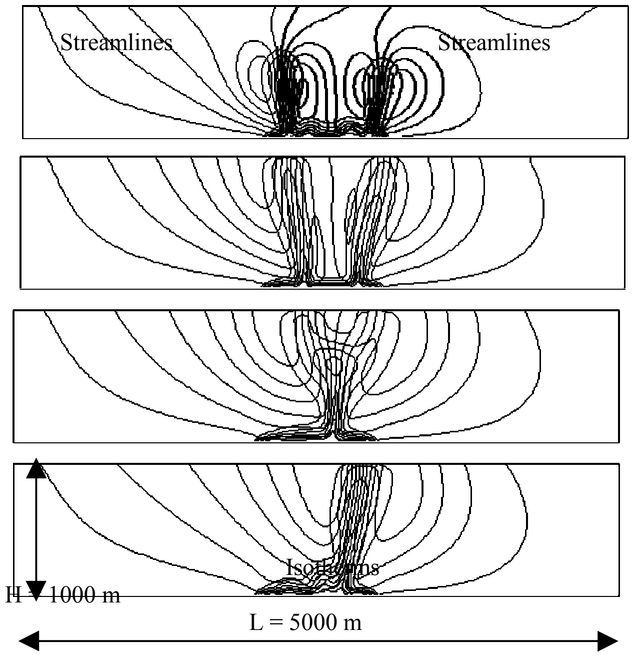

Figure 3. The development of temperature and flow patterns for D = 1.6×10^{-6} m^2/s and $\partial h/\partial x$ = 1%.

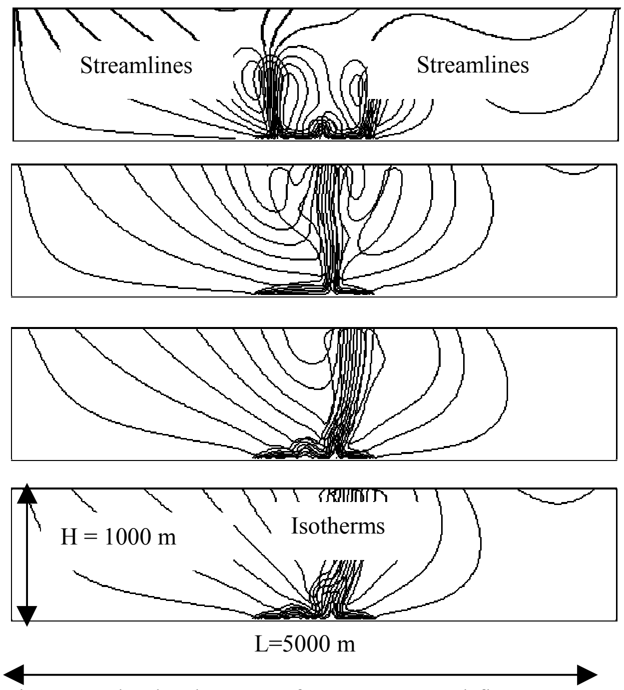

Figure 4. The development of temperature and flow patterns for D = 1.8×10^{-6} m^2/s and $\partial h/\partial x$ = 2%.

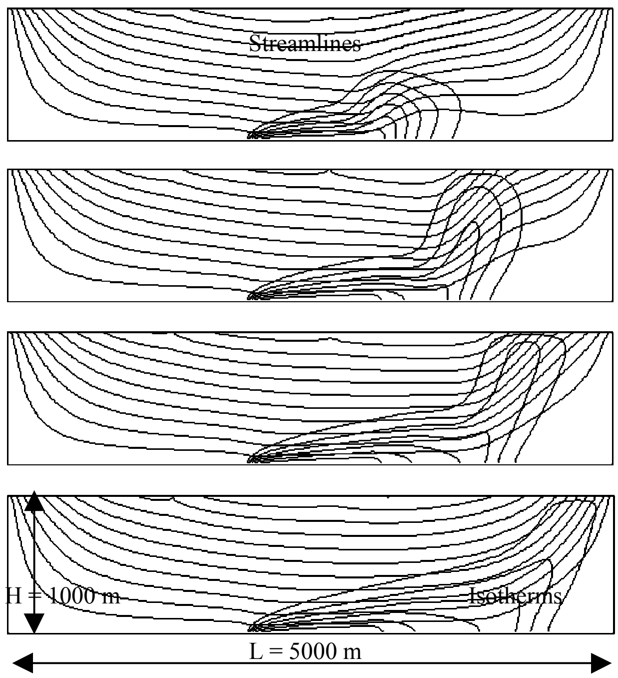

Figure 5. The development of temperature and flow patterns for D = 1.4×10^{-6} m^2/s and $\partial h/\partial x$ = 4%.

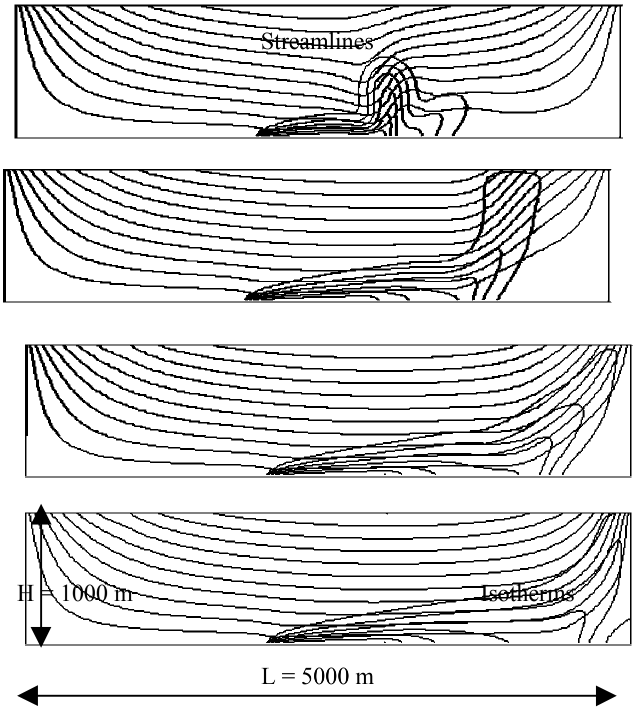

Figure 6. The development of temperature and flow patterns for $D = 1.3 \times 10^{-6}$ m^2/s and $\partial h/\partial x = 7\%$.

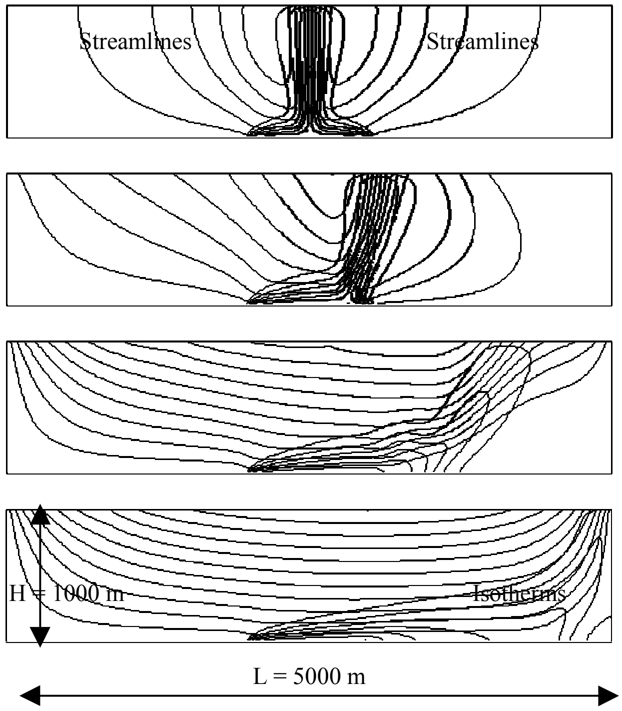

Figure 7. The final state of temperature and flow patterns for $D = 1.3 \times 10^{-6}$ m^2/s and $\partial h/\partial x$ = 0%, 1%, 4%, 7%, respectively.

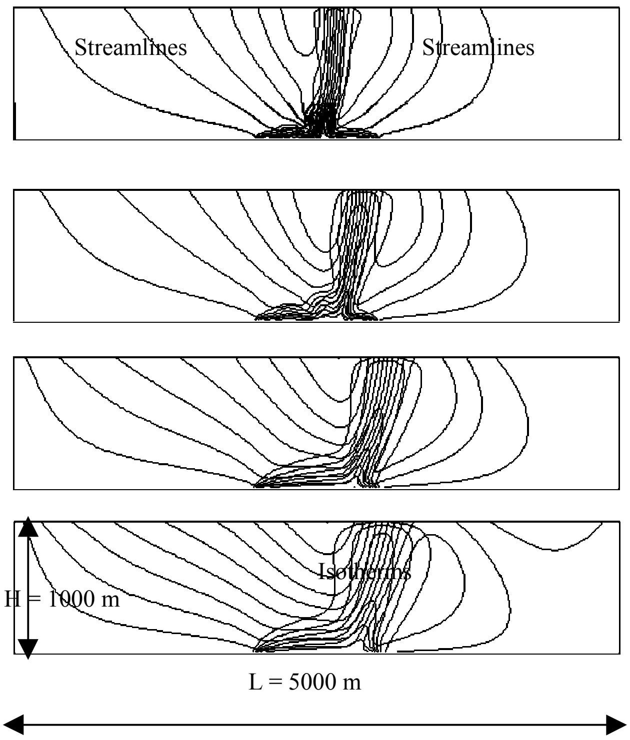

Figure 8. The final of temperature and flow patterns for $\partial h/\partial x = 1\%$ and $D = 1.8 \times 10^{-6}$, 1.6×10^{-6}, 1.4×10^{-6}, 1.3×10^{-6} m^2/s, respectively.

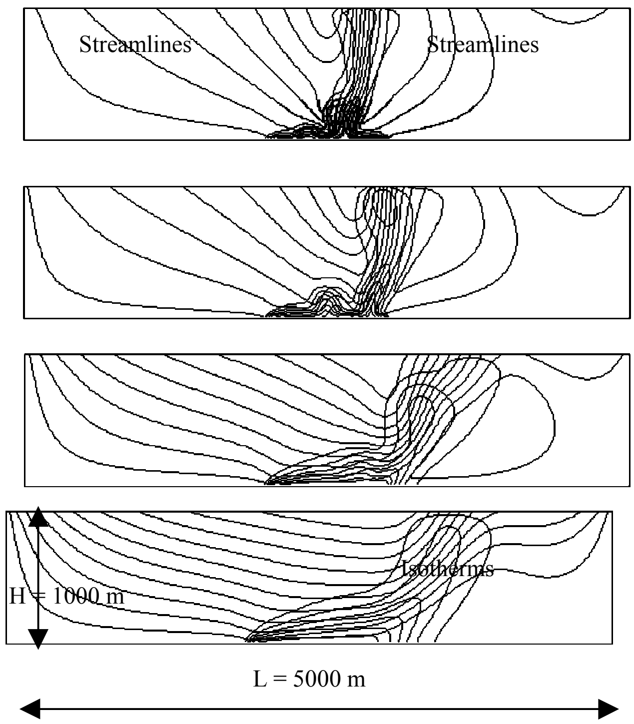

Figure 9. The final of temperature and flow patterns for $\partial h/\partial x = 2\%$ and $D = 1.8 \times 10^{-6}$, 1.6×10^{-6}, 1.4×10^{-6}, 1.3×10^{-6} m^2/s, respectively.