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Mathematical optimization of borehole heat exchanger fields under variable conditions

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ABSTRACT

Borehole heat exchangers (BHE) are often applied in multiple BHE fields. In current planning practice, interaction between adjacent BHEs is rarely considered and all BHEs are operated in the same mode. This means, potential adverse effects from superimposed cold or heat plumes, which simultaneously evolve around individual neighbouring BHEs, are neglected. Long-term heat extraction over decades, however, may lead to significant local cooling, especially in the interior of the field. As a consequence, the performance of the complete ground source heat pump (GSHP) system is attenuated, and ground temperatures below regulation thresholds may develop. In our work, we employ mathematical optimization techniques to strategically operate and arrange BHEs in such fields. Linear programming and an evolutionary algorithm are applied in combination with analytical equations to solve realistic problems. The presented methodology is flexible, robust and it can be applied to various conditions. The two scenarios studied in this paper represent conditions with negligible and significant groundwater flow. We inspect a field with 36 BHEs, which has a seasonally variable heating energy demand. It is demonstrated, by taking the maximum temperature decline in the ground as objective, that the BHE field performance can be improved both by case-specific ideal arrangement and time-dependently regulated individual BHE operation. It is found that instead of standard lattice arrangements, optimized geometries are favourable, with BHEs concentrated along the fringe of a field. Apparently, this enhances lateral conductive heat provision into the field. Groundwater flow means additional energy provision by advection towards the field.

1. INTRODUCTION

The combined use of multiple BHEs in large-scale low-enthalpy geothermal applications is common practice, for example to supply district heating systems or big office buildings. Multiple simultaneously operating BHEs can access the ground over a larger area than single BHEs, and it is not required to drill deep boreholes, which may be costly and/or face regulative barriers (Sanner et al. 2003, Katsura et al. 2009, de Paly et al. 2012). A common approach is to arrange the BHEs in a lattice, line(s) or L-shape with sufficient distance between them (e.g., Fuji et al. 2005, Koohi-Fayegh and Rosen 2012, Choi et al. 2013). For example, Signorelli et al. (2004) recommend a distance of 7 – 8 m, which is considered adequate to avoid substantial interaction among neighbouring installations. However, even smaller distances of around 5 m are chosen in practical applications (e.g., Fossa and Michio, 2013).

BHEs are commonly operated for a long time. Mostly, an annually balanced operation with the same amount of heat extracted as injected is not achieved. When full regeneration is not possible, permanent thermal anomalies develop and grow in the ground (Rybach and Eugster 2010, Zanchini et al. 2012). Their size mainly depends on the thermal conditions of the ground, potential hydrogeological factors, intensity of heat extraction, and operation mode. If, as is often the case, the prevailing application of a BHE field is for heating, the ground is being cooled. In the long term, the thermal anomalies from individual BHEs expand and potentially overlap. Even with a distance of several meters, such interaction could be relevant, as conductive heat supply to the individual BHEs is mitigated. With unchanged heat extraction rates, further local temperature decline in the field could become significant (Hecht-Mendez et al. 2013, Beck et al. 2013). Strong cooling of the ground is not desirable, as an altered ground(water) temperature influences the seasonal performance factor of the heat pump(s), and for ecological reasons it is often not even allowed. In several countries, regulations set or recommend minimum temperature thresholds for ground and groundwater (Haehnlein et al. 2012).

Our goal is to find a strategy that minimizes the local cooling of the ground. For a given volume of the ground, this can be either achieved by applying different heat extraction rates to the BHEs in the field, or by positioning the BHEs in an optimal way. In the following, we will first present a methodology, which integrates superimposed BHE line source models in an optimization framework. The objective is to achieve a balanced cooling from the ground by avoiding extreme temperatures, and to improve system performance and to promote sustainable operation this way. Then based on our previous work (de Paly et al. 2012, Beck et al 2012, 2013, Hecht-Mendez et al. 2013), we inspect the optimization potential of two scenarios with 36 BHEs.

2. METHODOLOGY

2.1 Analytical simulation

The temperature evolution, T, in the ground during BHE operation is modeled by the infinite moving line source model (Sutton et al. 2003, Diao et al. 2004, Molina-Giraldo et al. 2011). This analytical solution of heat transport is a fast and efficient means to simulate simplified homogeneous conditions with conduction and advection. The thermal effect from heat extraction is described as temperature change ($\Delta T = T_u - T$) in comparison to undisturbed conditions with natural background temperature, T_u. For a specific heat extraction rate (or load) per unit borehole length, q, we obtain for a single BHE at time t at distances Δx and Δy:

$$\Delta T(\Delta x,\Delta y,t,q)=\frac{q}{4\pi L\sqrt{\lambda_u\lambda_v}}\ exp\left(\frac{\rho_w c_w n_p v_a \Delta x}{2\lambda_u}\right)\times \int_0^b exp\left[-\left(\frac{\Delta x^2}{\lambda_u}+\frac{\Delta y^2}{\lambda_v}\right)\times\frac{(\rho_w c_w n_p v_a)^2}{16\lambda_u\Theta}-\Theta\right]\frac{d\Theta}{\Theta} \quad [1]$$

where:

$$b=\frac{(\rho_w c_w n_p v_a)^2 t}{4\rho_m c_m \lambda_u}$$

The effective thermal conductivity in the longitudinal and transverse direction is given by λ_u and λ_v. Parameters c_w and c_m are the volumetric heat capacities of water and porous matrix, and ρ_w and ρ_m are the densities; n_p is the porosity and v_a the horizontal groundwater flow velocity. Eq. [1] is linear with respect to q assuming that the thermal and hydraulic properties of the ground are independent from the temperature. This allows spatial superpositioning and we obtain for n BHEs in a borehole field:

$$\Delta T_{i,j}(t,q_{k=1,\ldots,n})=\sum_{k=1}^{n}\Delta T_k(i-x_k,j-y_k,t,q_k) \quad [2]$$

The total temperature change at location (i, j) is the sum of all changes ΔT_k by each BHE k at position (x_k, y_k), with an energy transfer rate q_k. Energy extraction varies during the year, typically in a seasonal mode and higher demand during cold winter periods. To describe time dependency of q, m time steps with constant q_l for time step l, with $q_0 = 0$ and $t_0 = 0$ are distinguished for each BHE. By spatial and temporal superposition, the temperature change by operation of multiple BHEs can be calculated by (de Paly et al. 2012):

$$\Delta T_{i,j}(t,q_{k=1,\ldots,n,1,\ldots,m}) = \sum_{l=1}^{m}\sum_{k=1}^{n} q_{k,l}\,\omega_{k,l}^{t,i,j}(i-x_k,j-y_k) \quad [3]$$

with

$$\omega_{k,l}^{t,i,j}(\Delta x,\Delta y)= \frac{1}{4\pi L\sqrt{\lambda_u\lambda_v}}\ exp\left(\frac{\rho_w c_w n_p v_a \Delta x}{2\lambda_u}\right)\times \left(\int_0^{b_{l-1}} exp\left[-\left(\frac{\Delta x^2}{\lambda_u}+\frac{\Delta y^2}{\lambda_v}\right)\times\frac{(\rho_w c_w n_p v_a)^2}{16\lambda_u\Theta}-\Theta\right]\frac{d\Theta}{\Theta} - \int_0^{b_l} exp\left[-\left(\frac{\Delta x^2}{\lambda_u}+\frac{\Delta y^2}{\lambda_v}\right)\times\frac{(\rho_w c_w n_p v_a)^2}{16\lambda_u\Theta}-\Theta\right]\frac{d\Theta}{\Theta}\right) \quad [4]$$

with $b_{l-1}=\frac{(\rho_w c_w n_p v_a)^2 t_{l-1}}{4\rho_m c_m \lambda_u}$, $b_l=\frac{(\rho_w c_w n_p v_a)^2 t_l}{4\rho_m c_m \lambda_u}$.

Coefficient $\omega_{k,l}^{t,i,j}$ is the response factor of BHE k on (i, j) at time step $l\in\{l,\ldots,m\}$, in relation to the current time step $t\in\{l,\ldots,m\}$ and $l\le t$. In vectorized form, Eq. [3] reads

$$\Delta\vec{T}_{i,j}(t,\vec{q})=\vec{q}(\vec{\omega}^{t,i,j})^T \quad [5]$$

with

$$\vec{\omega}^{t,i,j}=(\omega_{1,l}^{t,i,j},\ldots,\omega_{n,1}^{t,i,j},\ldots,\omega_{n,m}^{t,i,j})$$
$$\vec{q}=(q_{1,1},\ldots,q_{n,1},\ldots,q_{n,m}).$$

2.2 Multiple BHE scenarios

We demonstrate the combined simulation-optimization approach for a hypothetical BHE field (Beck et al. 2012). In total, 36 BHEs are arranged on a regular grid on a 50x50 m square (Fig. 1). Accordingly, a grid distance of 10 m between neighboring BHEs is chosen, which is several meters above thresholds we find in common practice and recommendations. The depth per BHE is set fixed to 100 m. The field is operated with time-dependent loads to supply a given and annually constant seasonal energy demand over an operation time of 10 years (Fig. 2). For this demonstration example, only two periods are subdivided per year, but this could be easily expanded and a finer time-discretization could be used (e.g. Hecht-Mendez et al. 2012).

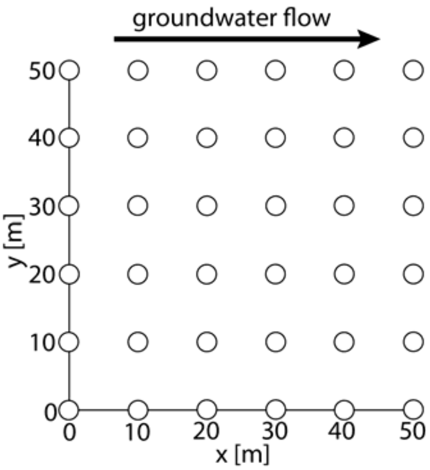

Figure 1: Lattice arrangement of 36 BHEs on regular grid with a step size of 10 m.

Two scenarios are distinguished, oriented at realistic conditions (Beck et al. 2012):

(1) Conduction-dominated case: The BHEs are operated in water saturated ground with low groundwater flow velocity of $v_a = 2.4 \cdot 10^{-10}$ m/s. Such conditions reflect a silt/clay dominated ground, and accordingly we specific the other thermal parameters as follows: volumetric heat capacity is $2.4 \cdot 10^6$ $Jm^{-3}K^{-1}$, thermal conductivity is 1.7 $Wm^{-1}K^{-1}$, and porosity is 46%.

(2) Advection-domination case: The BHEs are installed in a sand aquifer with substantial groundwater flow of $v_a = 2.0 \cdot 10^{-7}$ m/s. Volumetric heat capacity here is $2.6 \cdot 10^6$ Jm^{-3} K^{-1}, thermal conductivity is 2.4 W $m^{-1}K^{-1}$, and porosity is 30%.

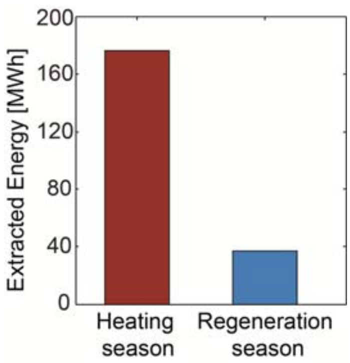

Figure 2: Constantly extracted energy during winter (heating) and summer (regeneration) season for each year of operation.

2.3 Optimization problem formulation and solution procedure

We choose the temperature change of the ground as the optimization criterion. The purpose is to keep the induced thermal anomalies as small as possible in order to maximize the heat pump efficiency and to avoid extreme local cooling in the field. This is implemented in the optimization procedure by minimization of the strongest cooling, simulated by Eq. [4]:

$$min\left[max(\Delta T_{i,j})\right] \qquad [6]$$

The sum of all loads $q_{k,l}$ has to match the energy demand E_l for each time step l:

$$E_l = \sum_{k=1}^{n} q_{k,l} \quad l = 1,...,m \qquad [7]$$

Four reference points at distance of 0.5 m around each BHE k at position (x_k, y_k) are defined to measure the ground temperature changes.

There are two ways to optimize the field. Either individual loads of the BHEs are separately adjusted (de Paly 2012, Hecht Mendez et al. 2013) or their positions are optimized (Beck et al. 2011, 2013). Load optimization is possible by linear programming, and consequently, this is applied for minimization of the maximum temperature change during each time step l, combining Eqs. [5] and [6]:

$$argmin\left(w \cdot max(\Delta \vec{T}_{i,j}(t,\vec{q}) + \sum_{l=1}^{m} max(\Delta \vec{T}_{i,j}(l,\vec{q})) \right) \qquad [8]$$

$$\forall (i,j,t) \in S$$

where S is the set of all spatial and temporal reference points, defined by (i,j,t); w is introduced as weighting factor to assign a high priority to the primary objective (Eq. [5]) by $w = 100$.

A general formulation is possible by introducing virtual variables $z_{0,...,m}$. The max-norm terms of the combined objective expressed by Eq. [6] can be formulated as m+1 linear programs, which minimize $z_{0,...,m}$. For every z, the linear program reads

$$min(z)$$

$$\Delta \vec{T}_{i,j}(l,\vec{q}) - z\vec{e} \prec 0$$

$$-\Delta \vec{T}_{i,j}(l,\vec{q}) - z\vec{e} \prec 0 \qquad [9]$$

with $\vec{e}$ being the unity vector. With the linear relationship between $\Delta \vec{T}$ and q, respectively between $\Delta \vec{T}_{i,j}$ and $\vec{q}$, the linear optimization problem can be stated as:

$$min\left(w\cdot z_0+\sum_{l=1}^{m}z_l\right) \qquad [10]$$

subject to

$$\Delta\vec{T}_{i,j}(t,\vec{q})-z_0\prec 0$$

$$-\Delta\vec{T}_{i,j}(t,\vec{q})-z_0\prec 0$$

$$\Delta\vec{T}_{i,j}(l,\vec{q})-z_l\prec 0$$

$$-\Delta\vec{T}_{i,j}(l,\vec{q})-z_l\prec 0 \qquad [11]$$

$$\forall(i,j,t)\in S$$

with l = 1,…,m and $z_{0,\ldots,m}$. and q as optimization variables. The linear program is constrained by the energy demand for each time (Eq. [6], see Figure 2). As only heating is considered, the loads are positive:

$$q_{k,l}\geq 0 \quad l=1,\ldots,m \text{ and } k=1,\ldots,n \qquad [12]$$

Aside from load optimization, another option is to adjust the positions of the BHEs. Since the temperature $\vec{T}_{i,j}$ at a position (i,j) is non-lineary dependent on the position (x_k, y_k) of a borehole, a non-linear optimization method is needed. For this objective, we selected a heuristic method, Differential Evolution (Storn and Price 1997). This is an established evolutionary algorithm, and we chose standard settings with population size of 30, algorithm-specific tuning parameter F = 0.8, and cutoff probability of C_r = 0.7. This variant revealed to be favorable to alternative evolutionary global search methods (Beck et al. 2012).

3. RESULTS

The results are depicted in Figures 3 and 4. In Figure 3, we show the maximum temperature changes for the two different scenarios. For scenario 1, with small groundwater flow velocity, the same equal load operation would result in a stronger maximum temperature decline. ΔT of 9.75 K is computed for this variant, whereas the relative temperature change reaches only ΔT = 5.6 K for scenario 2. This reflects the more pronounced additional advective heat component, which is characteristic for higher groundwater flow velocities and which promotes seasonal regeneration.

It is demonstrated that any mathematical optimization mitigates the maximum temperature change. Applying linear programming for load optimization only means a substantial reduction to ΔT = 7.25 K and 3.52 K for scenarios 1 and 2, respectively. This is a relative improvement of around 30%. When only positions are adjusted by Differential Evolution, the resulting BHE arrangements perform better than the grid shaped BHE field. However, for both cases, load optimization is favourable to position optimization. The boxplots also show that repeated applications of the heuristic algorithm generate slightly variable results. This reflects that Differential Evolution is a stochastic search procedure and needs to be employed several times to arrive at a reliably good solution. In contrast to linear programming, the stochastic search is computationally more demanding and does not guarantee to find the global optimum. Even so, experience with this and alternative evolutionary algorithms indicates that the detected "optima" are close to the global optimum (Beck et al. 2012).

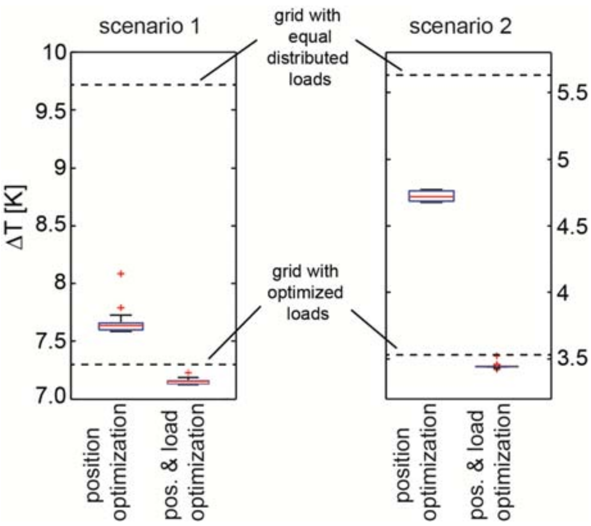

Figure 3: Results for the two scenarios in comparison to default case with lattice arrangement and equal loads. Both boxplots show the resulting maximum temperature changes for exclusive position optimization, load optimization, and combined position/load optimization.

Combining position and load optimization in a sequential procedure turns out to be the best choice, which exploits all degrees of freedom. However, as shown in Figure 3, the improvement potential is only minor. Apparently, by concentrating on the loads only, already potentially satisfactorily close-optimal solutions are generated.

The best solutions for the two scenarios are illustrated in Figures 4 and 5. BHEs tend to concentrate on the fringe of the given area, where lateral heat conduction is not insulated by adjacent BHEs (see also Beck et al. 2012, 2013). As expressed in the objective function, strong local thermal anomalies are avoided and a balanced cooling within the BHE field is achieved. For the second scenario, cooling is less pronounced due to stronger advection. What is also shown in Figure 5 with substantial groundwater flow, the thermal plume mainly generated during the winter moves downstream. This means advection does not only provide additional energy, but also moves cooled regions away from the interior of the BHE field. This accelerates regeneration but may be critical for neighbouring geothermal applications in the downstream.

3. CONCLUSIONS

With the two scenarios, we demonstrated the potential of combined simulation-optimization methods to properly adjust positions and/or extraction rates of individual boreholes in multiple BHE fields. This is shown for conditions with mainly conduction and negligible groundwater flow, as well as for conditions with pronounced advection.

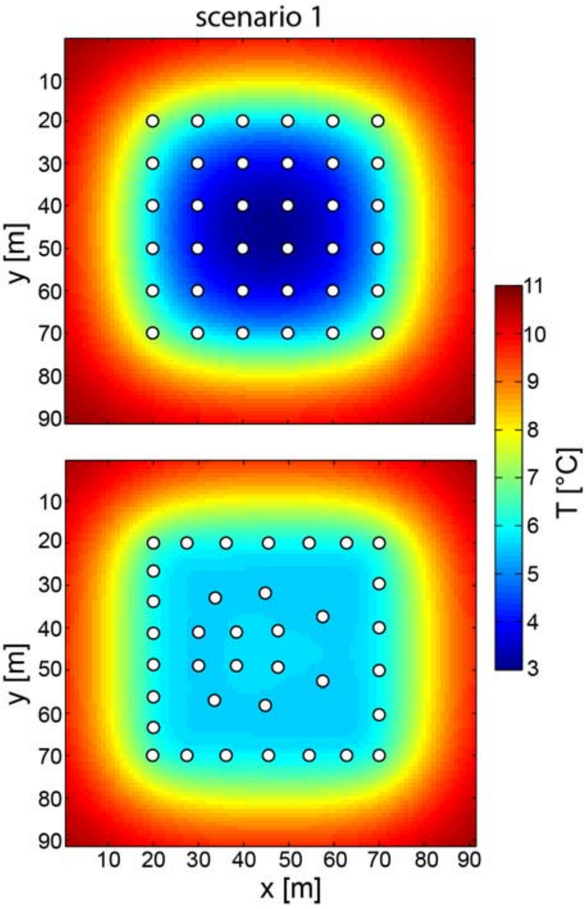

Figure 4: Scenario 1: Absolute temperatures for position optimized BHE fields with groundwater flow from left to right (90 m x 90 m area). Top figure shows equal loads, and bottom one shows load optimized results.

The selected scenarios are idealized examples, but clearly show that mathematical optimization can be employed to achieve a more balanced heat extraction from the borehole field than is typical for standard application without BHE regulation. The non-regulated reference case with equal loads, however, only approximates conditions in practice. For given total extraction rates, the BHE-specific extraction rates typically are not the same. In fact, by equal flow rates of the heat carrier fluids in the BHEs, heat extraction will depend on the local temperature gradient at each borehole. This can be interpreted as a self-regulating mechanism which automatically mitigates extreme local cooling. Still, further work in this direction (de Paly et al. 2012, Hecht-Mendez et al. 2013, Beck et al. 2013) with numerical models shows that such equal flow conditions are commonly far from being optimal. Using load-based assessment and equal loads as proxy thus, is a viable and computationally efficient workaround. Future work will concentrate on real-case applications and integration of combined heating and cooling modes.

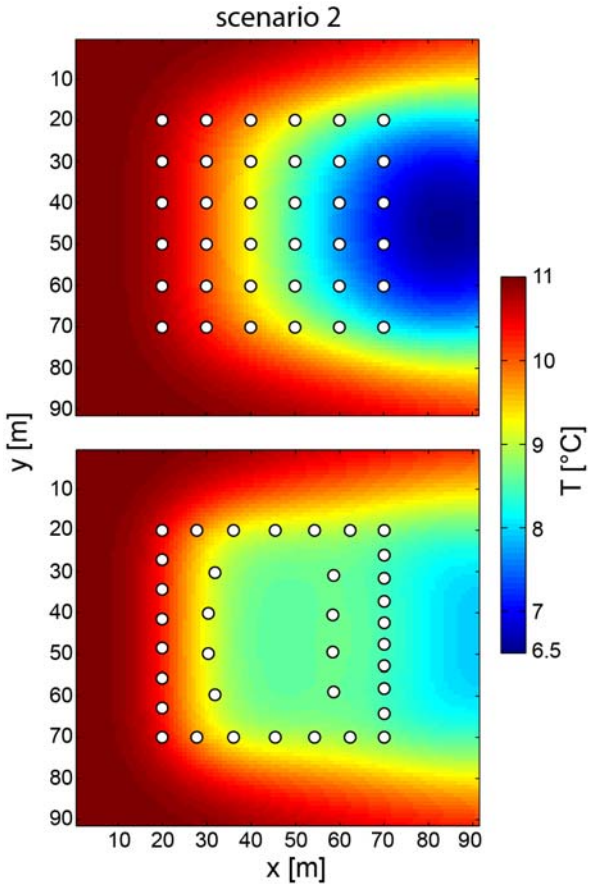

Figure 5: Scenario 2: Absolute temperatures for position optimized BHE fields with groundwater flow from left to right (90 m x 90 m area). Top figure shows equal loads, and bottom one shows load optimized results.

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