Derivation of a "thermal conductivity-log" out of petrophysical correlations

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ABSTRACT

Thermal conductivity is one of the key properties of geothermal studies and other applications, like petroleum geology, applications to geothermal energy, civil engineering applications and hydro-geological studies. Due to difficult measurements of the thermal conductivity in boreholes, in most cases only laboratory values are available. Therefore the knowledge of correlations between the thermal conductivity and other petrophysical properties measurable in wells or from the surface could deliver indirectly thermal conductivity. Our approach was to correlate thermal conductivity with compressional wave velocity starting with magmatic rocks and followed with sedimentary rocks (sandstone).

Compressional wave velocity was determined with an ultrasonic laboratory device. At each sample from 3 measurements the mean value was determined. Thermal conductivity was measured using the Tk04 thermal conductivity meter from TeKa (Berlin, Germany) with a half-space line-source (transient method). At each sample from 15 measurements the mean value was determined.

Two models are applied in order to formulate the basic structure of a correlation between compressional wave velocity and thermal conductivity: a defect and an inclusions model. The solid mineral composition values are taken from the literature. The types of rock are divided into: granite and gneiss, gneiss respectively granite with higher content of quartz, basalt/gabbro/diorite and sandstone. Groups indicate a petrographic code as a property which controls the correlation.

Both models give good correlation of measured data. With the derived equations a calculation of the thermal conductivity out of a sonic log was possible. For verification an example was chosen where cores were taken and thermal conductivity measurements were available. The calculated "thermal conductivity log" fits good to the core data.

Summarized it can be said that correlations with both models for the laboratory values worked well for the selected rocks. They show the two important factors that influence the thermal conductivity and the velocity: the effect of mineral composition and cracks/fractures. The calculations of the thermal conductivity from the sonic log with both models worked well. Both models show nearly the same results. The values fit to the measured values from the cores in the laboratory.

1. INTRODUCTION

Thermal rock properties are of increasing interest for various problems in fundamental and applied geoscience like geothermal applications, civil engineering and hydrogeological studies. Thermal conductivity can hardly be determined in the borehole, so in most cases it is determined from core measurements. Intensive research is focussed on an indirect determination via correlations of thermal conductivity and other petrophysical properties.

Mainly empirical correlations are publsihed by many authors. Vacquier et al. (1988), derived from log data of two oil wells in France regression between thermal conductivity, density, slowness, neutron porosity, and shale content. Goss et al. (1975) derived an empirical correlation between thermal conductivity, porosity, and compressional wave velocity. Thermal conductivity was determined at sandstone, some claystone, siltstone and carbonate samples from Imperial Valley (California). Evans (1977) derived an equation for Jurassic North Sea sediments (39 samples) and implemented additionally the density. Brigaud et al. (1992) derived the rock composition ("electrofacies mineralogy and porosity)" from logs and used a four component (sandstone, carbonate, shale, pore fluid) geometric mean equation for thermal conductivity calculation. Popov et al. (2003) gives an overview over different correlations for 6 different types of rocks (silt- and sandstones, carbonates, granites and gneiss). These show the known trends between thermal conductivity, porosity and electrical resistivity. Another correlation is given by Hartmann et al. (2005) for shaly sandstones and marls for thermal conductivity, porosity as well as compressional wave velocity. They sum up that these correlations depend only on the local conditions. Sundberg et al. (2009) described a correlation for density and thermal conductivity for igneous rocks.

The following tendencies control the character of expected relationships between elastic wave velocity, thermal conductivity and also density for igneous and metamorphic rocks:

- velocity decreases with increasing fracturing or porosity and increases from acid/felsic (granite) to basic/mafic (dunite) types,
- thermal conductivity decreases with increasing fracturing or porosity but decreases from acid/felsic (granite, high quartz content) to basic/mafic (dunite) types,
- density decreases with increasing fracturing or porosity and increases from acid/felsic (granite) to basic/mafic (dunite) types.

2. MEASURING METHOD AND SAMPLES

For the derivation of the data, thermal conductivity and compressional wave velocity were determined in the laboratory.

The thermal conductivity was measured with a thermal conductivity meter Tk04 (TeKa, Berlin), which is a transient method. A half-space line-source is pressed with a constant pressure of 5bar on the sample. A contact agent (here: Nivea) is used to establish an optimal heat flow between probe and sample. The line-source is heated up and the temperature increase is measured. Out of the resulting heating cycle the thermal conductivity is determined directly (Erbas, 2001).

The compressional wave velocity was determined with an ultra-sonic device. The sample gets fixed between a transmitter and a receiver (both piezoceramic systems) with a contact agent (ultrasonic gel). A singular impulse (frequency=10kHz, amplitude=5V) is sent through, the signal is then displayed on the computer screen with a storage oscilloscope where a self-made program picks the first arrival and calculated the compressional wave velocity.

Samples are taken all over Austria, there are basalts from Klöch (lower Styria), granites from upper and lower Austria, sandstonde samples are from Germany and Paraguay and different samples from two projects (THERMTEC and THERMALP) of the geological survey of Austria. Table 1 gives an overview of the samples and measured data.

Rock type	**Location**	**λ**	**vp**
		[$Wm^{-1}K^{-1}$]	**[ms^{-1}]**
Granite	Lasberg (A)	2.77	5206
Tonalit	Ulrichsberg, Aigen (A)	2.60	5456
Granite	Perg, Mauthausen (A)	2.49	4613
Granite	Aigen (A)	2.26	4394
Granite-gneiss	Kindberg (A)	4.71	4468
Granite-gneiss	Übelbach (A)	2.58	3689
Granite-gneiss	Zauchen, Villach (A)	2.79	3304
Metagabbro	Koralpe (A)	2.96	6171
Gabbro	Nondorf (A)	2.43	6010
Gabbroider Diorite	Juhlbach (A)	2.62	5743
Diorite	Wölsau (Bayern) (D)	2.85	6297
Diabase	Saalfelden (A)	2.67	5166
Basalte	Puliberg, Kobersdorf (A)	2.61	5754
Basalte (shoshonit)	Weitendorf, Wildon (A)	1.67	4773
Basanitlava	Steinberg, Feldbach (A)	1.25	4316
Migmatite Granite	Vienna Basin (A)	2.74	5103
Weinsberger Granite	Vienna Basin (A)	2.83	5735
Granite-gneiss fine grained	Hintermuhr (A)	2.68	4718
Granite-gneiss coarse grained	Hintermuhr (A)	2.55	4731
Granite	Mühlviertel (A)	2.98	5246
Granite	Waldviertel (A)	3.53	5653
Granite	Lasberg (A)	2.81	
Granite	India	3.01	3843
Granite	India	2.80	3587
Granite	India	2.97	3947
Granite	India	2.89	3951
Sandstone	Oberfranken (D)	2.64	2892
Sandstone	Seckau (A)	2.81	2667
Sandstone	Deutschgoritz (A)	2.77	3790
Sandstone	Paraguay	6.25	5166
Sandstone	Paraguay	6.25	4922
Sandstone	Paraguay	4.20	3603
Sandstone	Paraguay	4.20	3361
Sandstone	Pirna (D)	3.70	3696
Sandstone	Pirna (D)	3.30	3030

Table 1: Measured data from the laboratory (v_p=compressional wave velocity, λ=thermal conductivity).

3. PETROGRAPHIC CODED MODELS

The two main controlling factors for thermal conductivity are the mineral composition (or petrography) and porosity and/or fractures. Among the petrophysical properties, compressional wave velocity (v_p) shows similarities. A model concept with two steps is used for the following derivations:

Step 1: Modelling of the solid matrix properties of the host material. Here mainly input data are from literature. This step also considers the mineral composition with its petrographic code.

Step 2: Implementation of pores/fractures in the host material with two model types: an inclusion model and a defect model.

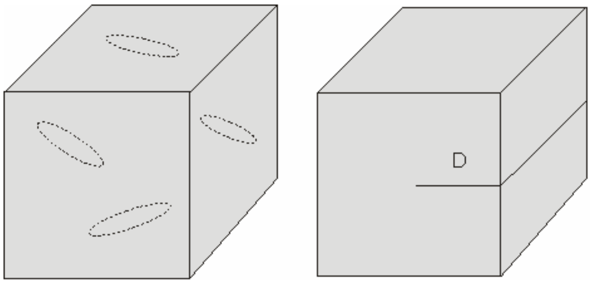

Figure 1: Illustration of the inclusions (left) and defect model (right)

Input parameters for the solid matrix for both models are displayed in table 2.

	V_p [ms^{-1}]	λ [$Wm^{-1}K^{-1}$]
Granite/Gneiss (high quartz)	4900	4.5
Granite/Gneiss (low quartz)	5600	3.5
Basalt/Diorite/Gabbro	6800	3.2
Sandstone	5000	6.5

Table 2: Input data for the inclusions model (v_p=compressional wave velocity, λ=thermal conductivity).

2.1 Inclusions Model

Budiansky and O'Connell (1976) derived a self-consistent algorithm for the elastic properties assuming a penny-shaped crack medium. Compressional and shear modulus result as:

$$k_{SC} = k_S * \left[1 - \frac{16}{9} * \frac{1-v_{SC}^2}{1-2*v_{SC}} * \varepsilon\right] \quad [1]$$

$$\mu_{SC} = \mu_S * \left[1 - \frac{32}{45} * \frac{(1-v_{SC})*(5-v_{SC})}{2-v_{SC}} * \varepsilon\right] \quad [2]$$

ε is a "crack density parameter" ($\varepsilon = \frac{N}{V} * r^3$), defined as the number of cracks (N) per unit volume (V) times the crack radius (r) cubed, k_s is the compression modulus for the solid material and μ_s is the shear modulus for the solid material (Mavko et al., 1998).

The equations of Clausius-Mosotti (see Berrymann, 1995, Gegenhuber, 2011 and Schön, 2011) are used for the calculation of the thermal conductivity.

$$\lambda_{CM} = \lambda_S * \frac{1-2*\phi*R^{mi}*(\lambda_S-\lambda_i)}{1+\phi*R^{mi}*(\lambda_S-\lambda_i)} \quad [3]$$

$$R^{mi} = \frac{1}{9} * \left(\frac{1}{L_{a,b,c}*\lambda_i+(1-L_{a,b,c})*\lambda_s}\right) \quad [4]$$

λ_i is the thermal conductivity of the inclusions

λ_s is the thermal conductivity of the solid mineral composition

R^{mi} is a function of the depolarization exponents L_a, L_b, L_c where the subscript a,b,c refers to the axis direction of the ellipsoid. Depolarization exponents are related to the aspect ratio. There are also values and approximations for some extreme shapes:

sphere $L_a=L_b=L_c=1/3$

needle $L_c=0$ (along needle long axis), $L_a=L_b=1/2$ (along needle short axes)

disk $L_c=1$ (along short axis), $L_a=L_b=0$ (along long axes).

Sen (1981) recommends the following approximation for plate-like objects (a=b>>c)

$$L_c = 1 - \frac{\pi}{2} * \frac{c}{a} = 1 - \frac{\pi}{2} * \alpha \quad [5]$$

where $\alpha=c/a$ is the aspect ratio. This can be applied for an estimate of L_c.; then for the other exponents results in:

$$L_a = L_b = \frac{1-L_c}{2} = \frac{\pi}{4} * \alpha \quad [6].$$

2.2. DEFECT MODEL

The second used model is a simpler model, the defect model. The defect parameter D in a solid matrix is characterized by its relative length. The decrease of the parameters can be calculated with:

$$k_{rock} = k_s * (1-D) \quad [7]$$

$$\mu_{rock} = \mu_s * (1-D) \quad [8]$$

$$\lambda_{rock} = \lambda_s * (1-D) \quad [9]$$

k_s, μ_s, and λ_s are the values for the compressional modulus, shear modulus and thermal conductivity, respectively of the solid matrix block. These result in the following relationship for the calculation of the thermal conductivity:

$$\lambda_{rock} = v^2_{p,rock} * \left(\frac{\lambda_s}{v^2_{p,s}}\right) = v^2_{p,rock} * A_{solid} \quad [10]$$

The equation reflects the correlation between thermal rock conductivity and the square of elastic wave velocity as result of the defect influence. The rock type ("petrographic code") is expressed as the parameter A_{solid} (solid matrix value), which is controlled only by mineral composition and properties (same position as host material in case of inclusion models).

3. RESULTS OF THE MODELS

This chapter will present the results of the calculations of the models in comparison to the measured data in the laboratory. Figure 2 shows thermal conductivity versus the compressional wave velocity. Points show measured data for different rock types (granite, gneiss, diorite/gabbro/basalt and sandstone). Lines are calculated with the inclusions model. The four curves fitting the different rock types are calculated for different input values for the host material (table 2) representing step 1 and different aspect ratios characterizing the inclusion shape representing step 2.

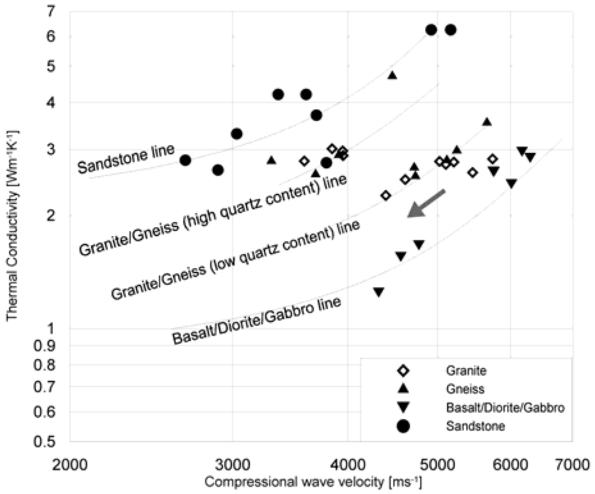

Figure 2: Results of the model calculations with the inclusions model. Points show measured data.

For the inclusions model the curve parameter is the aspect ratio. Aspect ratio α for best fit is 0.20 for granites and gneiss with higher and lower content of quartz. This aspect ratio represents fractures and pores with an axis ratio of 1:5. Basalt/diorite/gabbro obviously show not such a flat shape with aspect ratio of 0.25 and axis ratio of 1: 4. The aspect ratio for sandstone with 0.2 results in an axis ratio of 1:5.

For a practical derivation of thermal conductivity from a velocity measurement (Acousticlog, Soniclog), the calculated model curves are approximated by regression functions and result in:

Rocktype	Regression equations	R^2
Sandstone	λ=1.123exp0.0003*v_p	0.967
Granite/Gneiss-lower quartz content	λ=9E-07*$v_p^{1.756}$	0.996
Granite/Gneiss-higher quartz content	λ=5E-08*$v_p^{2.14}$	0.994
Basalt/Diorite/Gabbro	λ=6E-07*$v_p^{1.747}$	0.981

Table 3: Resulting regression equations from the inclusions model for the calculation of the "thermal conductivity log"

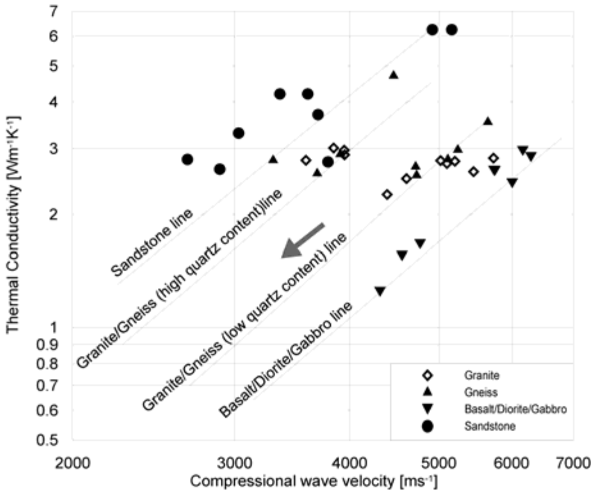

Figure 3: Results of the model calculations with the defect model. Points show measured data.

Figure 3 shows the results of the defect model. Again points show measured data and lines show calculated results of the defect model. The lines are controlled by the factor A_{solid} (eq. 10), which depends only on the solid material properties (mineral composition) (table 2). Along the curves the defect parameter changes (porosity increases).

Rocktype	Defect Model
Sandstone	λ=vp²*2.60E-07
Granite/Gneiss-lower quartz content	$\lambda=v_p^2$*1.12E-07
Granite/Gneiss-higher quartz content	$\lambda=v_p^2$*1.95E-07
Diorite/Gabbro/Basalt	$\lambda=v_p^2$*6.29E-08

Table 4: Resulting equations for the defect model for the calculation of the "thermal conductivity log"

4. "THERMAL CONDUCTIVITY LOG"

The derived equations (Table 2 and 3) allow a direct transformation of acoustic log data into a thermal conductivity log for defined petrographic types. This is demonstrated for a borehole (KTB/Continental Deep Drilling Project) with metamorphic rocks.

The continental deep drilling project is situated in Germany and was carried out from 1986 till 1992. A wide spectrum of methods (logs and core analysis) have been measured. All data are still available on the internet. So this borehole was a good choice for an application and a direct verification by core data.

The rocks are metabasites and gneisses (alternately). The metabasite sections show lower thermal conductivity than the granites. Therefore two equations for the two different petrographic types (granite and basalt/gabbro/diorite) must be applied. Figure 4 shows the results for the inclusions model in comparison to the defect model from the acoustic log, once with the equation for basalt/diorite/gabbro and once with the equation for granite always for the whole borehole section, to give an idea what would be the result when only one of the equations without respect to the petrographic type would be used. Using the petrographic relevant equations for the different sections gives good results (Figure 5). Here only the equations for the inclusions model are used. Values are in the same range as the measured thermal conductivity from the cores (black dots).

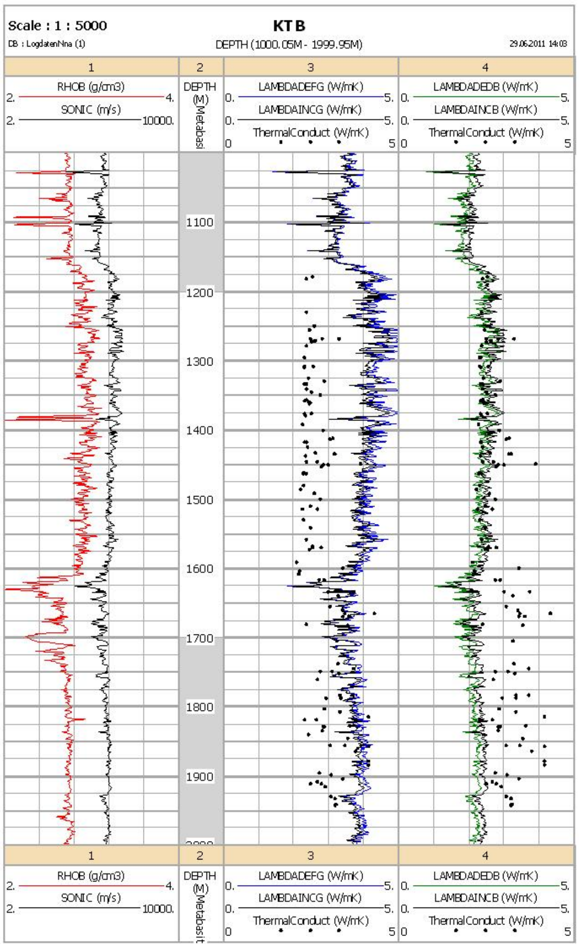

Figure 4: Well KTB (1000-2000m): Trace 1: Sonic and density log; Trace 2: lithlogy Grey=gneiss, white=metabasite, Trace 3: "thermal conductivity log calculated from the sonic log for the granite"(blue: Defect model; black: Inclusion model); Trace 4: "thermal conductivity log calculated from the sonic log for the basalt"(green: Defect model; black: Inclusion model); Additionally: dots show core data

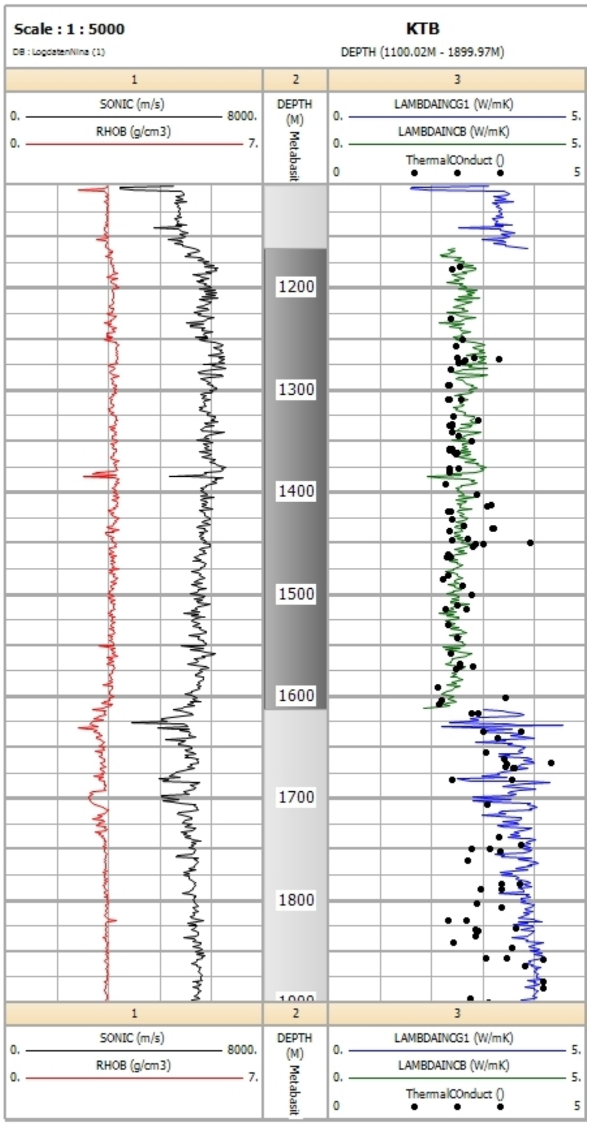

Figure 5: Results of the model calculations with the inclusions model. Points show measured core data. Blue: for granite, green: for basalt/diorite/gabbro with the inclusions model

5. CONCLUSION

In detail the comparison of measured and calculated data show:

- Correlations are controlled by mineral composition and fractures/pores.
- Inclusions models are one possibility to derive model-based relationships. They implement both properties (mineral composition and fractures).

- A mathematical simplification of the derived curves from the inclusion model by a regression is possible.
- The simpler defect model works good for a derivation of the “thermal conductivity log”

In order to implement these influences, a modular concept of model architecture was developed. It has two main steps:

Step 1: Modeling of mineral composition – this controls the petrographic code or rock type

Step 2: Modeling or implementation of fractures, pores etc.

For step 1 “mixing rules” or averaging equations give a possibility of forward calculation; as a result of the variation of rock composition within one rock type in some cases a pure empirical assumption of the “solid parameters is a more practical way and comparable to the practice of “matrix properties” in log interpretation.

For step 2 the inclusion model and the defect model are a powerful basis for correlation between thermal conductivity and compressional wave velocity. The application on experimental data shows

- the models deliver the general correlation very well
- correlation is strongly influenced by the aspect ratio, particularly for fractured rocks with the inclusions model

Application on acoustic logs to derive a thermal conductivity log work really well. Taking the petrographic code into account calculated log data fit to the measured core dat.

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