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Analytical and numerical models for determining geothermal energy potential

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ABSTRACT

Comparison of analytical theories on heat extraction from hot dry rock (HDR) has been presented in the literature by various authors..

In this paper a physical model, it is proposed to analyze the phenomenon of heat exchange between water and hot dry rock as a homogeneous and isotropic impermeable medium. Cold water enters a vertical fracture from below, extracts heat from the rock, during the ascent of through the fracture. The equations governing the heat exchange are the conduction equation in the rock, and convection in the fluid. The study of this phenomenon, is conducted in two parts, a single fracture through the rock, and the case of multiple fractures with an infinite series of parallel fractures with equal distance. In addition to comparing the analytical solutions in the literature, the comparison was extended with numerical solutions solved by numerical methods based on reverse – Laplace transform, and the resulting solutions from software models built on finite element method (FEM). These solutions, are given in dimensionless terms, describing the trend of the temperature as a function of time, and in the case of multi – fracture as a function of the spacing between the fractures themselves.

Finally, a comparison between single fracture, and multi – fracture is presented, to show that in the case of multi – fracture the system is much more efficient for heat extraction than in the case of single fracture, for the same flow rate, as the multi – fracture presents a greater surface area of heat exchange between rock and fluid.

1. INTRODUCTION

The intense development of residential and industrial activities increased in recent decades, consuming mainly fossil fuels, resulting in climate change, greenhouse effect and environmental pollution.

World Climate Conferences and agreements signed by all the countries, have lead to an agreement in reducing greenhouse gas emissions, improving the efficiency of systems and processes, developing new technologies and diversifying as much as possible renewable energy sources.

Among the energy sources, geothermal energy is considered sustainably affordable, non-polluting, and renewable. It can be exploited for power generation, as well as for district heating.

The simple concept behind the Hot Dry Rock (HDR), is to exploit the thermal energy of the rock underground, heated over time by the presence of magma below.

The rock transfers heat to the water, which flows through the fractures of the rock, which are formed by the hydraulic pressure of the water, injected artificially.

In the zones of thermal anomaly, where faults occur, or in volcanic areas, the water temperature reaches high values of temperature, such as to provide hot water, steam, hot springs, fumaroles.

In HDR systems, two wells are installed below the ground, allowing the extraction and the re - injection of hot water (cooled by enthalpy exchange in a turbine and/or in a heat exchanger).

Many studies have been conducted by various authors, who presented the first simple theories of heat exchange between water and rock (single fracture), and then the generalized phenomenon of multi - fractures and randomly fractured rock, where a parameter that identifies the density of fractures of the rock is necessary.

The principal authors who worked on this field are:

- Carslaw and Jaeger (1959);
- Lauwerier (1955);
- Bodvarsson (1969 - 1970 - 1972 - 1974);
- Gringarten et al . (1975).

2. PRINCIPAL THEORIES ON THE EXTRACTION OF HEAT FROM THE ROCK THROUGH THE HEAT EXCHANGE BETWEEN THE ROCK AND WATER

A simplified solution of the problem was carried out by Carslaw and Jaeger (1959). Afterwards Lauwerier in 1955 modelled the heat transfer of a hot fluid injected into a fracture of thin, porous medium containing petroleum high density, in order to increase extractability of oil, reducing its viscosity. The mathematical study of Carslaw and Jaeger was resumed by Bodvarsson as well in his various studies (1969 - 1970 - 1972 - 1974), to quantify the extraction of heat from a hot dry rock, in the simple case of a single fracture. Similarly Gringarten et al . (1975) has extended the problem from a single rock to a rock with multiple vertical fractures, parallel and at the same distance. The model of Gringarten is much closer to reality, as in a geothermal reservoir rock multiple fractures are present in order to achieve a high heat exchange surface, assuring the heat throughout the life cycle of the geothermal reservoir.

2.1 Carslaw and Jaeger (1948 – 1959)

The study of the extraction of heat in a single fracture, was conducted initially by Carslaw and Jaeger (1959) based on the following equations (Figure 1):

$$\dot{m} c \frac{\partial T}{\partial y} = 2\lambda \left(\frac{\partial T_r}{\partial x}\right)_0 \qquad [1]$$

$$\lambda \frac{\partial^2 T}{\partial y^2} = (\rho c)_r \frac{\partial T_r}{\partial t} \Rightarrow a \frac{\partial^2 T}{\partial z^2} = \frac{\partial T}{\partial t} \qquad [2]$$

$$\frac{T_r - T_{(y,t)}}{T_r - T_{W0}} = erfc\left[\frac{\lambda H}{\dot{m} c \sqrt{at}}\right] \qquad [3]$$

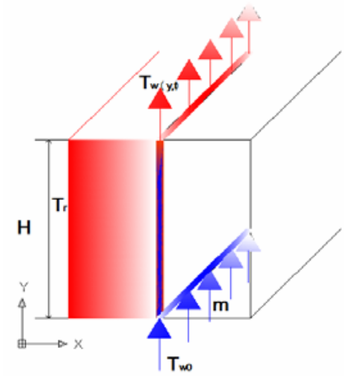

Figure 1: Heat extraction in a single fracture

2.2 Lauwerier (1955)

Lauwerier proposed a mathematical model for the injection of hot water in a porous medium saturated with oil by studying the problem in partial differential equations.
The hypotheses considered are: uniform thickness, permeability and porosity of the reservoir, constant flow, infinite conductivity in the direction normal to the flow, absence of axial conduction, thermal equilibrium between water and rock. The energy balance equation used by Lauwerier is:

$$b\rho_1 c_1 \frac{\partial T_w(x,t)}{\partial t} + b\rho_w c_w v \frac{\partial T_w(x,t)}{\partial x} - \lambda_r \left(\frac{\partial T_r}{\partial y}\right)_{y=b} = 0 \qquad [4]$$

The general equation for the heat conduction of the rock in the HDR is:

$$\lambda_r \frac{\partial^2 T_r}{\partial y^2} = \rho_r c_r \frac{\partial T_r}{\partial t} \qquad [5]$$

2.3 Bodvarsson (1969 - 1970 – 1972 – 1974)

The study conducted by Bodvarsson (1969) defines a simple theoretical model for the evolution of the temperature of the water passing through a single fracture of impermeable rock. The assumptions made by the author are:

- Model fracture consisting of a flat open space of constant width *h* between two large blocks of homogeneous, isotropic, impermeable rock;
- The plane $y = 0$ coincides with one of the two edges of the fracture, with the y-axis oriented towards the adjacent rock;
- The *x*-axis follows the development of the fracture;
- Along the fracture, a constant mass flow per unit depth *z* of the fracture;

- Temperature of the rock $T\ (x, y, t)$ does not depend on z;
- The temperature range is symmetrical with respect to the fracture and the width h is small in order to consider constant the temperature of the fluid; one considers then the temperature of the fluid equal to that of the rock in $y = 0$;
- Transport of heat by conduction and convection in the rock along the x axis in the fracture;
- Absence of conduction along the x-axis.

The temperature field in the rock can be expressed by means of the heat equation:

$$\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad [6]$$

The effect of convective transport is considered as a boundary condition on the surfaces of the fracture:

$$\rho_w c_w b \frac{\partial T}{\partial t} + c_w q \frac{\partial T}{\partial z} = 2\lambda \frac{\partial T}{\partial y} \quad at \quad y = 0 \qquad [7]$$

where:

ρ_w [kg/m^3] = water density

c_w [J/(kg K)] = Specific heat of the water

b [m] = width of the fracture

q [kg/(s m)] = specific mass flow rate along the fracture

λ [W/(m K)] = thermal conductivity of the rock

An interesting case study is presented by Bodvarsson considering constant flow and sinusoidal temperature change, assuming the temperature T=Aexp($i\omega t$) in x =0, y = 0, where A is an arbitrary, real amplitude.

The solution of the dimensionless temperature, is equal to:

$$T_D = A\, erfc\left[\frac{(\alpha x + y)}{\sqrt{\alpha t}}\right] \qquad [8]$$

2.4 Gringarten, Witherspoon and Ohnishi (1975)

The authors have presented a more complete model on the extraction of heat from HDR, starting from the analytical solution of Carslaw and Jaeger and from Bodvarsson. They extend the problem of heat transfer from single fracture to multi - fracture, as, in general, a HDR geothermal reservoir presents multi – fracture. This allows, assuming the same flow rate, to increase the heat exchange surface, allowing to extract water at higher temperatures over the time, thus increasing the life of the reservoir. (see Figure 2).

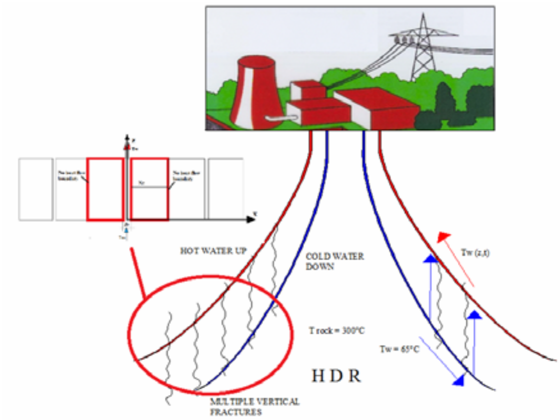

Figure 2: Scheme of a multi-fracture HDR.

The assumptions underlying the model are:

- linear model;
- infinite vertical fractures parallel and equidistant;
- width of fractures are uniform;
- rock is homogeneous, isotropic and impermeable;
- width of fracture are negligible compared to the distance between fractures.
- density and specific heat of rock and fluid are constant;
- constant thermal conductivity of the rock;
- volume flow of the fluid is constant;
- the water temperature $T_w\ (z, t)$ is uniform in each section of the fracture and for each height z equal to the temperature of the rock on the edge $x = b$;
- no conduction in the vertical direction of the fracture and in the rock; all the heat is transferred by conduction horizontal by into the rock and by forced convection in the fracture;
- initially, both the water in the fracture and the rock are at the same temperature; this temperature is not uniform along the fracture but it depends on z. At a given height z, the initial value of the temperature is calculated is: $T_{ro}' = T_{ro} - z\omega$ where T_{ro} is the temperature of the rock at the injection point and ω is the temperature gradient of the rock [K / m] ;

- no flow of heat is exchanged with the contour at a distance $x = xE + b$.

The differential equation that governs the temperature of the water is obtained by writing the thermal balance of an element of fracture with a volume $dV = dz * db * 1\ m^3$.

The heat equation in the rock is described as:

$$\frac{\partial^2 T_r(x,y,t)}{\partial x^2} = \frac{\rho_r c_r}{\lambda_r}\frac{\partial T_r(x,y,t)}{\partial t} \qquad [9]$$

The heat transfer between water and rock assumes pure convection of water and pure conduction in the rock:

$$b\rho_w c_w\left[\frac{\partial T_w(y,t)}{\partial t} + v\frac{\partial T_w(y,t)}{\partial y}\right] = 2\lambda_r \left.\frac{\partial T_r(x,y,t)}{\partial x}\right|_{x=b} \qquad [10]$$

Where:

ρ_w [kg/m^3] = water density

c_w [J/kg K] = water specific heat

b [m] = fracture width

T_w [K] = water temperature

t [s] = time;

v [m/s] = water velocity

λ_R[W/m K] = rock thermal conductivity;

T_R [K] = rock temperature.

The equations of the model studied by Gringarten for the single fracture can be expressed as:

$$T_{wD}\left(t_D^{'}\right) = 1 - 2\beta\left(\frac{t_D^{'}}{\pi}\right)^{0,5}\left[1-\exp\left(-\frac{1}{4t_D^{'}}\right)\right] - (1-\beta)erf\left[\frac{1}{\sqrt{t_D}}\right] \qquad [11]$$

Where β is dimensionless parameter of the geothermal gradient.

For multi – fracture and single fracture the equation for $\beta = 0$ can be written as:

$$\bar{T}_{wD}(y_D, s) = \frac{1}{s}\exp\left(-y_D s^{0,5}\tanh\frac{\rho_w c_w Q x_E}{2\lambda_r H}s^{0,5}\right) \qquad [12]$$

These two equations can be solved by means of both Papoulis (1957) and Gaver – Stehfest (1979) methods.

3. APPROACH TO THE STUDY BY MEANS OF THE FEM MODEL

The finite element method (FEM), is a suitable numerical technique to find approximate solutions of boundary value problems, and the initial values described by partial differential equations, reducing them to a system of algebraic equations.

The problem domain is discretized to form a grid (mesh). On each element (triangles, quadrilaterals, tetrahedra, hexahedral), the solution is assumed to be a linear combination of basic functions or shape functions.

Given a strong formulation in the edge of the domain, and assigned to the conditions, they may be of the type:

- *Neumann condition:* the derivative of the function (flow) takes on values imposed on the edge of the domain;
- *Dirichlet condition:* the solution (temperature) takes on values imposed on the edge of the domain;
- *Condition of Robin:* it imposes a link between flow and temperature on the edge of the domain.

The model used here is COMSOL Multiphysics which considers heat equation for conduction in solids and both conduction and convection in fluids. Heat equation in the rock:

$$\rho_r c_r \frac{\partial T}{\partial t} = \nabla(\lambda_r \nabla T) \qquad [13]$$

General equation of conduction and convection in the fluid:

$$\rho_w c_w \frac{\partial T}{\partial t} + \rho_w c_w v \nabla T = \nabla(\lambda_w \nabla T) \qquad [14]$$

4. RESULTS

The size of the rock and fracture are those considered by the example of Harlow and Pracht (1972) and Gringarten et al. (1975): rock size is 1000 m height and 1000 m depth; the other data are listed in table 1.

In the case of single fracture flow rate is equal to Q = 0,145 [m^3/s]; in the case of multiple fracture N = 10 fractures were considered, where in each fracture Q_N = Q/N = 0.0145 [m^3/s], thus leading to the same overall flow rate.

In Figures 3 and 4, the FEM models of single fracture and multi - fracture respectively are presented, , with the boundary conditions and the mesh used.

As regards the mesh, in the fracture, where the water flows, the rectangular shape has been chosen, while in the rock the mesh was based on triangular finite elements. This choice ensures an accurate solution, and leads to time saving.

Table 1 – Project data by Harlow and Pracht (1972).

Q [m^3/s]	0.145	Volumetric flow rate
ρ_w [kg/m3]	1000	Density water
c_w [J/(kgK)]	4184	Specific heat water
ρ_r [kg/m3]	2650	Density rock
c_r [J/(kgK)]	1046	Specific heat rock
λ_r [W/(mK)]	2.6	Conductivity rock
T_r [°C]	300	Initial temperature of the rock
T_{w0} [°C]	65	Water inlet temperature

The results in the case of single fracture, are represented in Figure 5. In this graph the dimensionless temperature of the outlet water from the rock, as a function of dimensionless time, is shown. The results show the analytic theory of Lauwerier the numerical solutions of Gringarten et al., resolved by both the methods of Papoulis and Gaver – Stehfest as well as FEM solution with COMSOL Multiphysics. All the solutions follow the same trend. The numerical solution of Gringarten et al. resolved by the method of Papoulis differs 3.5% compared to the other methods.

In Figure 6, instead, the dimensionless temperature of the outlet water from the rock as a function of dimensionless time is shown, in the case of single fracture, considering the dimensionless geothermal gradient β, defined by Gringarten et al. Two values of gradient ($\beta = 0$ and $\beta = 0.1$) were considered. Also in this case the numerical solution resolved by the method of Papoulis differs by + 3.5% compared to FEM method. The presence of the geothermal gradient allows initially to extract water at a higher temperature; then all the cases show similar trends.

Gringarten et al. defined a simplified method for the preliminary design of a geothermal power plant, capable of producing electricity using the heat extracted from the rock to the water. For this purpose, a series of data are imposed, such as the thermal properties of the rock and the water present in situ, while another set of data must be fixed depending on the technical and economic considerations, which are realized, as a function of the useful life of the geothermal reservoir. In this case the choice will be based on the number of vertical fractures, the size of individual fractures, and the spacing between fractures. These parameters are chosen considering the graph of Figure 7 presented in the article of Gringarten et al (1975), which represents the dimensionless temperature of the outlet water from the rock as a function of the dimensionless time, by varying the spacing of the dimensionless fracture in the rock. This graph, in the article was obtained via the numerical solution of Papoulis.

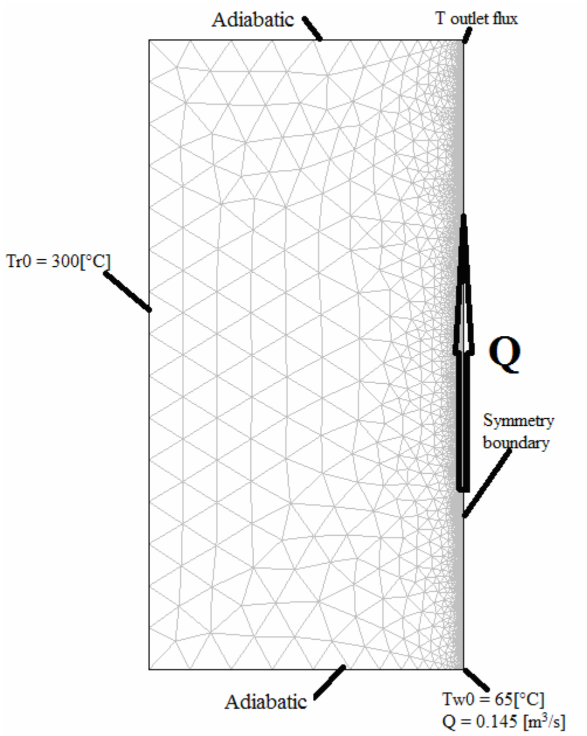

Figure 3: FEM Model for the single fracture.

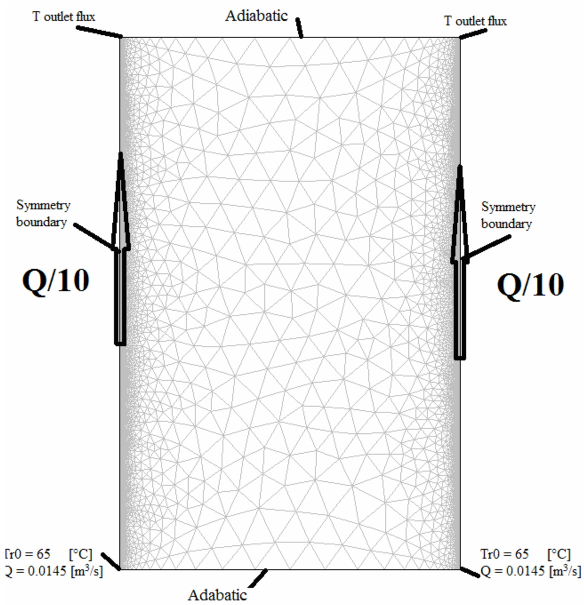

Figure 4: FEM Model for the multy – fracture.

For comparing the different methods the calculations using different fracture spacing have been carried out.

In Figure 8, the results reported by Gringarten et al. (1975), the numerical method of Gaver - Stehfest used in this work and the FEM model in COMSOL

Multiphysics are shown for a temperature gradient $\beta = 0$ as a function of multi - fracture spacing.

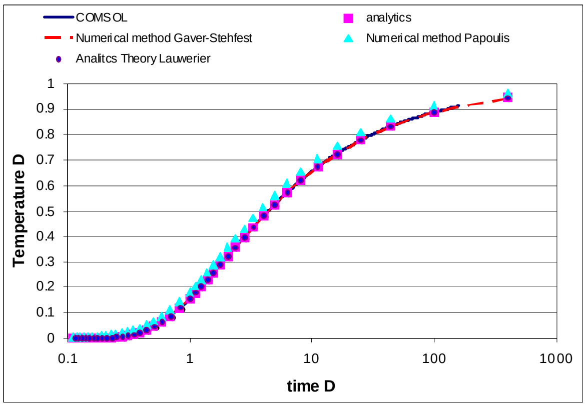

Figure 5: Dimensionless water outlet temperature versus dimensionless time for the single fracture theory.

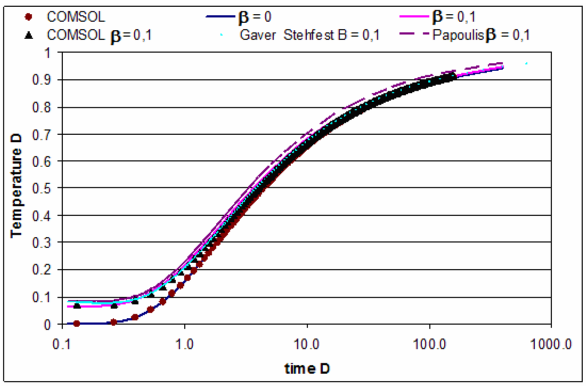

Figure 6: Dimensionless water outlet temperature versus dimensionless time theory with geothermal gradient β for the single fracture theory.

The results of Gringarten et al. (1975) based on the numerical methods of Papoulis has been compared to the Gaver – Stehfest method: the two methods have the same trend of dimensionless temperature as a function of dimensionless time, depending on the dimensionless spacing between the fractures. The small differences between the two solutions depend on the approximations made by the two methods, which lead to a maximum error of 7.5% in the case of a single fracture.

The method of Gaver - Stehfest is currently most used, in the field of hydrogeological and geothermal problems, as it is easier to implement compared to the method of Papoulis, which approximates the solutions with polynomials of n - degree.

The solutions obtained by FEM models are very interesting, because the solution depends on the calculating step and on the mesh used.

Papoulis and COMSOL curves follow the same trend, with a maximum difference in the results in the case of single fracture of about 7%.

The numerical solutions based on Gaver – Stehfest and the FEM model have a maximum difference of 3%. This allows to say that the solution of Gaver - Stehfest is preferable, compared to the method of Papoulis, used by Gringarten et al., since it is easier to implement, even in a spreadsheet, offering results closer to an accurate solution as the one of the FEM models.

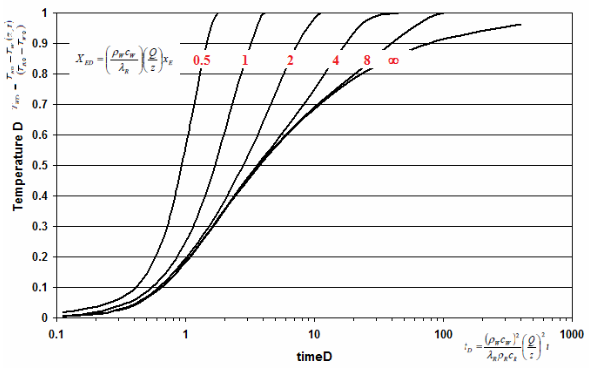

Figure 7: Dimensionless water outlet temperature versus dimensionless time depending on the of fracture spacing (solved with Papoulis method).

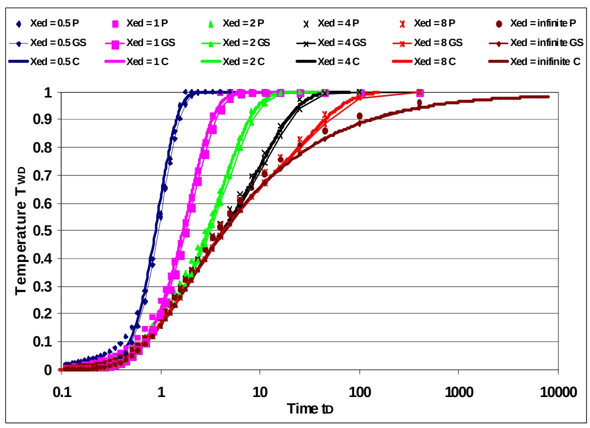

Figure 8: Dimensionless water outlet temperature versus dimensionless time for Gringarten, Papoulis method and the FEM model.

The input data defined by Harlow and Pracht (1972), have been applied to the example of Gringarten et al. (1975). In this case temperature a function of time for various values of vertical fractures spacing have been calculated as shown in Figure 9:

$$T_w = T_r - T_{wD}\left(T_r - T_{w0}\right) \qquad [15]$$

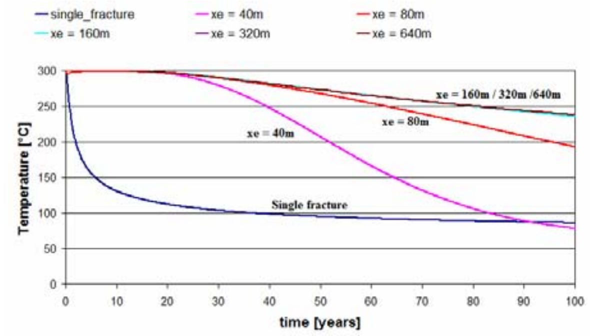

Figure 9: Water outlet temperature versus time for different fracture spacings xe, with T_{r0} = 300 °C, and T_{w0} = 65 °C.

Finally, as an example of the trend of temperatures in the different cases, the temperature of the rock in various time steps are reported for a single fracture (Figure 10), and multi - fracture (*xe* = 640m) (Figure 11).

4. CONCLUSIONS

A collection of the main theories of heat extraction in Hot Dry Rock has been presented.

The analysis of this study started with the simplest case: the extraction of steam and hot water from the rock in a single fracture, comparing the analytical theories of various authors, with numerical relations using the anti - Laplace transform, and the results with a FEM model, considering the presence of geothermal gradient in the case of thermal anomaly.

The authors Gringarten et al. (1975) studied a model, more simplified but more realistic, with heat extraction in the case of HDR with multiple fractured rock, fractures, vertical and parallel, by dividing the total flow of water in N fractures in the rock. The numerical equation which is resolved only by numerical methods using anti - Laplace transforms, were considered. In this work the both numerical methods of Papoulis and Gaver – Stehfest have been used. Results have shown that the multi fracture technique permits to extract more energy compared to single fracture, increasing the lifetime of the thermal resource.

The results of multi fracture were then compared with FEM models, highlighting that the numerical method proposed by Gaver - Stehfest, in addition to being easier to implement than the method of Papoulis, presents fewer errors of approximation.

Finally, it is important to consider another aspect, in reality fractures are not really vertical and parallel, but the crack propagation is changed due to the effects of thermal stress cracking, in fact, some authors have studied this aspect confirming: an extraction of geothermal energy greater than the model proposed by Gringarten et al. (1975). Further research will be addressed to consider the effects on the thermal performance of a geothermal reservoir.

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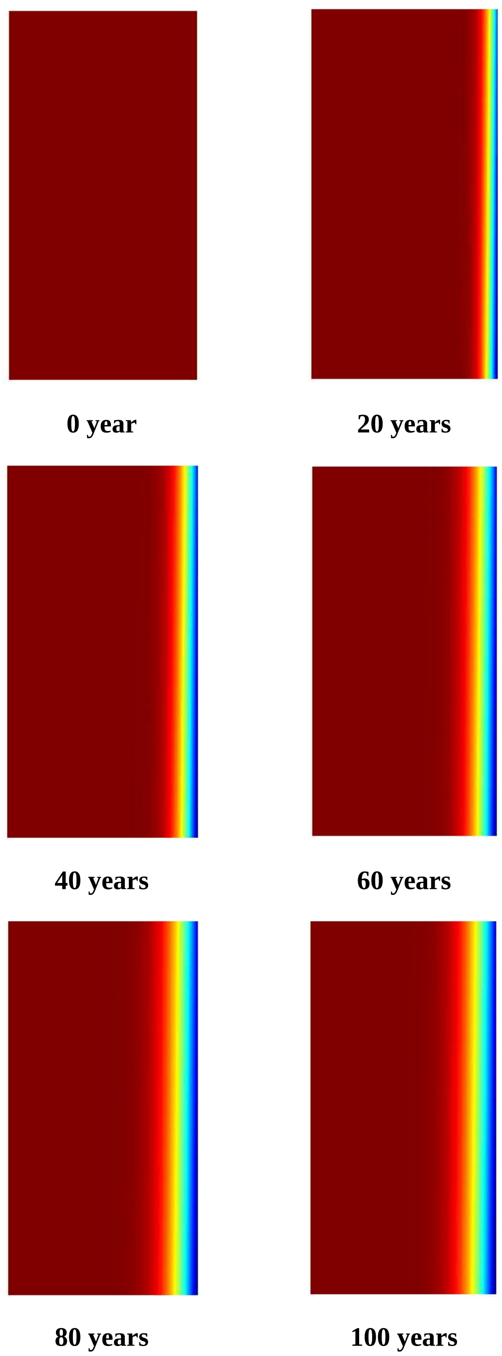

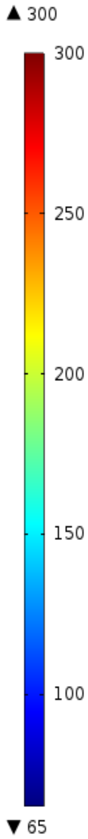

Figure 10: Temperature maps over the time for a single fracture

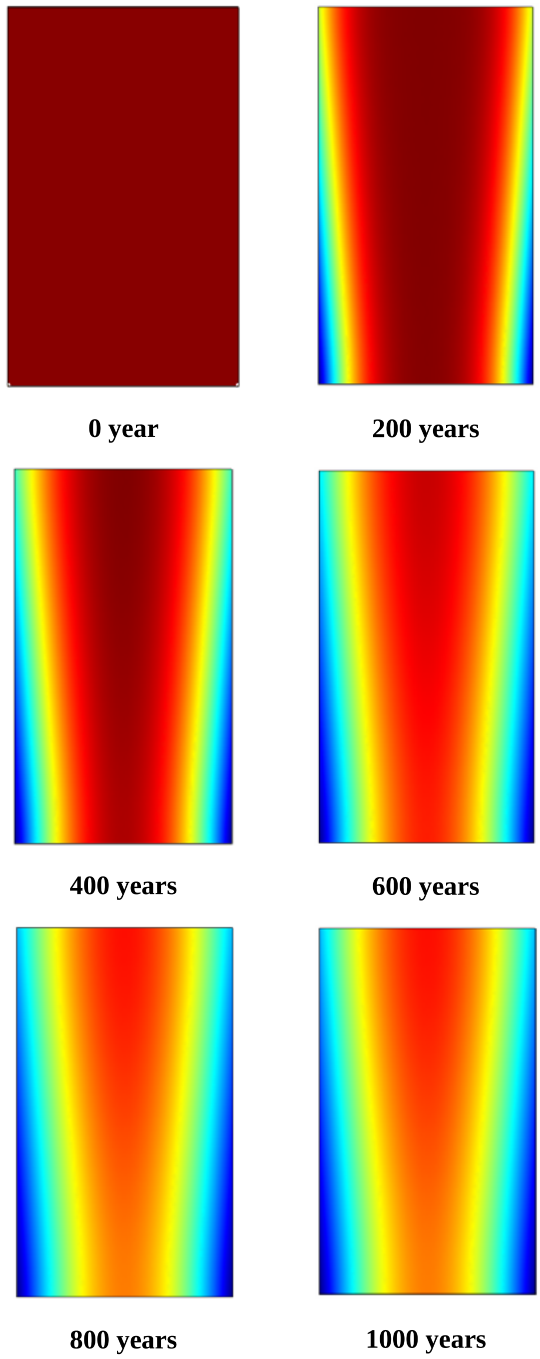

Figure 11: temperature maps over the time for a multi – fracture xe = 640 m.