EGS & "The Law of Averages"

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Enhanced Geothermal Systems (EGS) seek to produce high permeability flow systems where Nature has provided only low permeability flow systems. We take a science-based modelling look at the flow heterogeneity character of high permeability geothermal systems to better understand the *in situ* systems EGS is dealing with. Our flow modelling is based on percolation through grain-scale fractures, with grain-scale fracture density η(*x,y,z*) allowed to fluctuate in space on all scale-lengths in line with well-log power-law scaling phenomenology recorded in both sedimentary and crystalline basement rock. Massive hydrofracturing in EGS drill holes can in principle introduce high permeability quasi-planar mega-fractures into our natural poroperm medium but almost certainly EGS induced fractures cannot be expected to completely define the hydraulic connectivity between an induced fracture and the remaining drill holes in the EGS plumbing system. At some point in an EGS volume, the natural fracture hydraulic connectivity is likely to control the fluid system throughput We therefore suggest that it is highly relevant for EGS developments to target crustal volumes in which the natural fracture permeability is higher rather than lower.

For spatially variable porosity φ(*x,y,z*) proportional to fracture density η and spatially variable permeability κ(*x,y,z*) proportional to fracture connectivity factor η!, then the identity δη ~ δ(log(η!)) reproduces the poroperm spatial fluctuation relation δφ ~ δlog(κ) documented in oil and gas field well-core data. We model 3D fluid flow in numerical media constructed for 3 degrees of spatial correlation of grain-scale fracture density: (1) no spatial correlation -- grain-scale density does not cluster and "the law of averages" holds; (2) intermediate spatial correlation -- grain-scale density clusters are diffuse and unpredictable whence the law of averages fails; (3) strong spatial correlation -- grain-scale clustering compartmentalizes rock volumes and the law of averages has rough validity over limited spatial domains. Our flow simulations indicate that geothermal reservoir pressure and flow data are far more likely to resemble case (2) than cases (1) or (3). We infer that, at least for scale-lengths characteristic of geothermal reservoirs, the *in situ* low permeability flow systems that EGS seeks to enhance into high permeability flow systems are likely to be spatially erratic and unpredictable at all relevant scale-lengths, and hence that the resulting EGS flow systems are also likely to be more spatially erratic and unpredictable than expected from "the law of averages". Supplementary fracture-sensitive data, e.g. microseismic and/or MT surveys, can thus be useful in completing EGS projects.

Keywords: fractures, heterogeneity, percolation, permeability, flow modelling

Introduction

To date flow models for both geothermal and hydrocarbon reservoirs have had little predictive value. The failure of reservoir modelling can in good part be attributed to ignoring the pervasive fracture-heterogeneity of crustal rock attested by well-log and well-core fluctuation systematics. *In situ* fluctuation systematics indicate that geofluids flow via spatially erratic fracture-percolation networks that cannot be predicted from traditional small-scale reservoir sampling (Leary 2002).

In geothermal reservoirs, spatially erratic *in situ* percolation flow networks can potentially be mapped using flow data observed at suitably large scale lengths. A possible means of flow-structure mapping is afforded by systematically recording and interpreting inter-well flow connectivity data. Such reservoir model-building tactics supersede the standard approach based on "the law of averages".

The law of averages supposes that for every earth property, variations in one direction are sooner or later balanced out by variations in the opposite direction. The law of averages thus says that reservoir geological formations can be described in terms of "effective medium properties" about which reservoir physical properties fluctuate benignly. Moreover, effective property values can be determined by a few small scale samples. Typically the small-scale samples of formation porosity, say, are taken from well logs and formation permeability samples are taken from well core.

Formation property averaging, however, leads to accurate reservoir flow models only if *in situ* fluctuations of relevant properties are spatially uncorrelated. The necessary and sufficient condition for spatially uncorrelated *in situ* geophysical properties is that spatial fluctuations in these properties have a "flat" or "white" Fourier power spectrum in spatial frequency k, $S(k) \sim 1/k^0$.

Well-logs show, however, that the characteristic spectrum of *in situ* geophysical property fluctuations is $S(k) \sim 1/k^1$ rather than $S(k) \sim 1/k^0$. The necessary condition for "the law of averages" to apply *in situ* is universally and systematically violated at all relevant wavelengths for virtually all physical properties of crustal reservoirs (Leary 2002).

Percolation phenomenology for spatially correlated grain-scale fracture-density fluctuations

We compute percolation flow of geofluids in a numerical framework capable of embodying well-log fluctuation systematics. In this heterogeneity framework, the spatially fluctuating numerical density $\eta(x,y,z)$ of grain-scale fractures attested by well-log data is modelled by power-law-scaling numerical fluctuations within the model volume.

Numerical fluctuations $\eta(x,y,z)$ are connected to geofluid flow by the empirical relation $\delta\varphi \approx \delta\log(\kappa)$ observed to hold for well-core porosity φ and well-core permeability κ. In this expression, $\delta\varphi$ and $\delta\log(\kappa)$ are respectively the zero-mean unit-variance reductions of the porosity and log(permeability) fluctuation sequences for cored reservoir intervals (Leary & Walter 2008).

The empirical well-core poroperm fluctuation relation $\delta\varphi \approx \delta\log(\kappa)$ is conceptually equivalent to a mathematical identity $\delta\eta \sim \delta(\log(\eta!))$ if fluctuations in porosity are proportional to fluctuations in grain-scale density $\eta(x,y,z)$ and fluctuations in permeability are proportional to the fluctuations in the combinatorial factorial term $\eta(x,y,z)!$ representing the number of ways $\eta(x,y,z)$ grain-scale fractures can combine to produce a percolation pathway. If a rock volume has η grain-scale fractures per unit volume at location (x,y,z) and $\eta + \delta\eta$ grain-scale fractures at location $(x,y,z)+(\delta x,\delta y,\delta z)$, the percolation-related permeability in the two volumes can be expected to vary as the combinatorial terms $\eta!$ and $(\eta+\delta\eta)!$. Stirling's formula, $\eta! \approx (\eta + ½)\log(\eta) - \eta$, applied to the two fracture connectivity expressions reduces to the empirical well-core fluctuation expression $\delta\eta \approx \delta\log(\eta!)$.

Percolation flow in spatially correlated fracture networks

Single phase flow in heterogeneous media is simulated using SUTRA (Voss and Provost 2008), a finite-element solver for Darcy's law in permeability-heterogeneous media, $\partial_t P = \underline{\nabla}\bullet(\kappa(x,y,z)\underline{\nabla}P)$, P = geofluid pressure. Model porosity $\varphi(x,y,z)$ is proportional to numerical fracture density $\eta(x,y,z)$, and model permeability $\kappa(x,y,z)$ is given by the well-core relation $\delta\varphi \approx \delta\log(\kappa)$.

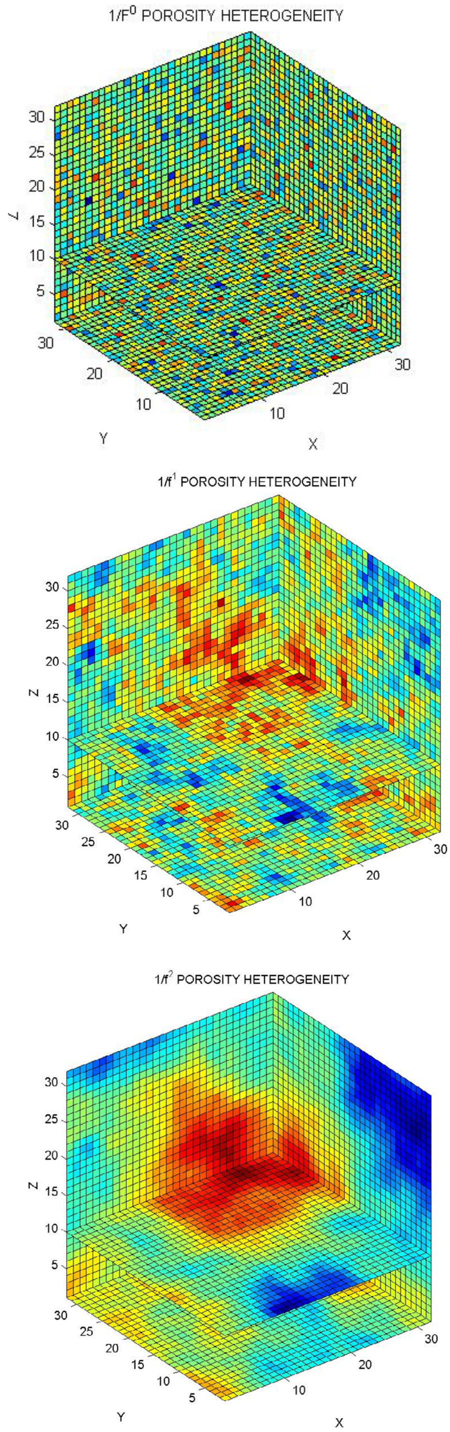


Figure 1a-c: 3D numerical spatial noise distributions with increasing degrees of correlation: (top) uncorrelated white noise, fluctuation power spectrum $S(k) \sim 1/k^0$; (mid) intermediate correlation $1/f$-noise, power spectrum $S(k) \sim 1/k^1$; (bottom) strong correlation Brownian noise, power spectrum $S(k) \sim 1/k^2$.

To see the effect of spatial correlation on reservoir flow, we compute flow for three degrees of spatial fracture correlation parameterized by power-spectral exponent p, $S(k) \sim 1/k$p, k = spatial frequency. The degrees of spatial connectivity are: (*i*) $p = 0$, spatially uncorrelated or white noise fluctuations (Figure 1a); (*ii*) $p = 1$, moderately spatially correlated "1/*f*" noise fluctuations (Figure

1b); (*iii*) p = 2, strongly spatially correlated Brownian noise fluctuations (Figure 1c). Each of these 3D numerical heterogeneity constructs has the same mean and standard deviation. The only distinguishing feature is the degree of spatial correlation/clustering of the numbers representing grain-scale fracture density.

Finite-element solutions to fluid flow in and out of elementary digital volumes or cells are robust against spatial variations in the porosity and permeability of cells. The fact that these property spatial variations occur at all scale lengths throughout numerical medium has no effect on the speed or stability of the flow computations. Of course, the greater the number of cells, the more realistic will be a specific flow simulation, but it is currently feasible to run simulations with 64x128x128 = 2^6x2^7x2^7 cells, giving six octaves of vertical fluctuation power and seven octaves of lateral fluctuation power, equivalent to factors 8 and 11, respectively, in amplitude fluctuations.

Flow systematics for $1/k^0$, $1/k^1$ & $1/k^2$ grain-scale fracture density spatial correlation

Simulation pressure histories plotted in Figures 2a-d reflect the distinct character of 1/*f*-noise heterogeneity flow evolution (red traces) compared with those of white noise (black traces) and Brownian noise (blue traces). Each figure represents a different source well flow simulation for the three heterogeneity types. For each source well, a colour-coded quartet of flow history traces shows time-evolving pressure signals recorded at observation wells at four offsets from the source well.

The Figure 2 colour-coded pressure curves show that the different heterogeneity types result in substantially different pressure histories. All pressure histories evolve more or less consistently within a heterogeneity class. The uncorrelated heterogeneity (white noise = black traces = Figure 1a) and strongly correlated heterogeneity (Brownian noise = blue traces = Figure 1c) both generate essentially monotonic pressure evolution. The black curves of uncorrelated heterogeneity are relatively subdued in pressure history difference as expected from a medium that is essentially uniform within a standard deviation about a mean permeability; there is no evidence of flow complexity that might be associated with significant spatial trends in poroperm distribution. The blue curves of Brownian-noise heterogeneity have a stronger degree of temporal evolution that is the same for three of the source-sensor well pairs, but substantially different for the fourth source-sensor well-pair. The amplitude discrepancy is due to three of the sensor wells being inside the Figure 1c high-porosity volume while the fourth sensor well is on the edge of the high porosity volume. In both amplitude cases, however, pressure evolution at the sensor well is monotonic.

These simulation flow data are understandable in terms of simple considerations. Flow in spatially uncorrelated media conforms to the standard statistical expectation that randomness tends to average out. Flow in strongly spatially correlated media tends to be simply diffusive over each range of coherent spatial properties.

The red curves of 1/*f*-noise (Figure 1b) heterogeneity present an unfamiliar degree of flow complexity. The Figure 2a-d observation well pressure histories in red are responding to a consistent but complex spatially-correlated heterogeneity structure, and the spatial complexity generates temporally complex evolution signals. In conspicuous contrast with the monotonic well-pair histories for the uncorrelated and strongly correlated poroperm heterogeneity, 1/*f*-noise poroperm heterogeneity consistently exhibits large-amplitude temporal fluctuations about a diffusion trend history.

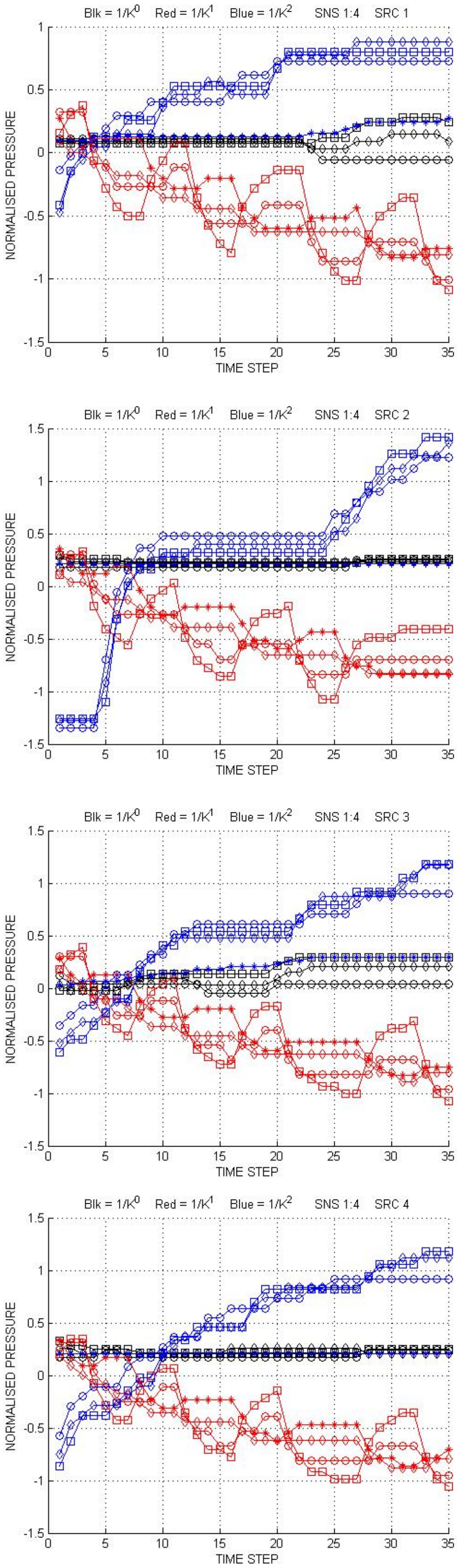
If we now compare the three Figure 2a-d pressure history types to a sample of *in situ* pressure histories in Figures 3a-d, we see little evidence in field pressure data for essentially monotonic diffusion-flow characteristic of uncorrelated $1/k^0$ and $1/k^2$ spatial correlation, but see ample evidence for pressure histories that systematically fluctuate about long-term diffusion trends that are modelled by flow simulations in $1/k^1$ poroperm heterogeneous media. The correspondence between *in situ* flow data fluctuations (Figures 3a-d) and model flow data fluctuations (Figures 2a-d) indicates that spatial poroperm heterogeneity based on well-core and well-core empirics has a general validity for *in situ* fractures and fracture-borne percolation flow.

Discussion

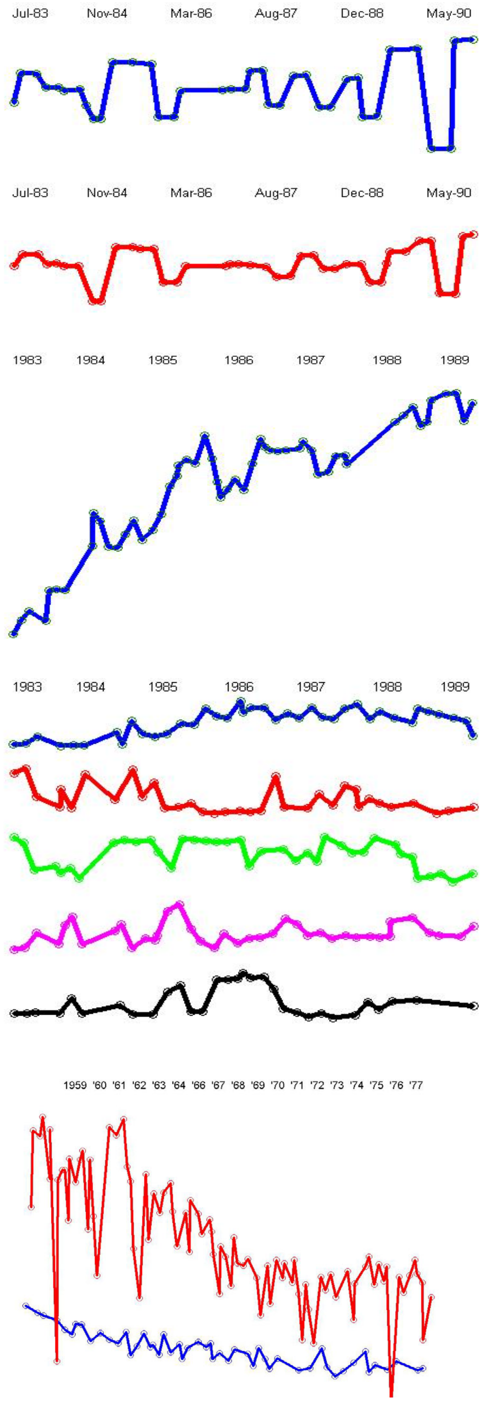
The “law of averages” incorporates three working assumptions:

- Fluctuations balance out on all scale lengths;
- No significant trends develop at any scale length;
- Small scale samples yield good indicators of large scale properties.

These working assumptions are highly convenient but are unfortunately formally invalidate for *in situ* spatial fluctuations in reservoir properties recorded by well logs; well logs have power-spectra that scale inversely with spatial frequency, $S(k) \sim 1/k^1$, rather than as the form $S(k) \sim 1/k^0$ required for the law of averages.



Figures 2a-d: Quartets of simulation flow pressure histories for (top to bottom) 4 different source well locations in Figure 1 numerical reservoir volumes: black traces = Figure 1a; blue traces = Figure 1b; red traces = Figure 1c.



Figures 3a-d: Geothermal field flow-related well data: (a-c) Chloride abundances, Palinpinon field, Philippines (Horne 2008); (d) Pressure and mass discharge (Sorey and Fradkin 1979).

The *in situ* geothermal reservoir flow data of Figures 3a-d go beyond mathematical formalism to give additional and perhaps more practical rebuttal to the law of averages applied to reservoir processes. The different types of *in situ* fracture distributions in Figures 1a-c are seen to have real meaning in terms of reservoir flow. Figure 3a-d *in situ* flow data imply that we can be more confident in understanding that long-range trends in fracture density and fracture connectivity are characteristic of *in situ* rock permeability over a range of scale lengths characteristic of reservoir flow. It is visually clear from Figures 1a-c that such *in situ* trends cannot be adequately modelled by data averaging (Figure 1a is not a valid smoothed version of Figure 1b), nor can valid statistical inferences about large-scale flow structures be made from acquiring small-scale samples (a few samples from Figure 1a fix the likely values of other samples; this is clearly not true of sample from Figure 1b).

As a general statement, fracture heterogeneity in the form of Figure 1b must be addressed by making suitable *in situ* measurements at the appropriate scale lengths.

Applying this statement to EGS projects seeking to enhance *in situ* fracture permeability, it seems clear that EGS fracture enhancement is best conducted in rock volumes that host large-scale fracture clustering. Even if EGS induced fractures are thought of only in terms of massive planar flow structures, such structures must naturally terminate in country rock. In terms of Figure 1b, country rock volumes with high fracture densities denoted by warm colours are far more promising EGS targets than are rock volumes with low fracture densities denoted by cool colours.

A practical implementation of this statement is to associate *in situ* fractures with earthquake failure and/or fracture-borne fluid electrical conductivity. While it is common to regard with alarm earthquake failure in a reservoir environment, this attitude is likely to be naïve and counter-productive. Enhanced reservoir flow is likely to be essentially concomitant with earthquake failure. Given the comprehensive failure of the law of averages applied *in situ* and the use of small-scale sample to infer large-scale *in situ* flow properties, a more informed point of view would be systematically use micro-earthquakes as an exploration tool to locate *in situ* fracture clusters that are likely to be large-scale permeability systems for which input and output flow can be engineered/enhanced. Microseismicity data can be supplemented by MT detection of electrically conducting volumes at depth.

A compelling logic associates earthquakes with fluid-rich fracture clusters. Such clusters are the weakest and more compliant part of the rock mass, and fluid pressures tend to lower normal frictional stresses that work against slip failure. Seismicity is almost universally induced by the modest crustal loads of water ponded behind dams, indicating that the crust is everywhere near fracture failure. The most fractured crustal volumes being the most likely to fail, microseismicity, either natural or induced, is a likely indicator of fluid-rich fracture systems. Conversely, absence of microseismicity implies a lower potential for *in situ* fracture systems. Exploration wells instrumented with seismic sensor arrays to detect the range and azimuth of background seismicity could be the most direct way to assess EGS prospectivity. Fibre optical seismic sensor technology is on the verge of making seismic assessment of elevated temperature EGS prospects a viable exploration option.

Summary

The law of averages does not apply to the spatial or temporal properties of *in situ* reservoirs. *In situ* reservoir fracture systems have a tendency to develop long-range spatial trends in fracture density that generate localised fracture/flow clusters that could be plausible EGS targets, but such trends can equally be towards low fracture content and therefore unpromising EGS prospects. Because the law of averages fails for reservoirs, small-scale sampling of *in situ* crust has little bearing on the location and/or properties of EGS prospects. EGS prospect assessment can logically be conducted through systematic large-scale microseismic and or MT data acquisition.

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