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Characteristics of extracting geothermal heat for one-hole systems

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ABSTRACT

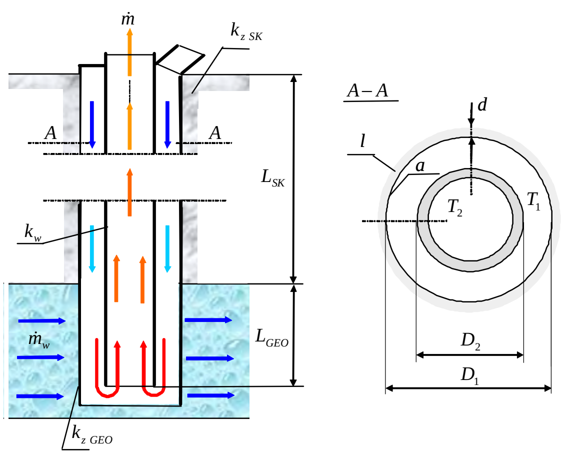
In the paper the author present a computational model of a Geothermal Heat Exchanger (GHE) and the characteristics of a geothermal heat energy flux in a function of the depth of a GHE and in a function of temperature of water injected into the heat exchanger as well as in a function of a volume flow of the water and kinds of rock deposit. When taking into consideration appropriate characteristics of heat receivers, the characteristics for one-hole systems allow one to chose the most effective system of extracting the geothermal heat energy for the designed system of heat removal and extracting the geothermal energy.

1. INTRODUCTION

Geothermal heat plants and power stations in most cases work in two-hole systems with injection and production wells. In these systems the temperature of geothermal water extracted to the earth surface may be estimated precisely using known computational models in a relatively easy way as well as the temperature of geothermal water pumped back to its original source can be estimate. When the flows of drown out water are great, changes of temperature of the geothermal water in injection and production conductors are relatively low. However, high expenditure on drilling of the hole in comparison to the total capital cost is the negative aspect of using this method of winning heat energy.

The capital cost may be reduced by employing the one-hole injection system. Therefore a double-pipe exchanger with intermediate fluid drawing heat from a geothermal deposit is located in the existing single hole. In available literature the double-pipe exchanger is called the Geothermal Heat Exchanger or Field's exchanger (Fig. 1a).

a)



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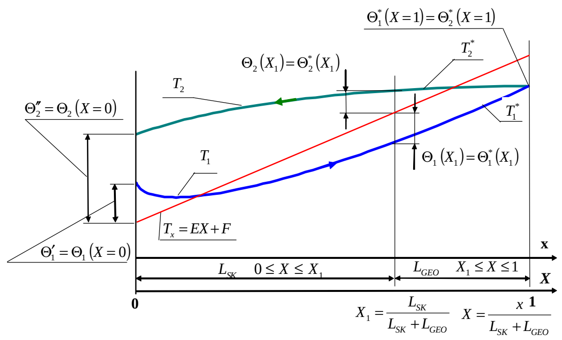


Figure 1: The scheme of the double-pipe heat exchanger (a) and the temperature field of a fluid with exchanger (b)

2. MATHEMATICAL MODEL OF A GEOTHERMAL HEAT EXCHANGER (GHE)

The exchanger is partly immersed in a geothermal deposit which heat of geothermal water is drawn from. The other part of the heat exchanger is located in impervious rock massif which temperature changes are linear. An overall heat transfer coefficient through of an outside surface $k_{z\,GEO}$ and inside surface k_w is calculated from known classical formulas. The amount of heat transmitted between water flown in outside part and a rock deposit is estimated by using a substitute overall heat transfer coefficient $k_{z\ SK}$, the function of time. Values of the substitute overall heat transfer coefficient may be estimated on the base of formulas presented in book Diad'kin and Giendler (1985).

The Geothermal Heat Exchanger (GHE), due to different conditions of heat exchange over the length of external surface, is divided into two parts. The fluid temperature field with adequate notations is shown in Fig. 1b.

In the papers Kujawa and Nowak (2000) and Kujawa, Nowak and Szaflik (1998) presented analytical method of calculation for the GHE with intermediate fluid.

On the basis of the balance equations of the first part of the exchanger for the fluid flown in a ring-shaped channel and an inside channel written with the aid of equations of heat transfer and introducing additional relations $\dot{W} = \dot{m} c_p$, $L_{GHE} = L_{SK} + L_{GEO}$, $K = k \pi D_2 L_{GHE}$, $a = K / \dot{W}$, $K_{z\,SK} = k_{z\,SK} \pi D_1 L_{GHE}$, $b_{SK} = \left(K + K_{z\,SK}\right) / \dot{W}$, we obtain the following system of differential first-order equations:

$$\mathrm{d}\Theta_{1SK} / \mathrm{d}X = a \Theta_{2\,SK} - b_{SK} \Theta_{1\,SK} - E \,. \qquad (1)$$

$$\mathrm{d}\Theta_{2SK} / \mathrm{d}X = a\Theta_{2SK} - a\Theta_{1SK} - E \,. \tag{2}$$

The system of equations can be solved using the d'Alambert's method and it can be written as follows Kujawa, Nowak, and Szaflik (1999):

$$\Theta_{1SK} = -C_{2SK} \exp\left(\nu^2_{2SK} X\right) + \\ + C_{1SK} \exp\left(\nu^2_{1SK} X\right), \tag{3}$$

$$\Theta_{2SK} = C_{2SK} \frac{q_{2SK}}{p_{2SK}} \exp\left(\nu^2_{SK2} X\right) - \\ + C_{1SK} \frac{q_{1SK}}{p_{1SK}} \exp\left(\nu^2_{1SK} X\right) + \frac{E}{a}, \tag{4}$$

where: $\nu^2_{iSK} = 0{,}5\left(K_{zSK} / \dot{W}\right)\left[-1 \pm \sqrt{1 + 4\left(K/K_{zSK}\right)}\right]$, $\left(q_{iSK} / p_{iSK}\right) = -\left(\nu^2_{iSK} + b\right)/a$.

Introducing additional signs $K_{zGEO} = k_{zGEO}\pi D_1 L_{GHE}$, $b_{GEO} = \left(K + K_{zGEO}\right)/\dot{W}$ and considering proper value of index "$_{GEO}$" equations (1) and (2) and solutions (3) and (4) may be used for the second part of the heat exchanger. The relations are as follows:

$$\Theta_{1GEO} = -C_{2GEO} \exp\left(\nu^2_{2GEO} X\right) + \\ + C_{1GEO} \exp\left(\nu^2_{1GEO} X\right), \tag{5}$$

$$\Theta_{2GEO} = C_{2GEO} \frac{q_{2GEO}}{p_{2GEO}} \exp\left(\nu^2_{2GEO} X\right) - \\ + C_{1GEO} \frac{q_{1GEO}}{p_{1GEO}} \exp\left(\nu^2_{1GEO} X\right) + \frac{E}{a}, \tag{6}$$

where:

$\nu^2_{iGEO} = 0{,}5\left(K_{zGEO} / \dot{W}\right)\left[-1 \pm \sqrt{1 + 4\left(K/K_{zGEO}\right)}\right]$,

$\left(q_{iGEO} / p_{iGEO}\right) = -\left(\nu^2_{iGEO} + b_{GEO}\right)/a$.

To get an explicit description of the temperature field, in both the first and the second part of the exchanger, we need to determine four integration constants in Eqs. (3), (4), (5) and (6), using four following boundary conditions:

$$\left.\begin{aligned} &\Theta_{1SK}(X = 0) = \Theta'_{1SK}, \\ &\Theta_{1SK}(X = X_1) = \Theta_{1GEO}(X = X_1), \\ &\Theta_{2SK}(X = X_1) = \Theta_{2GEO}(X = X_1), \\ &\Theta_{2GEO}(X = 1) = \Theta_{1GEO}(X = 1). \end{aligned}\right\} \tag{7}$$

The determined integration constants:

$$C_{1SK} = \Theta'_{1SK} + C_{2SK},$$

$$C_{2SK} = \left(N_2 M'_1 + M'_2\right)/\left(1 - N_1 M'_1\right),$$

$$C_{1GEO} = C_{2SK} N_1 + N_2,$$

$$C_{2GEO} = C_{1GEO} M_1 - M_2,$$

where:

$$M_1 = \frac{\left(1 + q_{1GEO} / p_{1GEO}\right)\exp\left(\nu^2_{1GEO}\right)}{\left(1 + q_{2GEO} / p_{2GEO}\right)\exp\left(\nu^2_{2GEO}\right)},$$

$$M_2 = E / \left[a\left(1 + q_{2GEO} / p_{2GEO}\right)\exp\left(\nu^2_{2GEO}\right)\right]$$

$$M'_1 = \frac{\exp\left(\nu^2_{1GEO} X_1\right) - M_1 \exp\left(\nu^2_{2GEO} X_1\right)}{\exp\left(\nu^2_{1SK} X_1\right) - \exp\left(\nu^2_{2SK} X_1\right)},$$

$$M'_2 = \frac{M_2 \exp\left(\nu^2_{2GEO} X_1\right) - \Theta'_{1SK} \exp\left(\nu^2_{1SK} X_1\right)}{\exp\left(\nu^2_{1SK} X_1\right) - \exp\left(\nu^2_{2SK} X_1\right)},$$

$$N_1 = \\ = \frac{\frac{q_{2SK}}{p_{2SK}} \exp\left(\nu^2_{2SK} X_1\right) - \frac{q_{1SK}}{p_{1SK}} \exp\left(\nu^2_{1SK} X_1\right)}{M_1 \frac{q_{2GEO}}{p_{2GEO}} \exp\left(\nu^2_{2GEO} X_1\right) - \frac{q_{1GEO}}{p_{1GEO}} \exp\left(\nu^2_{1GEO} X_1\right)},$$

$$N_2 = \\ \frac{M_2 \frac{q_{2GEO}}{p_{2GEO}} \exp\left(\nu^2_{2GEO} X_1\right) - \Theta'_{1SK} \frac{q_{1SK}}{p_{1SK}} \exp\left(\nu^2_{1SK} X_1\right)}{M_1 \frac{q_{2GEO}}{p_{2GEO}} \exp\left(\nu^2_{2GEO} X_1\right) - \frac{q_{1GEO}}{p_{1GEO}} \exp\left(\nu^2_{1GEO} X_1\right)}$$

The heat flow, which is collected by the GHE, is determined by:

$$\dot{Q} = \dot{W}\left[\Theta_{2SK}(X = 0) - \Theta_{1SK}(X = 0)\right] = \\ \dot{W}\left(\Theta''_{2SK} - \Theta'_{1SK}\right) = \dot{W}\left(T''_2 - T'_1\right). \tag{8}$$

3. CALCULATION RESULTS

Based on the presented mathematical model of heat exchange a computational programme which lets calculate a field of temperature of fluid and a heat flow in a geothermal deposit for the following parameters:

- the depth is equal to the sum of the thickness of a rock layer L_{SK} and the thickness of a geothermal layer L_{GEO}, $L_{GHE} = L_{SK} + L_{GEO}$ = 3200 + 800 = 4000 m
- temperature at the depth 4000 m is 145°C,
- temperature on the earth surface is 6°C,
- temperature of water injected into the exchanger $T_1 = T_{inj}$ = 10, 15, 20, 25, 30, 35, 40, 45 and 50°C,
- a water flow in the exchanger $\dot{V}$ = 1 – 60 m^3/h,

– the inner diameter of the outside pipe $D_1 = 0.6223$ m,

– the outer diameter of the inside pipe $D_2 = 0.2056$ m,

– an overall heat transfer coefficient of the inside pipe k_w =0.71 W/(m·K),

– an overall heat transfer coefficient through of the outside surface $k_{z\,GEO}$ =6.03 W/(m·K),

– the parameters of a rock deposit:

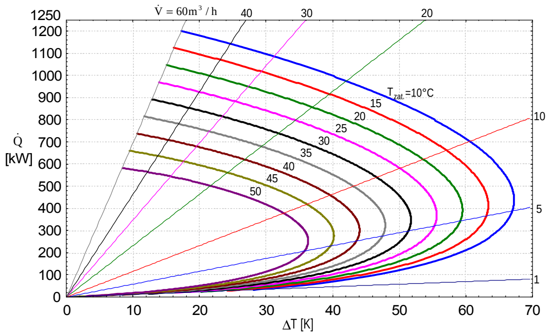
kinds of rock (1): $\rho = 1440$ kg/m^3, $\lambda = 0.346$ W/(m·K), $a = 0.283\cdot10^{-6}$ m^2/s, $c_p = 840$ J/(kgK),

kinds of rock (2): $\rho = 1600$ kg/m^3, $\lambda = 0.865$ W/(m·K), $a = 0.517\cdot10^{-6}$ m^2/s, $c_p = 1050$ J/(kgK),

kinds of rock (3): $\rho = 2100$ kg/m^3, $\lambda = 1.3$ W/(m·K), $a = 0.644\cdot10^{-6}$ m^2/s, $c_p = 960$ J/(kgK).

Results of the calculations in the appropriate heat-flow characteristics are presented in Figs. 2 and 3.

a)



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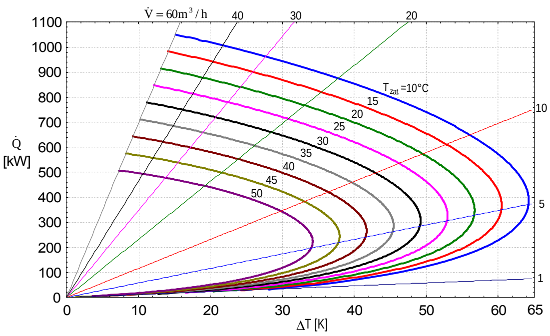

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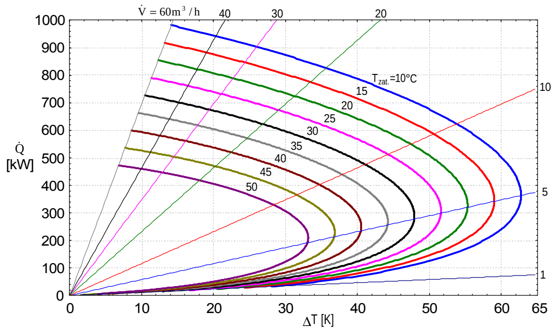

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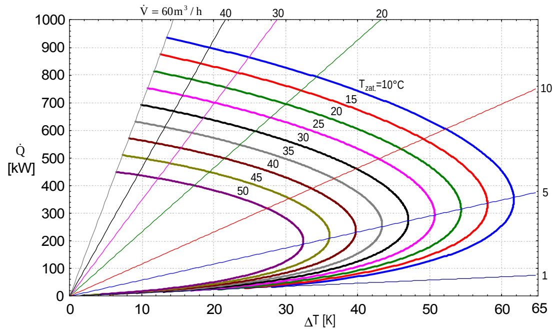

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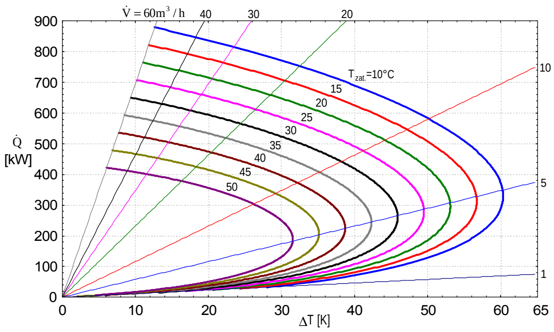

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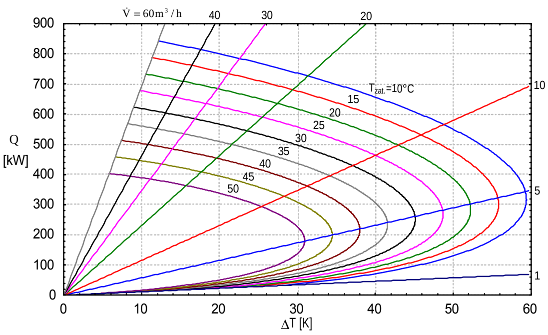
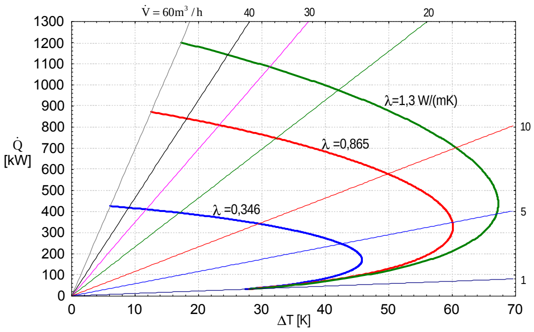


Figure 2: The impact of time of work GHE on flux of gained geothermal heat $\dot{Q}$ for kinds of rock deposit (3) – $\lambda = 1,3$W/(m?K), $\rho = 2100$ kg/m^3, $a_s = 0,644\cdot10^{-6}$ m^2/s, $c_p = 960$ J/(kg?K): a) 30 days, b) 60 days, c) 90 days, d) 120 days, e) 180 days, f) 240 days

a)



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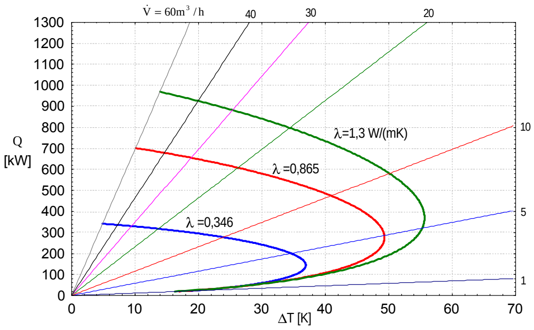

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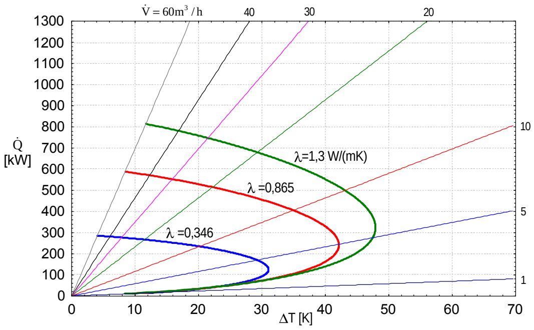

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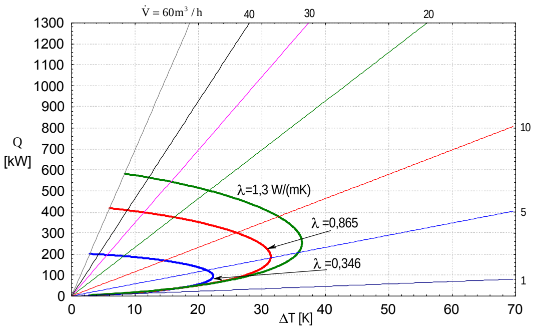

Figure 3: The impact of kinds of rock on flux of gained geothermal heat $\dot{Q}$ for assuming constant work within 30 days (temperature of water injected into the Geothermal Heat Exchanger $T_1 = T_{inj}$ are: a) 10°C, b) 25°C, c) 35°C, d) 50°C

The characteristics show how the change of a volume value of a water flow in a GHE (at the given temperature of injected water) influences the changes of the fluxes of gained geothermal heat with a real change of the temperature of the water on the output of the heat exchanger. The temperature depends on the characteristics of a deposit as well as on the kinds of rock deposit.

4. CONCLUSIONS

The analysis of the graphs (Figs. 2 and 3) shows the following conclusions:

- temperature of injected water, the thickness of a geothermal layer, kinds of rock deposit as well as the thermal capacity of the fluid flow, have a considerable influence on the quantity of input heat flow,
- with the increase in a volume flow of water in a heat distribution network there is the increase in a flux of gained geothermal heat whereas the temperature rise on a geothermal heat exchanger decreases and reciprocally.

REFERENCES

Diad'kin, Ju.D., and Giendler, S.G. Prociessy teplomassopierienosa pri izwlecenii gieotiermalnoj eniergii, Leningrad 1985 (in Russian).

Kujawa, T., and Nowak W.: Modified thermal calculations of a geothermal double-pipe heat exchanger with insulation immersed into geothermal deposit, *Proceedings*, Heat Transfer 2000, Madrid 2000.

Kujawa, T., Nowak, W., and Szaflik, W.: Mathematical model of a geothermal Field exchanger. In: Mathematics of Heat Transfer, ed. G.E. Tupholme and A.S. Wood, Clarendon Press, Oxford, pp. 205-212, 1998.

Kujawa, T., Nowak, W., and Szaflik, W.: Mathematical model of a geothermal Field exchanger, *Archives of Thermodynamics*, **20**, (1999), No. 1–2, pp. 39-50.