Peculiarity of Hydrodynamic Modeling of Fluid Flow in Porous Rocks under Precipitation and Compaction Conditions

Elena I. Suetnova , Vladimir M. Cherniavsky

Inst. Earth Phys. RAS Moscow Russia, Moscow State University Moscow Russia

elena_suetnova@mail.ru

Keywords: Earth crust, sediments, compaction, fluid velocity, permeability, precipitation, gas hydrate

ABSTRACT

In reservoir engineering and geophysics it is important to have a fundamental understanding of time dependent processes of fluid flow and species precipitation in the permeable rocks and their compaction. The porosity and effective stress evolution history of porous rock, involved into these processes, is described by a poro-visco-elastic (Maxwell-type) constitutive law. But such a process is disturbed by the precipitation and accumulations of a species in there P-T stability zones. This processes lead to decreasing of porosity and permeability. Mathematical model of coupled processes of sediment compaction and pore feeling by precipitation is developed. We formulate the coupled system of equations describing sediment accumulation and compaction, fluid and matter flow and dissolved species precipitation, and illustrate interdependence of the processes by numerical examples.

INTRODUCTION

During geological history of the Earth crust, subsurface fluid movement plays an important role in the process of geological media evolution, (Fife, Price, Thompson 1978). Modeling of subsurface flow has been recognized as an important for mineralogy, geochemistry, hydrology, petroleum geology, reservoir engineering and so on, (Barenblat, Entov and Ryzhik 1990). Subsurface fluid flow is the most powerful process of reactive solute transport, (Dagan and Cvetcovic, 1996). Problem, related to fluid movement in sediments during their accumulation and burying, is known as dewatering and compaction problem, has been studying widely; see Wangen (2001), Suetnova and Vaseur (2000) for review. Pervasive pressure solution transfer describes permanent porosity reduction in stressed porous aggregates saturated by an aqueous solution. This process includes dissolution of the grain contacts due to stress-enhanced solubility, solute transport by diffusion along the grain contacts, and precipitation of the solute on the free faces of the grains. Pressure solution transfer corresponds to a response of the aggregate in an attempt to increase the grain-to-grain contiguity so as to redistribute the local effective stress over a larger grain-to-grain contact area. The porosity and effective stress evolution history of porous rock, involved into this process, is described by a poro-visco-elastic (Maxwell-type) constitutive law, (Suetnova and Vaseur, 2000). But it was not considered the interdependence problem of compaction driven fluid movement and process of precipitations of dissolved matter transported by fluid to its P-T precipitation domain. In present paper we formulate the coupled system of equations, describing a sediment accumulation and compaction, fluid and matter flow and dissolved species precipitation, and illustrate interdependence of the processes by numerical examples.

SYSTEM OF GOVERNING EQUATIONS

It is considered the growing domain of saturated porous media, which represent accumulating fluid feeling sediments peel. In the frame of poro-visco-elastic (Maxwell-type) rheology law, porosity loss, fluid movement, effective stress and sediment compaction obey the following equations, (Suetnova and Vaseur, 2000). To outline the studying effects we consider 1D formulation. The volumetric stress-strain rate relation is:

$$\frac{dm}{(1-m)dt} = -\frac{m}{h}p_e - \frac{m}{K_p}\frac{dp_e}{dt} \qquad (1)$$

where m, $p_e = p_{tot} - p_f$, h are porosity, effective pressure and shear viscosity. Here, p_{tot}, p_f are total stress and pressure of fluid, h/m is the bulk effective viscosity of solid matrix, K_p is

the bulk modulus, the reciprocal of which is the pore compressibility $b = \left(\frac{\partial m}{\partial p_e}\right)/m$. The pore fluid movement is described by the Darcy's equation

$$V_f - V_s = -\frac{k}{mm}\left(\frac{\P p_f}{\P y} + r_f g\right) \qquad (2)$$

where V_f, V_s, t, y, r_f, r_s, k, m, g are velocity of fluid and rock matrix, time, depth coordinate, directed upward, density of fluid and matrix, permeability, viscosity of fluid, gravity acceleration, respectively.

The continuity equations for the fluid and the solid matrix require

$$\frac{\P r_f m}{\P t} + \frac{\P r_f V_f m}{\P y} = 0 \qquad (3)$$

$$\frac{\P r_s(1-m)}{\P t} + \frac{\P r_s V_s(1-m)}{\P y} = 0 \qquad (4)$$

Thermal conductivity equations for the fluid and the solid matrix require

$$\frac{\partial f}{\partial t} + A_1\frac{\partial m V_f f}{\partial y} + A_2\frac{\partial(1-m)V_s f}{\partial y}$$
$$= K\frac{\P^2 f}{\P y^2} \qquad (5)$$

$$A1 = \frac{r_f C_f}{r C}, \ A2 = \frac{r_s C_s}{r C}$$

where f, C_f, C_s, C, K temperature, thermal capacity of fluid, matrix and rock, thermal conductivity of rock, respectively.

In order to complete the system of governing equations (1)-(5), we formulate the boundary conditions. At the lower impervious boundary $y = b(t)$ velocities V_f, V_s are equal to V_1, the subsidence velocity of the base of the basin. Porosity at the upper boundary is equal to m_0. Temperature gradient at the base B are known constants, effective pressure p_e at the upper boundary is equal to zero

$$m(y,t)\big|_{y=0} = m_0, \qquad f(y,t)\big|_{y=0} = 0,$$
$$\frac{\partial f(y,t)}{\partial y}\bigg|y = b(t) = B, \ p_e(y,t)\big|_{y=0} = 0,$$

The matrix permeability is assumed to be power law functions of porosity $k = k_0 m^2$, (Barenblat, Entov and Ryzhik, 1990).

SOLUTION AND CONCLUSION

For convenience, we transform the fixed coordinate system into a moving one, in which the base of the basin corresponds to the coordinate $y' = 0$ and the upper boundary grows with a constant velocity $V = -V_1$. Such a transformation does not change the full derivatives in the equation (1) and allow reducing the number of variables, using boundary conditions.

$$m V_f + (1-m)V_s = 0 \qquad (6)$$

To reduce a number of coefficients we use a p-theorem and scaling procedure (Barenblatt, 1996) by introducing porosity value m_0, density difference $\Delta r = r_s - r_f$, velocity V and gravity acceleration g which determine characteristic scales of compaction length $L = \sqrt{\frac{hV}{\Delta r g}}$, pressure difference due to buoyancy on this length $P = \Delta r gL$ and compaction time $T = L/V$.

Governing system of equations written in dimensionless form and to be short using the same notations is the next

$$\frac{dm}{dt} = \frac{\partial V_s}{\partial y} \qquad (7)$$

$$-e V_s = m^2\left(\frac{\partial p_e}{\partial y} + 1\right) \qquad (8)$$

$$-\frac{\partial V_s}{\partial y} = m p_e + m a\frac{\partial p_e}{\partial t} \qquad (9)$$

$$A\frac{\partial V_s \Theta}{\partial t} - k\frac{\partial^2 \Theta}{\partial y^2} = 0 \qquad (10)$$

where $A = (A_1 - A_2)m_0$, $k = KT/L^2$, $\Theta = f/273^o$. Using the typical representative for porous rocks parameters values we evaluate non-dimensional coefficient $e = \frac{Vm}{\Delta r g k_0 m_0}$ as $0.8\cdot10^{-5} < e < 4.5\cdot10^{-2}$ (Suetnova, and Chernyavskii, 2001). Non-dimensional coefficient a is known as "Maxwell relaxation time" for non-dimensional visco-elastic problem. Calculated a vary as $10^{-3} \div 10^{-1}$ in the frame of representative physical parameter of sedimentation and values of assumed viscosity variations $10^{20} \div 10^{22} Pa \cdot s$. Dimensional "Maxwell relaxation time" $t = h b$ vary as max $10^3 \div 10^5 s$, that is time period of accumulation $1 \div 100 m$ of sediments. When taking in mind the geological time scale and slowness of process of overburden pressure growing, we can consider viscous process of sediment compaction, because of viscous effects dominant. In the domain corresponding P-T

condition of some species precipitation, the system of governing equations should be completed by the equation of pore feeling by precipitation

$$\frac{\partial \boldsymbol{r}_f m (1 - s) c}{\partial t} + \frac{\partial \boldsymbol{r}_f V_f m (1 - s) c}{\partial z} =$$

$$- \boldsymbol{r}_h (c_h - c) \frac{\partial s}{\partial t} \qquad (11)$$

and corresponding change $m \to m(1-s)$ in the equations (Suetnova, 2007). Here s is a part of pore space occupied by a precipitated matter, c_h, c are concentrations of species in pore feeling mater and in pore fluid. Provided that the kinetic barriers to crystal growth are small compared with other rate- limiting processes, we assume that precipitated matter is in local thermodynamic equilibrium with surrounding fluid. In the frame of assumption that permeability is power law function of porosity, it is clear that precipitation decreases free porosity. So, precipitation should affect fluid velocity, which is determined by compaction. Because of non-linearity of the system, we investigate this problem numerically. As an example, we study the process of methane solution in pore water and its precipitation as hydrate in hydrate stability zone. We performed calculations for different time moments with parameters $k_0 = 10^{-14} \, m^2$, $\Delta \boldsymbol{r} = 1.6 \cdot 10^3 \, kg / m^3$, $\boldsymbol{m} = 2.5 \cdot 10^{-3} \, Pa \cdot s$, $V = 5 \cdot 10^{-10} \, m / s$, $\boldsymbol{h} = 5 \cdot 10^{21} \, Pa \cdot s$, $B = 40^{o} / km$. The thickness of hydrate stability zone, c_h and the dependences of c_{eq} on P-T were taken using results of Davie, Zatsepina and Buffet (2004). Figures 1 and 2 illustrate the non-dimensional quantities s and V_f versus sediments thickness, normalized to its final thickness, resulting after 2 My of sedimentation.
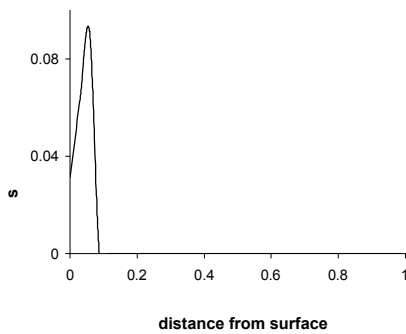

Figure 1. The resulting distribution of precipitated matter as a part of pore space.
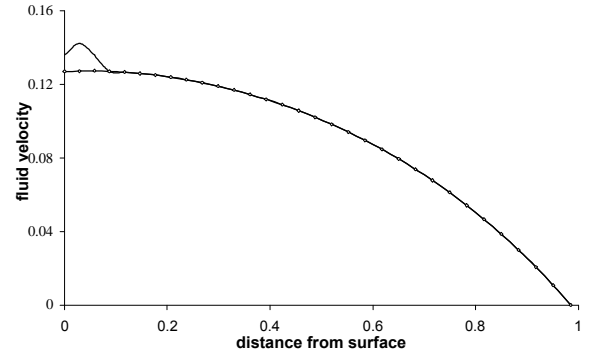

Figure 2. Fluid velocity inside sediments (unmarked curve represents case with precipitation, marked - without precipitation).

It is clear that precipitation affects fluid velocity in sediments. We conclude that, hydrodynamic modeling of porous rocks during geological time scale required accounting of variation free pore space and fluid velocity due to precipitation of species.

REFERENCES

Barenblat, G.I., Entov, V.M. and Ryzhik, V.M.: Theory of fluid flows through natural rocks, (1990), Kluwer, Dordrecht, 395 pp.

Barenblatt, G.I.: Scaling, self-similarity, and intermediate asymptotics, (1996), Cambridge University Press, 386 pp

Davie, M.K., Zatsepina, O.Ye. and Buffet, B.A. :Methane Solubility in Marine Hydrate Environments , *Marine Geology*, 203, (2004), 177-184.

Dagan, G. and Cvetcovic, V.: Reactive Transport and immiscible Flow in Geological Media.1. General Theory, *Proc. R. Soc.Lond. A.,* 452, (1996), 285-301.

Fyfe, W.S., Price, N.J. and Thompson, A.B.: Fluids in the Earth Crust, (1978), Elsevier. 333 pp.

Suetnova, E.I.: Gas hydrate accumulation and compaction of growing sediments: problem of feedback, *Dokl. Rus. Acad.Nauk*, (2007), in press.

Suetnova, E.I. and Vasseur, G.: 1-D modelling Rock Compaction in Sedimentary Basin Using Visco-elastic Rheology, *Earth Planet. Sci. Letters*. , 178, (2000), 373-383.

Suetnova, E.I. and Chernyavskii, V.M.: Free fluid flow through a visco-deformable porous medium with a moving boundary, *Fluid Dynamics,* 36, (2001), 121-129.

Wangen, M.: A Quantitative Comparison of Some Mechanisms Generating Overpressure in Sedimentary Basins, *Tectonophysics*, 344, (2001),211-234.