# Comparison of Multiple Load Aggregation Algorithms for annual hourly simulations of geothermal heat pumps

Marco Fossa and Elena Paietta

DIME, University of Genoa, via Opera Pia 15/A, 16145 Genova, Italy

e-mail: marco.fossa@unige.it

**Keywords:** BHE design, g-functions, hourly simulations, aggregation methods

## ABSTRACT

In most geothermal heat pumps installations, the vertical ground heat exchangers (BHE) represent the most important cost item and a careful analysis is needed to assure a long time performance together with the economical sustainability of the project. The most efficient way to predict the temperature evolution in time and space of a ground volume in contact with a system of BHE, is the recursive calculation of basic temperature response factors. Furthermore hourly load simulations along multiyear periods are considered the most reliable approach for simulating the thermal interaction between the ground and a system of BHEs. In this paper the Multiple Load Aggregation Algorithm is considered. In particular a detailed comparison with 5 year non aggregated data and an optimization analysis are performed in order to test and possibly enhance the original procedure. The proposal of enhancement is based on the introduction of an additional term, able to improve the accuracy of the aggregation method especially in case of non continuous building (and ground) heat load profiles, when the heat pump system is not operating for weeks or months.

## 1. INTRODUCTION

The ability to predict both the long-term and short-term behaviour of ground loop heat exchanger is critical for the design and energy analysis of ground coupled heat pump (GCHP) systems.

Heat could be extracted from a deep vertical borehole in the ground, where usually a single or double U pipe is inserted. The heat carrier fluid is circulated through the borehole and then returned, in heating mode, to the evaporator of the heat pump. The ground source can be used either for heating or cooling purposes. The diameter of the borehole is usually ranging from 0.09, to 0.15 meters.

In most applications the borehole heat exchangers (BHE) needed for the correct operations of the heat pump have to be more than one. The ground to BHE system interactions are quite complex, as it is well known: the long term cooling (or heating) of the ground around each borehole will influence the temperature at the surrounding boreholes. The heat extraction capacity of multiple boreholes will then be reduced in comparison with the single borehole, at the same specific transfer rate. This effect increases with time, heat transfer intensity, reduced space between the boreholes and it also depends on the ground properties.

The overall length of the BHEs (and hence their cost) and potential energy savings (with respect to traditional solutions) are the two main factors that establish the economic feasibility of a GCHP system. The required length of the boreholes can be determined by a number of methods based on the use of proper temperature transfer functions, able to describe the heat conduction problem in the ground. These methods include the Infinite Line Source solution (ILS), by Ingersoll et al. (1954), the Infinite Cylindrical Source solution by Carslaw and Jager (1947) and the family of the g-functions by Eskilson (1987). Some commercial codes are based on the exploitation of the properties of the g-functions, as the well known EED code.

EED like simulation tools need only few parameter for designing the BHE field; generally the approach is to refer to monthly buildings heat loads and to use seasonal average coefficients of performance (SPF).

The GCHP design process is more accurate if the simulations are made on a shorter time step base, typically on the hourly scale, while preserving a long term horizon. In such a way the fluid temperature to the heat pump can be evaluated at each time step (every hour), thus enabling a more accurate estimation of the SPF. Another important benefit of hourly analyses is the possibility to investigate and even implement sophisticated system control and operation strategies.

A number of researchers have used annual hourly simulations. Deerman and Kavanaugh (1991) used the cylindrical heat source (CHS) to predict heat pump entering water temperatures. This model is utilised in other studies like the one by Dobson et al. (1994): the cylindrical source is the solution for simulating a ground-coupled heat pump, with the cyclic behaviour of the GCHP determined by a thermal load model of the building structure. The most important hourly analysis is probably the DST one developed by Hellström (1991) and adapted for the first time in TRNSYS environment by Mazzarella (1993). The last DST TRNSYS version was updated by Hellström et al. (1996).

Different models have been compared for residential and commercial applications. Often DST was referenced as being the "benchmark", even if it was created for the simulation of big and regular ground storage and it is in principle unable to simulate layouts that differ significantly from compact rectangular spatial arrangements.

Shonder et al. (2000) used the DST benchmark in their comparison for commercial building applications.

Bernier et al. (2004) perform also another comparison between their Multiple Load Aggregation Algorithm (MLAA, Pinel 2003) and the duct storage model (DST) and concluded that the MLAA compare favourable well with DST for relatively small simulation periods in the selected test cases. In particular Bernier et al. (2004) demonstrated that long hourly series of thermal loads can be combined into few aggregated terms plus a number of non aggregated "recent" hourly loads. In the same study they optimized the aggregation period lengths with reference to given "continuous" heat load profiles.

Other strategies have been recently proposed for performing hourly analysis of ground response. Marcotte and Pasquier (2008) used the Fourier transform to solve the temporal superposition of heat extraction rates without any aggregation. The method allows simulation times over a period of 30 years with a reduced computational time but the drawback seems to be some lack of accuracy in predicting the time evolution of temperatures. Lamarche (2009) developed a semi-analytical method based on Duhamel's theorem. The author obtained a simulation time of the order of seconds for heavy simulation over a period of 10 years.

Cimmino et al. (2012) combine the use of g-functions with the method of the discrete Fourier transform for the simulation of BHE field behaviour. The temporal superposition is expressed as a convolution product and a spectral approach is used to simulate the evolution of the temperature of boreholes.

Although Fourier transform methods are a promising and quite effective solutions, these methods are not easy to be implemented and can be characterized by some lack of accuracy in the predicting detail of the time evolution of temperatures.

In this paper the Multiple Load Aggregation Algorithm is taken as a reference for hourly simulations of complex BHE arrangements based on multi-year hourly time series of building heat loads. In particular a detailed comparison with 5 year non aggregated data and an optimization analysis are performed in order to test and possibly enhance the original procedure. The proposal of enhancement is based on the introduction of an additional term, able to improve the accuracy of the aggregation method especially in case of non continuous building heat load profiles, when the heat pump system is not operating for weeks or months.

## 2. THEORETICAL BACKGROUND

The thermal interaction between the ground and a BHE arrangement, when underground water circulation can be neglected, is governed by the three-dimensional time-dependent conduction equation.

Due to its complexity the conduction equation is often solved numerically. However, a number of one-dimensional (in the radial direction) and two-dimensional (radial and axial) analytical solutions have been proposed, able to simulate the ground response to a single constant heat pulse. Combined with temporal superposition these solutions can be used to obtain the time varying solution to the heat transfer from BHE for any stepwise function describing the seasons.

Furthermore, spatial superposition allows quasi three-dimensional solutions to be obtained with relatively short computational time even for multi-annual hourly simulations.

Superposition techniques are often referred as hybrid models.

Ground Coupled Heat Pumps take advantage from the heat transfer between the ground and a fluid, typically water or water-glycol solution, which flows in pipes buried into the soil. In Figure 1 is shown a schematic representation of the problem.

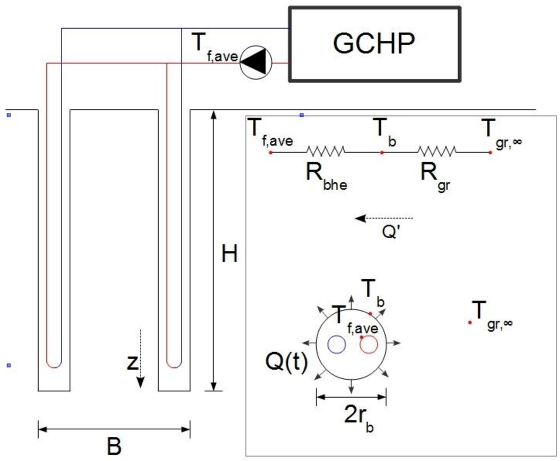


**Figure 1: Schematic representation of a GCHP system.**

The objective of any multiyear hourly simulation is to calculate the power required by the GCHP and by the circulating pumps. The success of these calculations depends on an accurate determination of the (average) fluid temperature leaving the ground $T_{f,ave}$.

This temperature depends on the configuration of the boreholes, on soil/grout proprieties and on total amount of energy (per unit time) rejected/absorbed in the ground, Q(t). The objective is to estimate accurate hourly values of $T_{f,ave}$ for a given history of hourly ground loads. In Figure 1 it is shown a vertical borehole system (H is the depth), its geometrical parameters and the two thermal resistance scheme frequently adopted in this kind of studies to describe the thermal interaction between the ground and BHEs.

Three temperature levels are involved: $T_{f,ave}$ the average fluid temperature; $T_b$ the borehole wall temperature and $T_{gr,\infty}$ the undisturbed ground temperature.

For a constant heat transfer rate to the ground, the borehole wall temperature at time t can be obtained using proper temperature transfer functions, say the g-functions solutions first proposed by Eskilson, (1987).

The model of Eskilson is based on the heat conduction equation and allowed a family of temperature transfer function (the g-functions) to be numerically calculated.

The following equation expresses the relationship between the borehole wall temperature $T_b$, the heat transfer rate per unit length Q' and the g-function pertinent to the BHE field consideration:

$$T_b = T_{gr,\infty} - \frac{Q'}{2\pi k_{gr}} g\left(9Fo_H, \frac{B}{H}, \left(\frac{r_b}{H}\right)_{ref}\right) \quad [1]$$

where $k_{gr}$ is the ground thermal conductivity. The g-function depends on dimensionless BHE spacing B/H and on dimensionless BHE radius $r_b$/H. $Fo_H$ represents the H based Fourier number. The ratio $r_b$/H was set in the original Eskilson work equal to the reference value of 0.0005. For other values of this dimensionless radius Eskilson suggested to correct the g-function according to the expression:

$$g(r_b/H)=g\,(r_b/H)_{ref} - \ln(r_b/r_{b,ref}) \quad [2]$$

In order to obtain $T_{f,ave}$ it is customary to neglect the thermal capacitance of the borehole. Under this steady-state assumption, the fluid temperature is given by:

$$T_{f,ave} = T_{gr,\infty} - Q' R_{bhe} - \frac{Q'}{2\pi k_{gr}} g\left(9Fo_H, \frac{B}{H}, \left(\frac{r_b}{H}\right)_{ref}\right) \quad [3]$$

where $R_{bhe}$ is the effective steady-state borehole thermal resistance.

The strength of the temperature response factors is that they can be employed to describe stepwise varying heat transfer loads by the way of the temporal superposition.

Unfortunately temporal superposition of a multiyear hourly series of thermal loads, is very computationally demanding, since the number of mathematical operations is very high ($10^{10}$ for 20 years of simulations). Yavuzturk and Spitler (1999) proposed to aggregate the hourly heat pulses into a single one when they are “far in time” from the current time step and they suggested to keep a meaningful number of hourly pulses for the “recent” period. They hence proposed the first aggregation algorithm able to perform hourly superposition while keeping the calculation time at a convenient level.

Pinel (2003) and Bernier et al. (2004) refined the aggregation concept introducing their MLAA algorithm. The method is based on a description of the 87600 hourly series (for a time horizon of 10 years) with just 16 terms, 12 of which are hourly “recent” pulses and other 4 are aggregated terms “far in time” (yearly load, monthly load, weekly load and daily load). The Authors tested and validated their method by performing non aggregated simulations for periods of 6 months.

The MLAA modified (hereafter, MLAA17 as in Fossa and Minchio, 2013) proposed in this paper is merely an extension of the original MLAA algorithm, where an additional aggregated term named “semestral load” has been introduced in the superposition scheme. The series of thermal loads to be superposed in time is described in the scheme of Figure 2.

In this paper a detailed comparison with 5 year non aggregated data and an optimization analysis are performed in order to test and possibly improve the original MLAA. The additional term in MLAA17 is aimed at better coping with yearly profiles of hourly thermal loads characterised by periods of no heat loads to the ground. In fact, Fossa and Minchio (2013) have demonstrated that the original MLAA algorithm can yield to some (small) errors in predicting the hourly temperature evolution if the heat load series is not continuous, say for example when it is describing the heat load history of a GCHP system working only in winter. To overcome this problem in the above paper the Authors demonstrated that a slight change in the MLAA original method, could improve the original algorithm. No optimum analysis in the above paper on MLAA17 had been performed.

Worth outlining, MLAA and its modified version are expected to yield the same temperatures for reference continuous hourly profiles.

In this paper the MLAA17 concept is addressed to the optimum search of best duration of the additional “semestral” aggregated period.

The remaining terms of the aggregation procedure are the same as in the original MLAA.

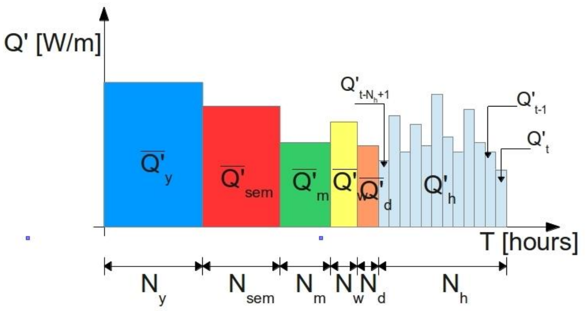


**Figure 2: Schematic representation of MLAA17 aggregation method.**

The non-aggregated part (hourly loads) is set to $N_h$ hours, where $N_h$ is 12. The "past" thermal history is subdivided in several periods of aggregation, as in the original Pinel's work.

Constant ground loads are assumed to prevail over a given time interval. For example, $Q'_t$ is the hourly ground load (heat transfer rate per unit length) prevailing during the period of time from t-1 to t hour. Ground loads in the "recent" thermal history, from $Q'_{t-Nh+1}$ to $Q'_t$ are not aggregated and they are the same as in the input complete hourly series.

$\overline{Q}'_d$ on the contrary is an aggregated load obtained as the average of all thermal loads of the last day, e.g. between the start of $N_y+N_{sem}+N_m+N_w+1$ hours at the end of $N_y+N_{sem}+N_m+N_w+N_d$ hours. Every periods, except the yearly one, has a fixed length (in hours) $X_i$. The detail of new aggregation scheme are very similar to the original MLAA ones and they are described in Table 1.

**Table 1: Evolution of multiple load aggregation algorithm according to Pinel scheme.**

| Time [hours] | $N_y$<br>$t-N_{sem}-N_m-N_w-N_d-N_h$ | $N_{sem}$ | $N_m$ | $N_w$ | $N_d$ | $N_h$ |
|---|---|---|---|---|---|---|
| $1<t<X_h$ | 0 | 0 | 0 | 0 | 0 | t |
| $X_h<t<X_h+X_d$ | $t-X_h$ | 0 | 0 | 0 | 0 | $X_h$ |
| $X_h+X_d<t<X_h+X_d+X_w$ | $t-X_h-X_d$ | 0 | 0 | 0 | $X_d$ | $X_h$ |
| $X_h+X_d+X_w<t<X_h+X_d+X_w+X_m$ | $t-X_h-X_d-X_w$ | 0 | 0 | $X_w$ | $X_d$ | $X_h$ |
| $X_h+X_d+X_w+X_m<t<X_h+X_d+X_w+X_m+X_{sem}$ | $t-X_h-X_d-X_w-X_m$ | 0 | $X_m$ | $X_w$ | $X_d$ | $X_h$ |
| $X_h+X_d+X_w+X_m+X_{sem}<t$ | $t-X_h-X_d-X_w-X_m-X_{sem}$ | $X_{sem}$ | $X_m$ | $X_w$ | $X_d$ | $X_h$ |

With reference to Figure 2 and Table 1 the MLAA17 scheme (again: very similar to that of Pinel) is described by the formula:

$$\theta_t = \frac{1}{2\pi k_{gr}}\Big(\overline{Q}'_{y,t}(A-A_1)+\overline{Q}'_{sem,t}(A_1-B_1)+ +\overline{Q}'_{m,t}(B_1-C)+\overline{Q}'_{w,t}(C-D)+\overline{Q}'_{d,t}(D-F_1)+ +Q'_{t-Nh+1}(F_1-F_2)+Q'_{t-Nh+2}(F_2-F_3)+ +\ldots+Q'_{t-1}(F_{Nh-1}-F_{Nh})+Q'_{t-1}(F_{Nh-1}-F_{Nh})+ +Q'_t(F_{Nh})\Big) \quad [4]$$

where $\overline{Q}'_{i,t}$ is the generic average thermal load at the related aggregation period, $Q'_j$ are the hourly non aggregated ground loads and the terms A, $A_1$, B, C, D and $F_i$ are calculated from g-function values at different times as:

$A=g(t)$
$A_1=g(t-N_y)$
$B_1=g(t-N_y-N_{sem})$
$C=g(t-N_y-N_{sem}-N_m)$
$D=g(t-N_y-N_{sem}-N_m-N_w)$
$F_1= g(t=N_h)$
$F_2= g(t=N_h-1)$ … $F_{Nh} = g(t=1)$ [5]

To sum up, the MLAA17 is hence composed by six distinct time intervals (aggregation periods) representative of the "far in time" heat loads plus $N_h$=12 (hourly values). Now the objective is to determine the best time length $X_{sem}$ (number of hours) in order to obtain the best estimates of the borehole or fluid temperatures with reference to a calculation made by the temporal superposition without any aggregation (5 year non aggregated).

## 3. VALIDATION AND OPTIMIZATION

The optimum search is done by comparing the results from simulations at different aggregation periods $X_{sem}$ to the results from simulations with a non aggregated superposition scheme. The time horizon selected for this analysis was one and five years.

Regarding the g-function employed for calculations, it refers to a rectangular borefield of 4x4 boreholes (16 boreholes, as in Fossa and Minchio, 2013). However, the present optimum analysis has a general validity which not depend on the g-function adopted. Table 2 shows the input data used for parameter refinement of the MLAA17 model.

In order to enlarge the test cases with respect to the original Pinel work, four ground load profiles have been used for the present optimum analysis. They are shown in Figures 3, 4, 5 and 6. Thermal loads profile (A) and profile (B) are depicted respectively in

Figures 3 and 4 and they refer to real building needs as calculated by TRNSYS simulations (again Fossa and Minchio, 2013). Profile (A) and (B) are the same concerning the positive heat load values (winter operations) but profile (B) has no summer heat loads. On the other hand, building of profile (A) is characterized by a cooling demand after an intermediate period during which the the heat pump is off. The third profile (profile (C), Figure 5) is the "synthetic" symmetric and continuous profile which is the original one adopted by Pinel (2003) when developing the MLAA algorithm and for the refinement of the aggregation period lengths. The synthetic profile is described by the following mathematical expressions:

$$Q(t) = q_1 \cdot q_2 + (-1)^{FL} \, | q_1 \cdot q_2 | + c_4 \cdot (-1)^{FL} \cdot SN \qquad [6]$$

where $q_i$, $c_4$, FL and SN are constants that can be found in the original paper by Pinel (2003).

The forth profile (Figure 6) is again a synthetic profile and it is the same of Figure 5 except for the fact that there is an interruption in the heat loads during the months from March to April and from August to September.

**Table 2: Input data used for parameter refinement of MLAA17 model.**

| Parameter | Values used for borefield |
|---|---|
| Number of boreholes | 16 (4x4) |
| Borehole depth, H | 100 m |
| Borehole diameter, $2r_b$ | 0.1 m |
| Distance between borehole, B | 5 m |
| Ground thermal conductivity, $k_{gr}$ | 2 W/mK |
| Thermal diffusivity of the ground, $\alpha_{gr}$ | 1.00 E-006 $m^2/s$ |

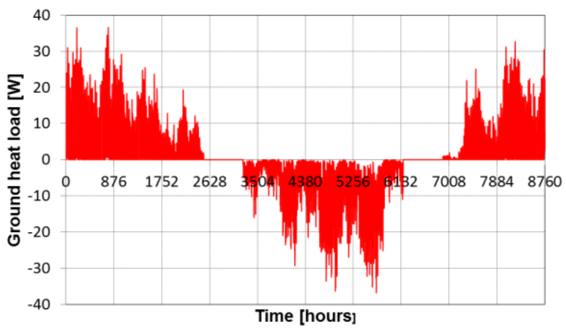


**Figure 3: Heat load profile (A) versus time.**

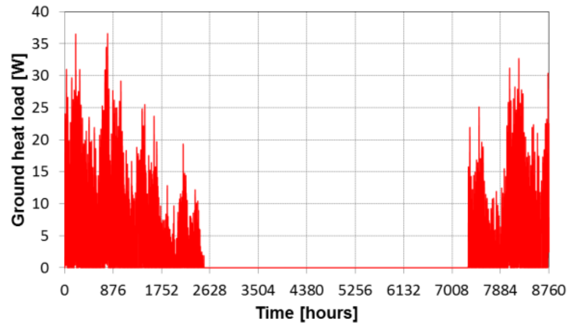


**Figure 4: Heat load profile (B) versus time.**

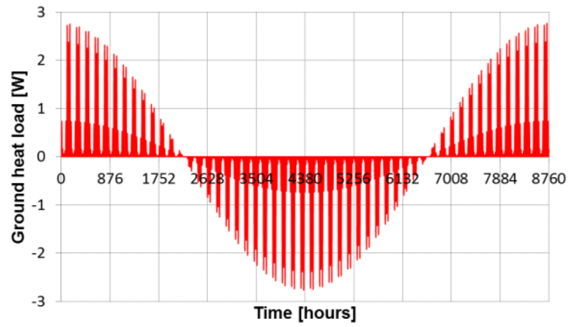


**Figure 5: Synthetic heat load profile (C) versus time.**

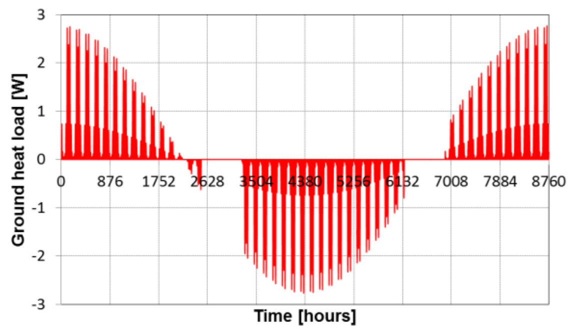


**Figure 6: Synthetic heat load profile (D) versus time.**

The comparisons and optimum search for the $X_{sem}$ duration were done after implementing a complete (non aggregated) superposition scheme and by calculating the excess temperatures with the MLAA17 model with different durations of the semestral period.

Differently from Pinel, here the optimum search and related error analysis is performed not only for single year simulations but also for 5 year ones. Furthermore the optimization is done by comparing results from simulations performed using a five years non aggregated superposition while, Pinel has done the same optimization just using a non aggregated period of 4900 hours.

Figures 7, 9, 11, 12 show the maximum difference in fluid temperatures as resulted from simulations using different $X_{sem}$ aggregation periods and with reference to the non aggregated simulations. Four different semestral lengths have been used, (720, 1440, 2880, 4320) for each of the eight test cases (4 heat profiles, 2 time horizons). Worth outlining, the aggregated period lengths of the original MLAA method have not been changed. In all the following figures where $X_{sem}$=0 applies, the original MLAA results are reported.

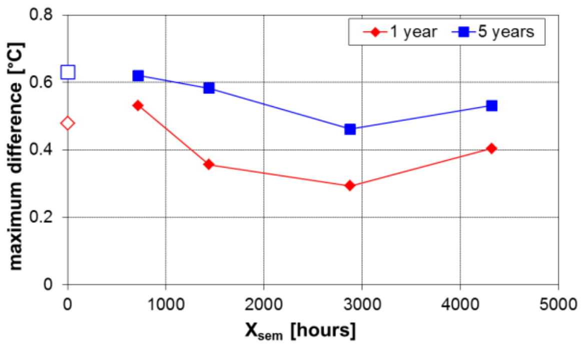


**Figure 7: Maximum temperature difference for heat load profile (A). Aggregated (17 terms) versus non aggregated.**

Figure 7 refer to heat profile (A) and shows the maximum difference in fluid temperatures for fixed periods of 360, 168, 48, and 12 hours for $X_m$, $X_w$, $X_d$, and $X_h$ respectively. Either the single year or the 5 year analysis exhibits a similar trend, with a minimum on error at $X_{sem}$=2880 hours. The improvement with respect to the original MLAA method is very small (lower than 0.2°C) as can be also noticed from the analysis of Figure 8, where the standard deviation of differences (again with respect to the non aggregated reference case) is depicted. Again the best value of $X_{sem}$ is 2880 hours.

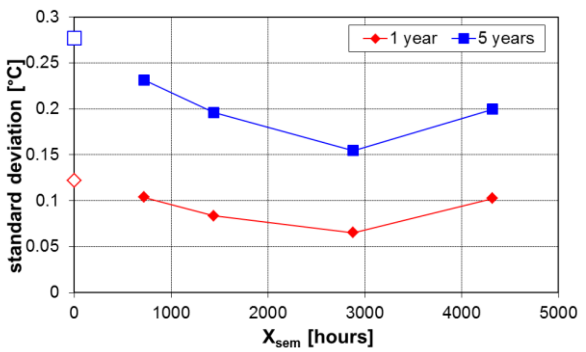


**Figure 8: Standard deviations of temperatures for heat load profile (A). Aggregated (17 terms) versus non aggregated.**

Figures 9 and 10 refer heat load profile (B). Figure 9 represent the maximum temperature differences while Figure 10 is the representation of the standard deviations of differences. The $X_{sem}$ best length again resulted 2880 hours, but again the improvement with respect to the original MLAA is reduced to some 0.2°C, for the five years simulation.

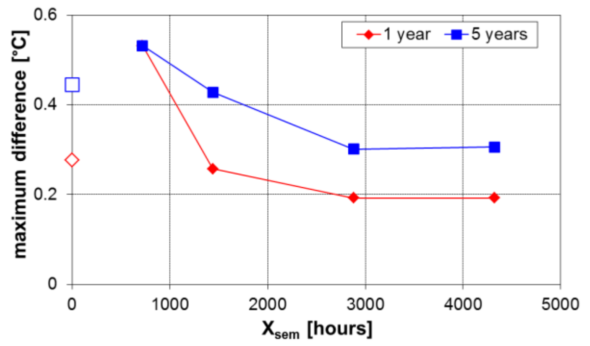


**Figure 9: Maximum temperature difference for heat load profile (B). Aggregated (17 terms) versus non aggregated.**

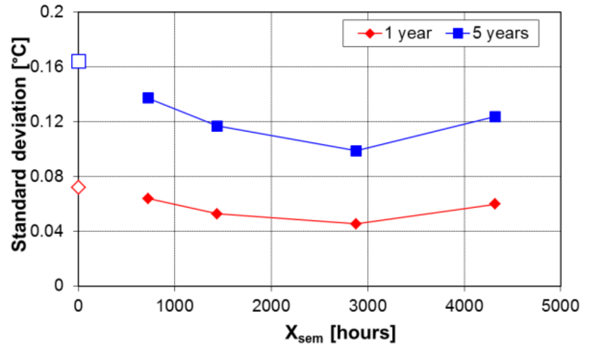


**Figure 10: Standard deviations of temperatures for heat load profile (B). Aggregated (17 terms) versus non aggregated.**

Figures 11 and 12 refer to the "synthetic" heat load profile (C), with reference to maximum temperature differences and related standard deviations respectively. Figures 13 and 14 are the corresponding figures with reference to load profile (D).

It can be observed that for both cases (C) and (D) the improvement due to the MLAA17 additional terms is irrelevant as it could be expected. Again the best $X_{sem}$ length resulted 2880 hours, but very similar results have been obtained for example with $X_{sem}$=1440 or for the original MLAA.

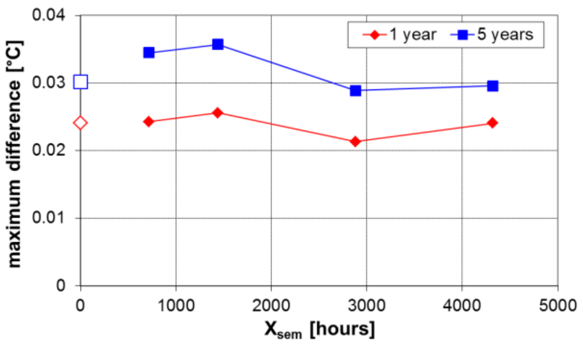


**Figure 11: Maximum temperature difference for heat load profile (C). Aggregated (17 terms) versus non aggregated.**

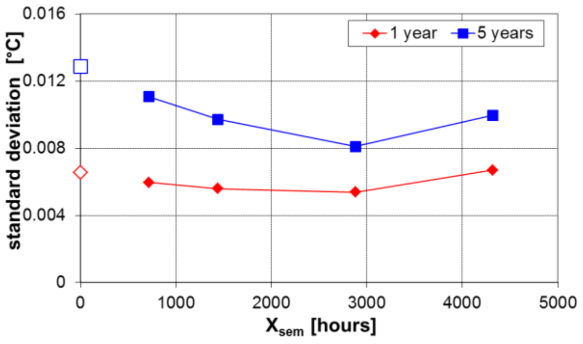


**Figure 12: Standard deviations of temperatures for heat load profile (C). Aggregated (17 terms) versus non aggregated.**

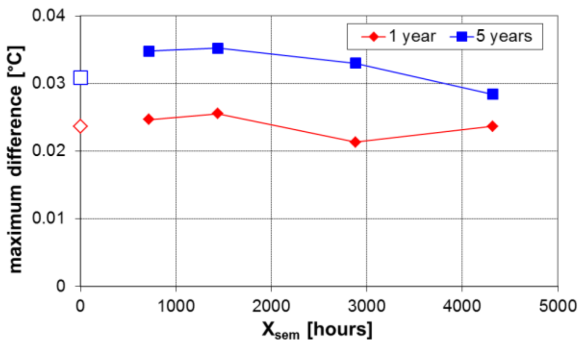


**Figure 13: Maximum temperature difference for heat load profile (D). Aggregated (17 terms) versus non aggregated.**

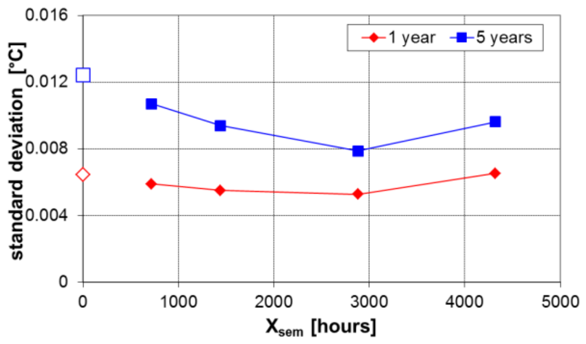


**Figure 14: Standard deviations of temperatures for heat load profile (D). Aggregated (17 terms) versus non aggregated.**

In order to draw a general comment, the improvement of the MLAA17 approach is minimum and it applies to non continuous heat load (e.g. cases A and B). On the other hand the comparisons among hourly methods for GCHP simulations (e.g. Spitler et al., 2009) are sometimes done by comparing differences in results lower than the degree Celsius, and in this direction the present study can represent a contribution to the problem. In addition the present analysis have been able to test the MLAA (original and modified versions) with reference to long (5 years) series of non aggregated hourly data, condition not considered in the original MLAA papers.

## 4. CONCLUSIONS

In this paper the techniques for aggregating heating loads for hourly simulations of ground coupled heat pumps have been presented and discussed. In particular the attention was focused on the MLAA method employed with proper precalculated g-functions.

A detailed comparison with 5 year non aggregated data and an optimization analysis have been performed in order to test the original method (originally built with reference to ”short” half annual non aggregated time series) and possibly to enhance the original procedure. The proposal of enhancement was based on the introduction of an additional term, able to improve the accuracy of the aggregation method especially in case of non continuous ground heat load profiles. Four different ground load profiles have been considered.

The comparison among non aggregated (5 years) and aggregated temperature data showed that the original MLAA method and the modified one are both in very good agreement (maximum differences of the order of 0.5°C) with reference case. Some improvement have been demonstrated to pertain to the modified aggregation method, when the additional term length is set to 2880 hours and the heat load series are discontinuous in time, such as in those practical cases when the heat pump is off in the intermediate season.

## NOMENCLATURE

| | |
|---|---|
| A, $A_1$, B, C, D: | g-function at different periods for the MLAA17 |
| B: | borehole spacing |
| $c_4$: | constant utilised for the determination of the symmetric profile |
| $F_i$: | g-function at different time for the MLAA17 i = 1, 2,…$N_h$ |
| FL: | constant utilised for the determination of the symmetric profile |
| $Fo_H$: | Fourier number in base H |
| g: | g-function analytical solution |
| H: | borehole depth |
| $k_{gr}$: | ground conductivity |
| $N_i$: | number of hours in aggregation period (i=h, d, w, m, sem or y) |
| Q: | heat rate |
| Q': | heat rate per unit length |
| $\overline{Q}'_{i,t}$ : | aggregated ground loads for different periods i=d, w, m, sem or y |
| $Q'_j$: | hourly (j= t-$N_h$+1...t hours) non-aggregated ground load at time j |
| $q_i$: | constant utilised for the determination of the symmetric profile i = 1, 2 |
| $r_b$: | borehole radius |
| $R_{bhe}$: | BHE thermal resistance |
| $R_{ground(t)}$: | ground thermal resistance |
| SN: | constant utilised for the determination of the symmetric profile |
| T: | temperature |
| $T_b$: | borehole wall temperature |
| $T_{f,ave}$: | mean fluid temperature at the borehole outlet |
| $T_{gr,\infty}$: | undisturbed ground temperature |
| t: | time |
| $X_i$: | pre-fixed numbers of hours in aggregation period i ( i=h, d, w, m or sem) |
| z: | Cartesian coordinate |

**Greek letters**

| | |
|---|---|
| $\alpha$: | ground thermal diffusivity |
| $\theta$: | difference of temperature between the borehole wall and the undisturbed ground |

**Subscripts**

| | |
|---|---|
| d: | aggregation period of the order of a day |
| h: | immediate thermal history |
| m: | aggregate period of the order of a month |
| sem: | aggregate period of the order of a semester |
| w: | aggregate period of the order of a week |
| y: | aggregate period of the order of a year |

## REFERENCES


Bernier M., Pinel P., Labib R. and Paillot R.: A Multiple Load Aggregation Algorithm for Annual Hourly Simulations of GCHP Systems, *HVAC&R*, Research V.10 N. 4, (2004), 471-487

Carslaw H.S. and Jager J.C.: Conduction of Heat Solids, *Claremore Press*, Okford U.K., (1947)

Cimmino M. and Bernier M.: A contribution towards the determination of g-functions using the finite line source, *Elsevier*, Amsterdam, (2012)

Cimmino M., Bernier M. and Pasquier P.: Utilisation des g-functions de Eskilson pour la simulation de systèmes géothermiques, *Proceeding of the Canadian Conference on Building Simulation 2012*, Halifax Nova Scotia, Canada, (2012), 283-295

Deerman J.D. and Kavanaugh S.P.: Simulation of vertical U-tube ground coupled heat pump systems using the cylindrical heat source solution, *ASHRAE Transactions*, 97, (1991), 297-295

Dobson M.K., O'Neal D.L. and Aldred W.: A modified analytical method for simulating cyclic operation of vertical U-tube ground-coupled heat pumps, *Energy Systems Laboratory,* Texas A&M University, (1994)

Eskilson P.: Thermal analysis of heat extraction boreholes, *Dep. of Mathematical Physics University of Lund*, Lund (1987)

Fossa M., Cauret O. and Bernier M.: Comparing the thermal performance of ground heat exchangers of various lengths, *Effstock Conference*, Stockholm, Sweden, (2009)

Fossa M.: The temperature penalty approach to the design of borehole heat exchangers for heat pump applications, *Elsevier*, Energy and Buildings 43, Amsterdam, (2011), 1473-1479

Fossa M. and Minchio, F.: The effect of borefield geometry and ground thermal load profile on hourly thermal response of geothermal heat pump systems, *Energy*, (2013), 323-329

Hellström G.: Ground heat storage, *Dep. of Mathematical Physics University of Lund*, Lund, (1991)

Hellström G., Mazzarella L. and Pahud D.: Duct Ground Heat Storage Model Lund DST. TRNSYS 13.1 Version January 1996, *Department of Mathematical Physics*, University of Lund, Sweden, (1996)

Ingersoll L.R., Zobel O.J. and Ingersoll A.C.: Heat Conduction with Engineering, Geological, and other Applications, *McGraw-Hill*, New York (1954)

Lamarche L. and Beauchamp B.: A Fast Algorithm for the Simulation of GCHP Systems, *ASHRAE Transactions*, Career and Technical Education 113, (2007), 470-476

Lamarche L. and Beauchamp B.: A new contribution to the finite line-source model for geothermal boreholes, *Elsevier*, Energy and Buildings 39, Amsterdam, (2007), 188-198

Lamarche L.: A fast algorithm for the hourly simulations of ground-source heat pumps using arbitrary response factor, *Elsevier*, Renewable Energy 34, Amsterdam, (2009), 2252-2258

Lamarche L.: Analytical g-function for inclined boreholes in ground-source heat pump systems, *Elsevier*, Geothermics 40, Amsterdam, (2011), 241-249

Marcotte D. and Pasquier P.: Fast fluid and ground temperature computation for geothermal ground-loop heat exchanger systems, *Elsevier*, Geothermics 37, Amsterdam, (2008), 651-665

Mazzarella L.: Duct Thermal Storage Model Lund DST. TRNSYS 13.1 Version 1993, *ITW*, University of Stuttgart, Germany, (1993)

Pinel P.: Amelioration, validation et implementation d'une algoritme de calcul pour evaluer le transfert termique dans les puits verticaux de systemes de pompes a chaleur, *Dep. De Génie Mécanique École Polytechnique de Montréal*, Montréal, (2003)

Shonder J.A., Baxter V.D., Hughes P.J., and Thornton J.W.: A Comparison of Vertical Ground Heat Exchanger Design Software for Commercial Applications, *ASHRAE Transaction*, 106 (1)N. 4, (2000), 831-842

Spitler J., Cullin J., Bernier M., Kummert M., Cui P., Liu X. et al. : Preliminary intermodel comparison of ground heat exchanger simulation models, *Effstock Conference*, Stockholm, Sweden (2009)

Yavuzturk C.: Modelling of vertical ground loop heat exchangers for ground source heat pump systems, *College of the Oklahoma University*, Oklahoma, (1999)

Yavuzturk C. and Spitler J.D.: A Short Time Step Response Factor Model for Vertical Ground Loop Heat Exchangers, *ASHRAE Transactions*,105, (1999), 475-485.