

Equation-Free Summary of the 2010 Mathematics in Industry Study Group on Geothermal Data Analysis and Optimization

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Abstract

The Australia and New Zealand Industrial and Applied Mathematics (ANZIAM) division of the Australian Mathematical Society (AustMS) held its annual Mathematics and Statistics-in-Industry Study Group (MISG) both at RMIT in Melbourne, and via telepresence at UWA using the access grid from the 7th through the 12th of February 2010. The Geothermal Working Group attracted a total of 18 academic, professional, post-doctoral, and post-graduate applied mathematicians and others to study problems of potential interest to the geothermal projects in the Perth Basin Hot Sedimentary Aquifer plays. In the interests of attracting the wider AGECC community's attention to the annual MISG efforts, reproduced below is the "Equation Free Summary" of the week-long intensive effort. The detailed summary paper and several flow-on papers are currently being prepared for publication. The author list of this current document are comprised of the so-called "Industry Representative", the "Project Moderators", and "Student Moderator". A full list of the participants in the geothermal study group are found in Appendix 1.

Equation-Free Summary

The aim of this project was to assess the economic feasibility of extracting geothermal energy from the deep sedimentary Perth Basin in Western Australia. The WA Geothermal Centre of Excellence (WAGCoE), based in UWA, has been collecting data from (groundwater and oil) boreholes in the area as well as examining remotely-sensed observational data of ground surface temperatures. The question was how to interpret the available data to gain information on the modes of thermally-driven convective flows, if any, in the groundwater aquifers, thereby determining which parts of the system would be most economical to use for energy extraction.

The 2010 MISG "Geothermal" study group concentrated on problems of advective transport of heat in stratified geological units appropriate for the conditions found in the Perth Basin of Western Australia. The main hypothesis examined – for several different sub-problems – is that advective transport of heat is an important contribution to the overall heat transport. Measured temperatures in the system indicate an approximate temperature increase through the main aquifers from about 40 °C to 80 °C. Clearly, conductive heat transport is present (as elsewhere over the Earth's surface), but we

desired to find out whether the system is susceptible to "unforced" or natural convection.

There is a well-developed theory in the fluid mechanics/porous media literature about the conditions required for convective instability in non-homogeneous permeable systems that are heated from below. The criterion manifests itself in a dimensionless combination of thermodynamic and rock matrix parameters known as the Rayleigh number, denoted Ra . This was seen as relevant for our study, so we concentrated on using the available field data to determine whether or not conditions in the Perth Basin would be right or not.

It is likely there is also an advective transport component due to a hydraulic gradient from a recharge zone assumed to be at an elevated height in the Darling escarpment to the east, westwards to a discharge zone assumed to be offshore. However, for the purposes of most of our sub-problems, we neglected the hydraulic gradient effects.

The geological background

Rocks of the Perth Basin consist of multiple layers of sedimentary strata, where matrix (as opposed to fracture) hydraulic permeability is expected to be dominant. Immediately below the ground surface, there is an unconfined aquifer consisting of rocks geologically known as the "superficial" formations. These rocks are underlain by various shallow confined aquifers consisting of rocks of the Kings Park Formation (locally), the Leederville Formation, and the South Perth Formation. For the study group's convenience, since these rocks are not in hydraulic communication with the deeper and hotter aquifers, we lumped all of these aquifers together into a group we labelled (perhaps confusingly to people familiar with the stratigraphy) as the "superficial" units and basically ignored them (other than the boundary conditions they provide) in our models.

At a depth assumed for our purposes to be about 600 m we find the top of a confined aquifer consisting of rocks from the Gage Formation. Those rocks are assumed to be about 200 m thick, with uniform physical properties. Below the Gage, but in hydraulic communication with it, we find rocks of the Yarragadee Formation. The contact between the two formations is an erosional unconformity, so the thicknesses of the two units vary with map position. The rocks of the Yarragadee are assumed to be about 1000 m thick, also with uniform physical properties.

(These assumptions of uniform rock properties were relaxed in various ways in several of our sub-problems.) Below the Yarragadee lie the rocks of the Cadda Formation, about 600 m thick, also in hydraulic communication with the Yarragadee. Indeed, in the groundwater community, rocks of all three formations (Gage, Yarragadee, and Cadda) are often called the "Yarragadee Aquifer".

Below the Cadda Formation lie rocks of the Cattamarra Coal Measures. These rocks are dominantly shales, with interbedded sandstone layers. Hence, we treat the basal contact of the Cadda with the Cattamarra as being a hydraulic seal.

The result is our basic model of three stratified layers, with physical properties internally uniform, all participating in a single hydraulically connected system.

Well Log constraints

The closest deep borehole to the sites of WAGCoE's proposed exploitation systems is the Cockburn Number 1 oil well. Cockburn 1 was drilled in 1967, to a total depth of 10,020 feet (3054 m), with cores recovered from several discrete sites within the bore. Results of laboratory measurements of porosity, permeability and other physical parameters from these cores are found in the well completion report. Such well logs have been used to try to determine a relation between porosity and permeability from the core measurements; it yielded two different estimates of the vertical distribution of permeability. Statistics from these estimates were used in the sub-problem concerned with estimating Rayleigh numbers and critical values thereof for our Perth Basin problem.

Mathematical modelling

The usual modelling paradigm for such problems is to use a spatially averaged continuum approximation for the rock matrix and fluid components of the system. Standard fluid mass, fluid momentum (Darcy's law in this case) and thermal energy conservation laws enable a set of differential equations to be formulated. With appropriate geometric and thermal boundary conditions, the mathematical problem may be solved; usually this has to be completed using numerical techniques since the equations and the thermodynamic parameters are non-linear. However, with simplified geometry and certain assumptions, the mathematical problem is tractable using analytic methods.

Some numbers

To get an idea of the times and fluid speeds involved, we estimated that a water particle descending into the aquifer in the Darling Range would flow underground to the west at a speed of

about 3.5 m year^{-1} , and take about 8,500 years to complete the 30 km journey.

If "waste" fluid from a geothermal energy installation was pumped into the aquifer at a rate of 100 l s^{-1} , after 1 year the injected fluid would have penetrated to a radius of about 100 m, with the fluid speed at the cylindrical front being about 50 m year^{-1} at that time. This speed is significantly greater than the estimated natural flow above, so the injected fluid would be likely to penetrate the aquifers "upstream" against the natural slow current.

Convective instability

Some effort was spent in attempting to determine whether the system was prone to convective instability; this would, according to theory and experiment, result in large-scale convective "rolls" of fluid motion that sought to augment the conductive heat transport. A series of aligned rolls, each in counter-rotation to its neighbours, would induce hot spots or alignments near the ground surface, as well as cooler areas near the downflows. A consequent question was: Could the associated perturbed isotherm patterns and non-uniform surface heat flux actually be measured? Even if there was convection, how would we know the pattern?

Figure 1 shows a simple idealized model of a porous slab that has a base maintained at a uniform constant temperature that is higher than that at the top. The hotter fluid near the bottom is less dense than the cooler fluid near the top. Provided the net buoyancy force can overcome viscous resistance everywhere, convective motion is induced. The most unstable mode is a set of longitudinal "rolls".

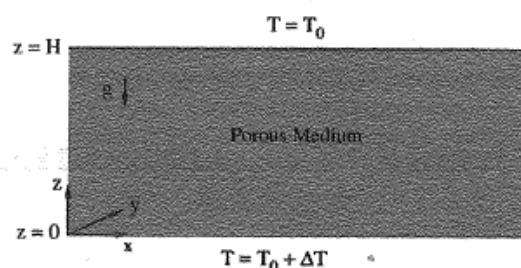


Figure 1: Idealized set-up for a permeable slab subjected to a vertical temperature gradient.

Linear perturbation theory for thermal convection in this idealized porous medium that is laterally unconfined determines that, for a uniform matrix, the critical value of the Rayleigh number is $Ra_{crit} = 4\pi^2$, with the most unstable mode being horizontal rolls with square cross-sectional boundaries. If the temperature gradient in the system is too small, so that Ra has a sub-critical value, then the heat transfer mechanism is conductive only. Above critical, heat transfer is by conduction and convection. The simplified theory assumes a

linearization of thermodynamic parameters around a certain temperature.

When the isothermal bounding surface above is at ground level some distance above the porous region, the above results are modified. The isotherms near the top of the convection region are then not horizontal. This is shown schematically in Figure 2, which also indicates the general form of the convective rolls.

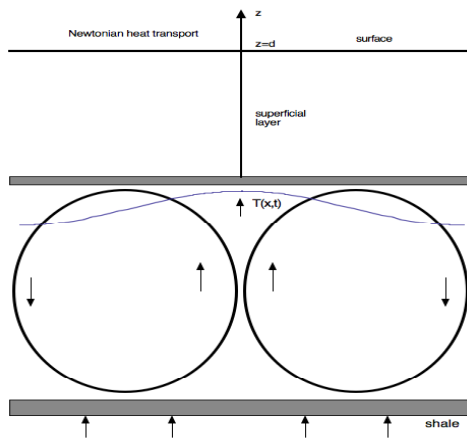


Figure 2: Schematic of longitudinal rolls in the permeable aquifer beneath a confining stratum, with an example perturbed isotherm near the top of the convection region.

Associated with supercritical conditions is an enhancement of the heat transfer. This increases with temperature gradient; the general trend of the so-called Nusselt number Nu , defined as the ratio of heat transferred by conduction plus convection to that transferred by conduction alone, is shown in Figure 3.

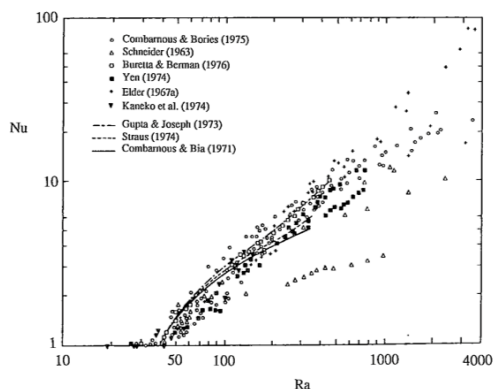


Figure 3: Trend of Nusselt number Nu with Rayleigh number Ra ; compilation of results from various theoretical and experimental studies (from Nield & Bejan, 2006).

First we considered a homogenized matrix system where the rock parameters were assumed uniform with somewhat arbitrarily-averaged values from the borehole core values. The

calculated values of Ra exceeded the critical value only for high enough (and perhaps unrealistic) values of the datum. However, the lack of euphoria was briefly tempered by the thought that the layering of the system may influence the calculation. It was further determined from the theory that anisotropy of thermal conductivity and permeability of the matrix produces an estimate of the critical Rayleigh number that is different from that for a homogeneous system.

Anisotropy in the thermal conductivity theoretically increases the critical Rayleigh number; however, the conductivity of rocks varies very little, so the anisotropy in that parameter is negligibly different from unity (measured data gave a value of around 1.06). Anisotropy in permeability is induced by material layering. Geological strata necessarily induce a permeability that is averagely greater in the bedding plane than perpendicular to it. What is more, such anisotropy generally reduces the (theoretical) critical Rayleigh number!

Amidst some excitement, an assessment of this anisotropy, from a rather complicated set of rock core measurements and well logs, was made as a stratum-thickness weighted average of intrinsic rock permeabilities, with the result that the horizontal value is about 4.6 times that in the vertical. These estimates reduced the critical Rayleigh number to 21.6, about half that computed from the homogenized parameters. Immediately, it became possible that conditions for convection may be met, even using the linearised theory with the lowest datum temperature (40 °C) for the linearised calculations.

Convective rolls and heat transfer

In optimistic anticipation of such a result, a small group had been working on how enhanced heat transfer in the Yaragadee Aquifer would manifest itself in the temperature distribution in the near-surface superficial formations. This is a fairly standard problem in heat transfer, and quick progress was made. However, it was shown that there would be a very small signature at ground surface level, and instrumentation would have to be sensitive and widespread to obtain significant measurements. So, this encapsulated the problem overall: Even if there was a convective process at work, how would measurements be able to be made to detect the upflow and downflow regions? So, how could advantage be taken of the convection even if the Ra estimates were correct?

The UWA Groupies

Over in WA, the small "at home" group was working; with the assumption that convection was occurring, they trawled the literature for information about theoretical work on layering effects on cell-size, and the stability of such convection cells. The idea was that significant

sub-layering caused by thick differently-permeable strata might produce vertically-aligned but largely separated co-rotating cells that, in tandem, would advect heat from the hot base to the surface. At the end of the week, no firm conclusions had been able to be made.

Simulation

Driven by concern that the thermodynamic properties of water were being approximated too severely in the linear stability theory, a numerical simulation of 2-D rolls was made using a commercial computer package. This allowed more accurate values for the water's temperature-dependent density, viscosity, etc. to be used. While the simulations were completed post-MISG, it turned out that there were only small qualitative differences in the flow patterns.

Slope-induced thermal convection

A late idea resulted from current work by one of the group members; it considers the possibility that longitudinal convection may be induced in a sloping aquifer by a near-vertical temperature gradient. The warmer fluid near the bottom would move up-slope (in this case, towards the east) while the returning cooler fluid near the top moves down-slope. It was not clear how this would affect the possible exploitation, because the temperature profile would remain conductive, and only be altered by convective rolls as were already considered above.

A small group of postgraduate students from the group seized upon some aspects of this, and their efforts will be rewarded by co-authorship of a paper to be submitted later in the year. A more general case will be considered, where induced convection in a layered sloping aquifer is investigated for optimal net convective heat transport by such a mechanism, as well as the stability of such motion to longitudinal rolls. The enthusiasm to participate in this way reinforces the benefits of MISG to engagement of postgrads in mathematical enquiry into useful applications.

Summary

The problem was tractable, but depended on suitable estimates of rock matrix properties from drill cores and well logs which were rather complex in structure. It also depended on good estimates of aquifer temperatures; the deeper values were elusive, and had to be deduced from extrapolations of shallow well measurements. However, by the end of the study week, there was a good increase of understanding of the problem, the issues to be resolved, and possible mechanisms at work in the Perth Basin.

Geothermal systems are complicated geological-geophysical-thermodynamical entities. A multi-disciplinary approach to understanding them is necessary. However, the quantification of their

attributes is well-handled by mathematically-able scientists and the MISG proved a suitable venue to tackle the WAGCoE problem.

Keywords: Convection, Hot Sedimentary Aquifers, Anisotropy

Reference

Nield, D. A., & Bejan, A. (2006). *Convection in Porous Media*. Springer, 3rd ed.

Appendix

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