European Geothermal Congress 2013
Pisa, Italy, 3-7 June 2013

Numerical generation of the temperature response factors for a Borehole Heat Exchangers field

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Keywords: borehole heat exchanger, g-function, long-term performance.

ABSTRACT

Ground Coupled Heat Pump (GCHP) systems connected to a set of vertical ground heat exchangers require short and long term dynamic analysis of the surrounding ground for an optimal operation. The thermal response of the ground for a multiple Borehole Heat Exchanger (BHE) field can be described by proper temperature response factors or "g-functions". This concept was firstly introduced by Eskilson (1987). The g-functions are a family of solutions of the transient heat conduction equation and each of them refer to a given borehole field geometry. Furthermore the g-functions are the core of many algorithms for simulating the ground response to a GCHP system, including the well-known commercial software EED.

Analytical approaches based on the Finite Line Source (FLS) model have been developed by Eskilson (1987), Zeng et al. (2002) and later by Lamarche (2007). Such solutions can be in principle applied together with space superposition to infer the thermal response for any BHE configuration.

This study is a continuation of the previous work presented in Acuña et al. (2012), and a further investigation is devoted to optimize a numerical model of a squared configuration of 64 boreholes using the commercial software Comsol Multiphysics©. Symmetry conditions and different Fourier numbers have been applied and explored together with the effects related to the dimensions of the calculation domain with respect to the BHE depth and BHE field width. Furthermore, a parametric analysis is addressed to boundary conditions, which points out possible limits on the calculation domain extension. The results of the proposed numerical model are compared with the g-functions embedded within the EED software as well as those calculated by FLS method through the spatial superposition. In a closer approximation to reality, the numerical model is also studied accounting for an adiabatic part at the top of the BHE.

NOMENCLATURE

Abbreviations

BHE	Borehole Heat Exchanger
EED	Earth Energy Design
FLS	Finite Line Source
GCHP	Ground Coupled Heat Pump
GLHEPRO	Professional Ground Loop Heat Exchanger Software
SBM	Superposition Borehole Model

Symbols

B	Distance between BHEs (m)
c_p	Specific heat (J kg^{-1}K^{-1})
D	Buried depth of the borehole field (m)
Fo_H	Fourier number defined in equation [1]
g-function	g-function, defined in equation [1]
H	Borehole length (m)
k	Thermal conductivity (W m^{-1}K^{-1})
$\dot{Q}'$	Heat flux per unit length (W m^{-1})
r_b	Borehole radius (m)
S0	Ground dimension of a borehole field, D/H=0
S1	Ground dimension of a buried borehole field, D/H=0.04 and 0.05
T_{ave}	Average temperature (°C)
T_{gr}	Undisturbed ground temperature (°C)
t	Time (s)
α	Thermal diffusivity (m^2 s^{-1})
ρ	Density (kg m^{-3})

1. INTRODUCTION

The characteristics of GCHP systems make this technology be recognized as one of the most sustainable for heating and cooling purposes in commercial and residential buildings. However, a well-designed GCHP system requires a short and long term dynamic analysis of the ground's thermal response.

In these systems, a heat pump is coupled to a secondary circuit consisting of a set of heat exchangers buried in the ground. Typically, the heat exchanger are set up in a vertical layout and connected in parallel. Thus, a carrier fluid circulates through them, while the heat is exchanged to or from the surrounding ground and transferred from or to the heat pump. Thus, the ground acts as a heat source or sink for heating or cooling purposes, respectively. An optimal design requires the study of the thermal response of a borehole field for long periods. This response, which is based also on building heat loads and ground properties, determines the choice of the BHE arrangement and its geometrical characteristics such as BHE length and spacing. The design of the borehole length is subject to a prescribed temperature limits. The ground temperature must be kept on such levels suitable for coping with the limits imposed or required (in terms of performance) by the heat pump according to the building demand over the years.

In the last two decades, different analytical and numerical methods have been proposed to determine the ground response to BHE systems. From the point of view of the numerical approach, probably the best known solutions are those developed by the Superposition Borehole Model (SBM), (Eskilson, 1986). This finite difference methodology (cf also Eskilson, 1987) allowed a set of response factors for certain BHE configurations to be generated: these solutions are known as g-functions. The g-function is a non-dimensional temperature response factor that relates the borehole temperature and the extracting/injecting heat rate from the ground through the ground conductivity. Any g-function is a particular solution of the BHEs configuration which depends on a non-dimensional time and two non-dimensional geometrical parameters. The non-dimensional geometrical parameters can be written as the ratio of the borehole spacing and the borehole radius with respect to its borehole length, i.e. r_b/H and B/H. Since the SMB finite difference method resulted in a high computational time, the g-functions were first calculated as single BHE temperature field in space and time and then superposed in space. In such a way a significant number of BHEs configurations could be described and a library of g-functions was made available for design as the commercial software, EED (Hellstrom and Sanner, 1994). Recently, other design software such as GLHEPRO (Spitler, 2000) used similar libraries of g-functions for specific BHEs configurations. By temporal superposition of the thermal loads requirements of the building, the EED family of softwares are able to predict the response of the ground to a system of BHEs as a function of the building time series of heating and cooling loads. One limit of this approach (together with the unknown uncertainty of g-function values) is the limitation on the predefined BHE configurations implemented in the software.

The fast semi-analytical approach based on the Finite Line Source developed by Lamarche (2007) presents a better flexibility to generate the g-function of any BHE configuration. In general, the analytical solution (with proper spatial superposition) is able to reproduce, with acceptable deviations, the response of the vertical BHE configurations calculated by Eskilson. Some authors as Sheriff (2007), Bernier and Cauret (2009) and Fossa (2011a, 2011b) applied the FLS solutions and their results confirm a good agreement with the original g-functions.

In this paper a numerical model for a squared configuration of 64 boreholes was properly developed in Comsol Multiphysics© environment, as a continuation of the work recently presented by Acuña et al. (2012). The choice of the present configuration was made due to the fact it represents one of those cases where the FLS generated temperature response factors show not negligible differences with respect to the original g-functions. In addition this configuration allows the possibility to exploit some symmetries and hence to reduce the dimension of the calculation domain. A number of steps during the design of the numerical model were carried out to optimize either the mesh or the time steps to enhance the computing time. For this purpose, symmetry conditions and different Fourier numbers have been applied, as well as different strategies to design the mesh and set the characteristics of the solver. The temperature distribution obtained from the numerical model was then employed to generate the g-function for the present geometry in a range of Fo_H values from -5 to 2. This g-function was then compared with the ones embedded within the EED software as well as with those calculated by FLS method. Moreover, in a closer approximation to a real case, the numerical model was also performed when an adiabatic part is considered at the top of the BHE, defined as buried depth of the borehole field in this study. Its response is also compared with the solution from the analytical FLS solution for the same buried depth in the borehole field.

2. THEORETICAL BACKGROUND

Of practical interest for the optimum operation of the heat pump is the estimation of the borehole wall temperature. In the GCHP systems, the borehole heat exchanger can be considered as a heat sink with respect to ground for winter operations. Some assumptions are behind any temperature response factor model, including homogeneous and isotropic surrounding medium. Another assumption is to consider only the heat conduction, hence neglecting any groundwater flow. The infinite line source theory represents the simplest method to evaluate the borehole wall temperature: in this case the heat source is considered as an infinite line in an infinite medium.

However, this model does not consider heat conduction in the vertical direction.

The line source theory can be refined in different ways, including the introduction of a boundary surface to the conduction domain. The temperature of this surface can be set at the undisturbed ground temperature, assumption which is reasonable also to describe a deep BHE field (H=100-200 meters). In the following a brief review of the most important contributions to the BHE conduction problem is presented.

Ingersoll et al. (1954) developed an analytical solution from the infinite line source (ILS) method to determine the thermal response factors for very simple BHE configurations. Later, Eskilson (1987) introduced the concept of g-function. To obtain the g-function of a borehole field, Eskilson solved the temperature field for a 2D case of a single borehole, in which the medium is limited by a top surface at zero temperature and constant heat flux is applied to the source. Spatial superposition is then performed to evaluate the temperature distribution at the BHE positions for the whole borehole field. Eskilson devoted an extensive research on both analytical and numerical solutions. However, the author inclined towards the finite difference solution for calculating g-functions. As mentioned before, the g-functions are nowadays implemented in commercial software, where the users can select the BHEs configuration and predict the response of the system for the specific energy requirements of the building. Even if current g-function libraries are quite comprehensive, the user cannot however specify a particular pattern of the borehole field in present commercial codes.

Zeng et al. (2002) proposed an analytical solution of the finite line source. This solution is a step forward with respect to the corresponding Eskilson one but it still involves important numerical issues, making it unattractive for practical applications. Recently, Lamarche and Beauchamp (2007) introduced a modification to the solution proposed by Zeng et al. (2002) which allows a faster calculation of finite length source temperature evolution. This solution is based on a mirror technique and on a double integration of the Fourier equation complete solution. The single FLS solution can be then superposed in space in order to calculate proper g-functions related to the particular geometry taken into account. In general, this approach allows the calculation of g-functions with values very similar to the ones calculated by Eskilson (1987). However, differences about 10% are found in the asymptotic part of the curve for large BHEs configurations. The difference in the boundary conditions applied in Eskilson (1987) and Lamarche and Beauchamp (2007) may explain these discrepancies in the g-function. Eskilson's (1987) solution considered a uniform wall temperature within all the boreholes when a constant heat flux is extracted from the borehole field, whereas the analytical solution in (Lamarche and Beauchamp, 2007) assumed a constant and uniform heat flux extraction at the wall in all the boreholes within the field.

In this paper, we developed a numerical model in accordance with the boundary conditions defined in (Lamarche and Beauchamp, 2007). Then, the g-function of the present borehole field is evaluated by calculating the average temperature of the boreholes along the time. The g-function can be related to the average borehole temperature as expressed in equation [1]:

$$T_{ave}(r_b) = \frac{\dot{Q}'}{2\pi k} \cdot g\left(\ln(9Fo_H), {}^{r_b}/_{H}, {}^{B}/_{H}\right) + T_{gr} \quad [1]$$

In equation [1], the Fourier number is used to define the non-dimensional time in terms of the borehole length when α is the thermal diffusivity of the ground. The Fourier number can be expressed as:

$$Fo_H = \frac{\alpha t}{H^2} \quad [2]$$

3. NUMERICAL MODEL

As a continuation of the work presented by Acuña et al. (2012), a numerical analysis is performed for a borehole field consisting of a squared pattern of 8x8 BHEs, including a number of improvements as compared to this earlier work where many simplifications were made. This section describes the new approach used for the numerical generation of the g-function.

3.1 The borehole field geometry

The geometrical characteristics of the borehole field are listed in Table 1.

Table 1 : Geometrical characteristics of the 8x8 borehole field

H (m)	100
r_b (m)	0.05
B (m)	5

The first simulations refer to the ground response when the borehole heat exchangers are thermally active from the top of the surface, as it is the case when boreholes are grouted -most borehole fields in central and south Europe. Moreover, in a closer approximation to real applications in north Europe, where groundwater filled boreholes with water table levels varying from 0 to several meters, and also to cope with some assumptions adopted by Eskilson (1987), a model was created according to the assumption that there is an adiabatic part at the top of the BHE. This condition was simulated by locating the heat sources in the Comsol model at a level below the ground top surface, as they were "buried" to a given depth of some meters. Eskilson did not specify clearly the buried depth of the borehole field in his studies. Our numerical model is built up for a buried depth of four and five meters. In Table 2, the study cases with regard to the buried depth of the borehole field are listed.

Table 2 : Study cases with regard to the buried depth of the borehole field

Study cases	D (m)
1	0
2	4
3	5

3.2 The heat transfer in the borehole field

The heat transfer problem is only accounted for the surrounding domain of the boreholes, i.e. not inside the borehole heat exchanger. Therefore, our system is simplified to one domain, which corresponds to the ground surrounding the boreholes. Then a tri-dimensional transient heat transfer problem is solved in this domain. To this aim, the following conditions are set up:

As an initial condition, the whole domain is supposed to be at an initial constant temperature. In our case, this temperature is set to T_{gr}=8°C, a typical value of the undisturbed temperature in southern Sweden.

In the surrounding boundaries of the borehole field, a temperature condition is set at the top surface and at boundary surfaces far enough from the borehole field. These boundaries are set in the radial direction and downwards of the borehole field, with a temperature corresponding again to an undisturbed level equal to T_{gr}.

Regarding the boundary condition at BHE surface, a constant and equal heat flux is imposed at the borehole wall. This study assumes a total heat flux of 6400 W, i.e. 1 W/m borehole. Thus, the heat transfer inside the borehole field is solved in agreement with the boundary conditions defined in the FLS-solution Lamarche and Beauchamp (2007), only differing from the fact that our geometry is fixed and the outer boundary (undisturbed ground) should actually move freely as the borehole field is loaded.

The numerical model has been solved according to different steps, which are explained in detail in chapter 4. The response of our numerical model is tested at each stage by comparing the g-function with the one from the analytical solution Lamarche and Beauchamp (2007) and the related superposition process for the 8x8 geometry. The numerical model allows generating the borehole temperature at every instant of time. By applying equation [1], the g-function is easily obtained for a known heat flux injection. It should be noted that for a borehole buried at a certain depth, the solution from the numerical model is verified with the FLS solution having the same buried distance, according to Table 2.

4. OPTIMIZATION STRATEGIES

A typical issue with numerical models is the reduction of the computational time. Since this problem applies in a severe way also to the present case, some strategies of optimization are applied here to speed up the solution of the problem. These strategies take into account the optimization of the geometry, the time step and the overall duration of the simulation.

4.1 Geometry optimization

First, the optimization strategies with regard to the geometry are applied. The borehole field geometry presented in Acuña et al, (2012) is reduced to a quarter of the total volume by considering symmetry in the system. Thus, an adiabatic condition is defined at the symmetry faces. To solve the heat transfer problem, the thermal properties are defined in this domain in accordance with typical values in the south of Sweden. Worth noticing that the derivation of the g-function values is not affected by the thermophysical data set employed for the calculation of the ground domain temperatures. These values are shown in Table 2.

Table 3: Thermal properties of the ground volume

c_p (J/kg K)	870
k (W/m K)	3.1
ρ (kg/m^3)	2300

Then, a parametric study is carried out with respect to the ground volume extension.

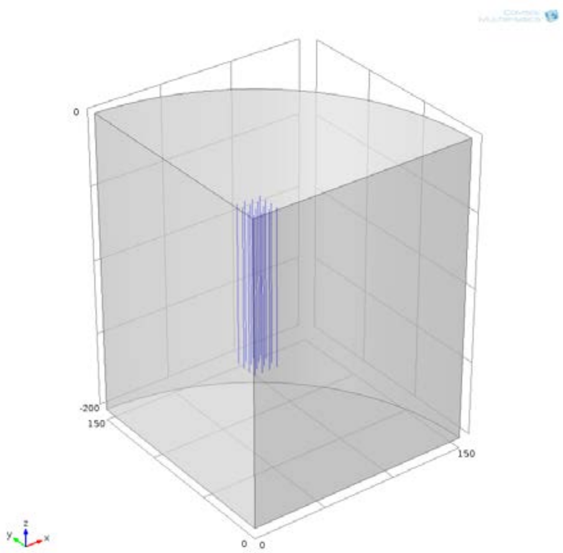

Figure 1: Computational domain and simplified geometry of the 8x8 borehole field

The volume has been increased in the radial direction and downwards the borehole field. The surrounding domain of the borehole field must be enlarged in such way that its volume is the minimum necessary to represent properly the thermal response of the borehole field. Thus, the heat transfer problem is solved by enlarging the ground volume until the solution does not change any more when the simulation time is selected in order to have $\ln(9Fo_H)=2$. This minimum volume was achieved using 150 m and 100 m to the side and downwards of the borehole field, respectively. The model generates a similar response to the one from the "minimum volume" when the ground volume increases up to 300 and 400 m in the radial and downwards the borehole field, respectively. Moreover, the numerical solution is compared with the analytical solution. Figure 1 shows a picture of a quarter of the borehole field and the final volume of its surrounding ground.

For a borehole field buried a depth of four or five meters, the size of the domain in our numerical model was increased until its response did not changed with the size of the ground volume. Therefore, a parametric study in relation to the ground volume is also carried out in case of a buried borehole field.

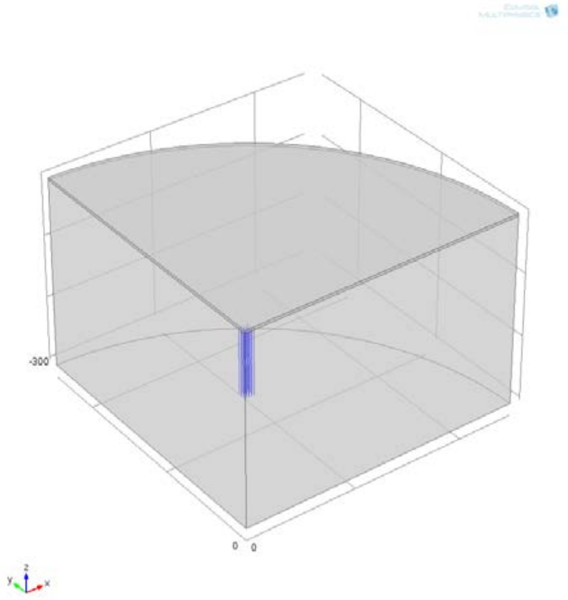

Figure 2: Computational domain and geometry of the 8x8 borehole field buried 4 meters in depth.

After a number of simulations, the minimum volume, in which the response from the numerical model does not change, is 500 and 200 meters in the radial direction downwards the borehole field, respectively, for values of $\ln(9Fo_H)$ around 2. The model presents a similar response to the one defined as "minimum volume" when the volume is enlarged to 500 and 300 meters to the side and downwards the borehole field, respectively. The numerical solution using a buried depth is compared with the FLS solution. The geometry for a buried borehole field is shown in Figure 2. This is a correct size choice, at least when the ratio D/H is equal to 0.04 or 0.05. Further work will be devoted to set a ground volume independent on the size, i.e. as an infinite element domain.

4.2 Simulation time optimization

When the strategies to optimize the geometry have been set up and verified to represent properly the response of the system for all the cases proposed in Table 2, a simulation time optimization is applied.

The simulation time optimization is carried out by modifying the thermal properties of the ground, since they affect the Fourier number which in turn is the independent variable of the g-function representation. Since the Fourier number is related to the thermal properties of the medium by means of the thermal diffusivity as expressed in equation [2], the overall duration can be decreased by modifying the thermal properties of the ground. This modification implies the increase of the thermal conductivity and the decrease of the specific heat and the density. The numerical model should represent the response of the system for values of the $\ln(9Fo_H)$ around 2, since this is the typical condition when a g-function (referred to any borehole field geometry) reaches its asymptotic value.

The starting point of the simulation time optimization is the part where the geometry is reduced to a quarter of the total volume and the thermal properties are those in Table 2. The thermal properties in Table 3 correspond to the values used in the first re-scaled problem study in this section. The end time of simulation can extend up to 200 years in the first investigation. In the second part of the analysis, which is performed using the values in Table 3, the time horizon is reduced to 35 years, again for $\ln(9Fo_H)$ values around 2.

Table 3: Thermal properties of the borehole field

c_p (J/kg K)	500
K (W/m K)	6
ρ (kg/m^3)	1500

In a third and final step, the time horizon is decreased to around 4 years thanks to the thermal property listed in Table 4.

Table 4: Thermal properties of the borehole field

c_p (J/kg K)	100
k (W/m K)	6
ρ (kg/m^3)	1000

In Figure 3, the study time is shown in relation to the three different thermal properties listed in Table 2, 3 and 4 against the $\ln(9Fo_H)$. However, it should be noted that the decrease of the end time of simulation implies a lower maximum time step in the solver. Therefore, a maximum time step of 120 days, 30 days and 10 days is set when considering the thermal properties of Table 2, 3 and 4, respectively.

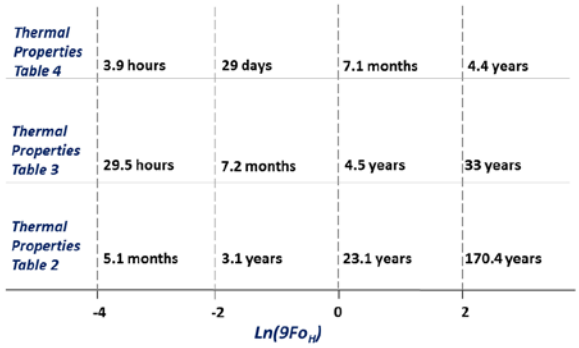

Figure 3: Strategy of optimization based on the Fourier number

In Table 5, the computing time is shown with respect to the thermal properties considered in each case. The computing times are similar for D equal to 4 and 5 m, so these are presented in a single column in Table 5 labelled as buried. The values presented in Table 5 correspond to the simulations run with a computer of 8 GB RAM with a processor Intel® Core ™ i7-860 CPU at 2.80 GHz.

Table 5: Optimization Strategy- Computing Time

Thermal Properties	*Time Stepping (days)*	*Computing Time (min)*	
		No Buried	*Buried*
Table 2	120	180	330
Table 3	30	150	240*
Table 4	10	70	120*

(*) the response is acceptable in a certain range of the $\ln(9Fo_H)$.

This optimization strategy shows that it is possible to decrease the computing time by increasing the thermal properties of the ground and setting proper time steps in the solver when the ratio is D/H=0. However, when a certain buried depth is considered this strategy is only valid for values of $\ln(9Fo_H)$ lower than about -1. For higher values of $\ln(9Fo_H)$, the numerical model was not able to properly represent the theoretical response of the borehole field. Consequently, this time optimization strategy is not applied in case of a buried borehole field. Further work will be devoted to this observation.

4.3 Mesh

The elements of the mesh play an important role in the results as well as in the simulation time. Thus, an optimization strategy is also applied to improve the mesh with respect to the one presented in Acuña et al, (2012).

Considering the geometry of Figure 1, where the borehole heat exchangers are not buried, the elements of the mesh are created by selecting a triangular mesh at the top face of the domain. Then, a swept is applied along the borehole length. Finally, the bottom part of the domain, under the boreholes, is meshed with tetrahedral elements. The domain consists of around 135256 elements in total. The mesh is created in such a way that small elements are chosen at the top and the bottom of the near region of the BHEs.

Since the geometry is different when having a buried borehole field, the mesh is constructed in a different way. A triangular mesh is set at the horizontal plane where the BHE are buried. The triangular elements are swept along the borehole depth. A free tetrahedral mesh is chosen in the upper part of the domain between the surface and the buried borehole field. The downward part of the borehole field is built in the same manner used in the non-buried case using tetrahedral free elements. The number of elements increases to around 440000 elements when the borehole field is buried. The volume above the borehole field, around the buried depth, consists of around 73000 elements.

5. RESULTS

Besides the results presented in section 4 concerning the optimization strategies, the resulting g-functions are presented here.

The g-functions obtained from the numerical model are labelled as Comsol for different ratios of D/H. These results are compared with those generated from the analytical FLS solution with proper superposition in correspondence to the same ratio D/H, referred as FLS, and those generated with the commercial software Energy Earth Designer, EED.

5.1 The borehole field is not buried

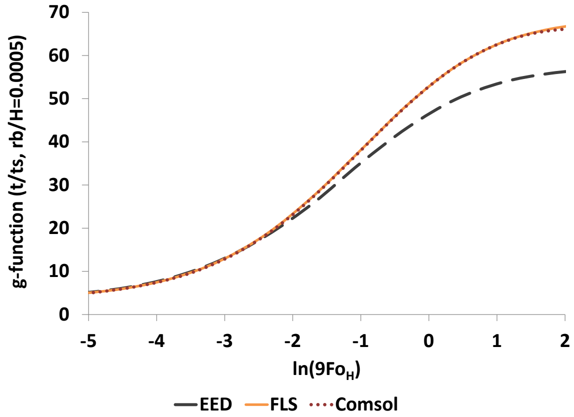

Figure 4: g-function generated with Comsol, the FLS method and EED, D/H=0

The g-function generated from the numerical model fits well with the solution obtained from the analytical solution, labelled in the graph as FLS. A small discrepancy can be appreciated between these in Figure 4 when the $\ln(9Fo_H)$ reaches values around 2. None of these g-functions is in good accordance with the one generated with EED, which possibly has to do with the different boundary conditions used in each of these approaches. Further work is being dedicated to generating a g-function showing better agreement with the Eskilson result.

5.2 The borehole field is buried 4 meters in depth

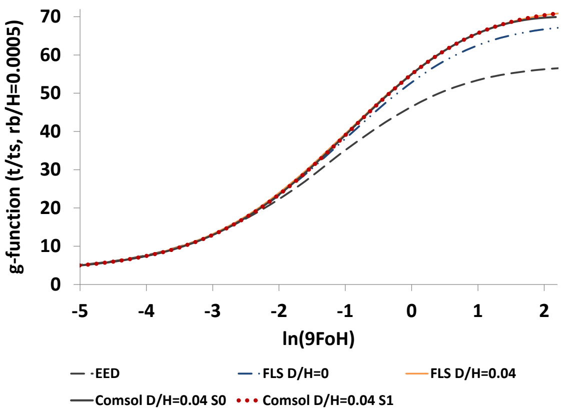

Figure 5: g-function generated with Comsol, the FLS method and EED, D/H=0.04

Figure 5 shows the g-function obtained from the numerical model when the surrounding volume is kept in the same size as in the initial case (Figure 1). This solution is labelled as *Comsol D/H=0.04 S0*, where S0 is referred to the initial volume dimensions. There is a slight better agreement between the Comsol and the FLS solution when the surrounding volume is increased (Figure 2). This solution is labelled as *Comsol D/H=0.04 S1*, where S1 makes reference to the increased surrounding. It can be observed that the two numerical solutions do not differ significantly and the percentage difference is below 1.5%. In Figure 5, the analytical solution when the borehole field is not buried is also included, *FLS D/H=0*. Comparing the solutions with respect to the buried depth, the g-function presents higher values when the borehole field is buried. This fact can be explained by recalling that all the models have a top surface boundary condition where undisturbed ground temperature is imposed. This condition cools down the ground and hence reduces the borehole field average asymptotic temperature, from which the g-function depends and it is calculated on. In the solution generated with EED, the buried depth is not well specified.

The comparison of the solution generated with EED with those from our numerical model and the analytical solution shows that Comsol and FLS functions agree well with each other, but differences with EED become higher when a certain buried depth is considered.

5.3 The borehole field is buried 5 meters in depth

Figure 6 shows the g-functions obtained when the borehole field has a buried depth of 5 m. By increasing the surrounding volume, the numerical solution presents a good agreement with the analytical approach for a similar buried depth. These solution are labelled as Comsol D/H=0.05 S1 and FLS D/H=0.05 for the numerical and analytical approaches, respectively.

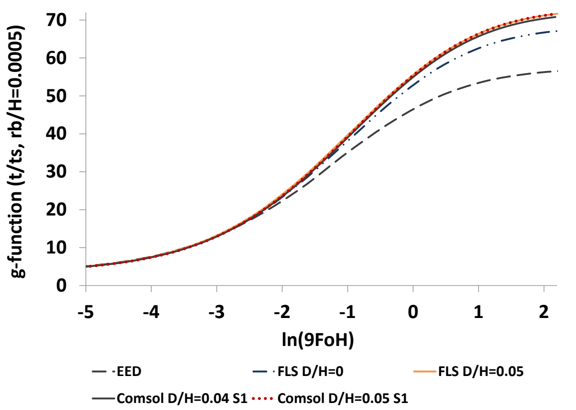

Figure 6: g-function generated with Comsol, FLS and EED, D/H=0.05

The FLS generated g-function for D/H =0.04 is also included in Figure 6.The g-function for a buried depth of 5 meters presents slightly high values than when a buried depth of 4 m is used, as could be expected from the considerations on the increasing distance from the imposed temperature top surface. This difference is more notable at the asymptotic part of the curve. Thus, the differences with EED solution become slightly higher than in the previous cases, which are also more relevant for a values of $\ln(9Fo_H)$ around 2.

6. CONCLUSIONS

A numerical model has been built in order to perform g-function calculation for complex BHE domains. In the present paper a 64 BHE square arrangement has been chosen. A constant heat flux is applied as a boundary condition on the borehole walls. The analyses have been performed by first assessing the effects of the dimensions of the calculation domain, mesh characteristics and time step. Once defined suitable calculation parameters, the model has been run to calculate the BHE field temperature transfer function (g-function) for a number of cases where the buried depth ratio D/H was changed while keeping other variables constant. The numerical g-functions been compared with the semi-analytical functions obtained by the FLS theory and proper superposition. The agreement was good, for all the D/H conditions here investigated. The comparison was extended to the g-function values that can be inferred from the commercial code EED. In this case both the numerical solution and the FLS one present a significant deviation from the EED solution, especially in the ln($9Fo_H$) range from -2 to 2. These differences with the EED solution become higher as the buried depth increases. Further investigation of the present research group will be addressed to assess and reduce these discrepancies, which may be ascribed to different boundary conditions.

ACKNOWLEDGEMENTS

Professor Louis Lamarche is acknowledged for his valuable support.

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